5 Conclusions

First principle investigations have shown different failure modes for liquid filled containers. These failure modes are depending on mass and velocity of the projectile, and geometry and material of the container. This is a basis for ongoing work to explain also the failure behaviour of concrete containers due to its specific material features.

References


Nonlinear vibrations of vertical asymmetrically-supported rotors under fluid confinement: theoretical results

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Abstract

This paper is concerned about vertical rotors immersed in fluid annulus of moderate confinement. Such rotors are subjected to the dynamical effects of the fluctuating co-rotating flows.

For high enough spinning velocities, the fluid-elastic forces become significant, and often lead to unstable dynamical regimes. These depend on the fluid gap and density, on the rotor eccentricity and spinning velocity, as well as structural properties.

We developed an improved linear model for rotors under moderate fluid confinement as well as an exact model for the corresponding nonlinear rotordynamics. Recently a symbolic-numerical formulation based on a spectral/Galerkin approach was also developed by the authors. Numerical results showed a quite good agreement between exact solutions and these formulations and experimental validation of the theoretical model has been provided for symmetrically-supported rotors.

Numerical simulations carried over immersed rotor configurations maintained by non-isotropic supports show that the rotor stability is affected by support stiffness-asymmetry.

In this paper, we briefly summarize the theoretical approaches used in the numerical simulations and present an analysis of the linear rotor-dynamics, as a function of the support stiffness-asymmetry and of the rotor eccentricity. Theoretical stability domains are computed from the eigenvalues of the linearized model.

Finally, we present time-domain numerical simulations of some stable solutions and nonlinear limit-cycles which stem from linearly-unstable solutions.

1 Introduction

Rotor dynamics under moderate fluid confinement — that is, in equipments with clearance ratios of about \( \delta = \frac{H}{R} \approx 0.1 \) (where \( H \) is the average gap and \( R \) the rotor radius) — have been studied since Black [6], Fritz [8] and Hirs [11]. Further relevant work was presented by Ramsden et al. [19] and [20], Childs [7], Nelson
The shear stresses at the rotor and stator walls, in equation (2), are given by
\[
\tau_x(\theta, t) = \frac{1}{2} \rho (\Omega \Delta u + \Omega R) \mu \frac{\partial}{\partial \theta} (\frac{\partial (\Delta u)}{\partial \theta} + \frac{1 \partial (\Delta u^2)}{\partial \theta}) + \tau_\theta + \tau_r = 0,
\]
where \( \tau_x \) and \( \tau_\theta \) are empirical friction coefficients, which depend on the flow Reynolds number and on wall roughness. Assuming \( f_r = f_s = f \) and adopting the simplifications discussed in [3], we can deduce \( \tau_x + \tau_\theta \approx \rho f R^2 \Omega / \mu \).

Two main directions can be followed to obtain rotordynamic models from this approach.

**Linearized flow equations/coupled system** The first one, completely described in Moreira et al. [14], uses classical perturbation analysis and leads to linearized model for the flow forces. Introducing the structural dynamic forces one can study the modal behavior of the coupled system as a function of \( \Omega \) and \( \epsilon \), by solving the complex eigenvalue \( \lambda = \sigma + i \omega \), and complex eigenvector \( \Phi_n \) problem of the set of five first order differential equations:
\[
N_2^* \dot{V} + N_2 V = 0
\]

where \( V = (X, Y, Z, W, C)^T \), \( Z = X, W = Y \) and \( C \) represents the first order fluctuating term of the average tangential flow velocity. Each eigenvalue \( \lambda_n = \sigma_n + i \omega_n \), the corresponding modal frequency \( \omega_n \) (Hz) and modal damping can be defined as \( \zeta_n \) and \( \alpha_n \). Note that 0, 1 or 2 complex conjugate pairs of eigenvalues (and eigenvectors) would be expected in the complete set of five eigenvalues (and eigenvectors) of the problem. One of these eigenvalues must be always real. Note that the coupling matrices depend on \( \Omega \) and \( \epsilon \) but are otherwise constant. One can find a detailed description of this approach in Moreira et al. [14].

**Nonlinear flow equations/coupled system** Using complex analysis techniques equations (1) and (2) can be exactly solved in order to deduce nonlinear variation of the flow forces as functions of the parameters \( \Omega \), \( X \) and \( Y \). Introduction the structural dynamic forces one obtain the nonlinear motion equations of the rotor-flow coupled system. Observe that, as in the linear case, the dynamics of the system depends not only on \( X(t) \) and \( Y(t) \) but also on a new variable called \( C(t) \) which represent here a quantity proportional to the average tangential flow velocity. This approach is completely described in Moreira et al. [14].

Alternatively, a symbolic-numeric formulation, using a spectral/ Galerkin approximation of the flow velocity and pressure fields can be adopted to obtain a satisfactory rotordynamic model. Following this approach, the gap-averaged tangential flow velocity \( u(t, \theta) \) and the gap-averaged pressure \( p(t, \theta) \) are approximated, in the equations (1) and (2), by truncated Fourier series with time-dependent coefficients. The Galerkin method (see, for instance, Zwillinger [22] and Reddy [21]) applied over the residuals so obtained, generate a set of simultaneous ordinary differential-algebraic equations for the time-dependent coefficients. Finally, the set of differential-algebraic equations deduced is solved numerically in parallel with the dynamics equations for the structure yielding an approximate solution for the motions \( X(t) \) and \( Y(t) \). This set constitute, with the equations for the rotor-flow coupled system, the approximate rotor dynamics model, called \( N \times M \) model. Note that the rotordynamics \( N \times M \) model developed using this methodology constitute a set of differential-algebraic equations (DAE's).

This class of problems arise naturally in many applications but present numerical and analytical difficulties which do not occur with systems of ordinary differential equations (see for instance Bremen et al. [4]).
Table 1: Main geometric, physical and modal parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (Rotor length [m])</td>
<td>0.250</td>
<td>Mass ratio, $M_r/M_n$</td>
</tr>
<tr>
<td>R (Rotor radius [m])</td>
<td>0.044</td>
<td>Reference struct. stiff., $K_{ref}^{stiff} (N/m)$</td>
</tr>
<tr>
<td>H (Annular gap [m])</td>
<td>0.0062</td>
<td>Damping in water (%)</td>
</tr>
<tr>
<td>$\delta = \frac{H}{R}$ (Reduced gap)</td>
<td>0.14</td>
<td>Friction coeff., $f = f_s = \delta$</td>
</tr>
</tbody>
</table>

The accuracy of such approximate model depends, respectively, on the number $N$ and $M$ of terms used on the truncated Fourier series of $u(t, \theta)$ and $p(t, \theta)$. However, numerical results showed a quite good agreement between exact solutions and their low-order Galerkin approximations, see Moreira et al. [15] and [16]. This approach is exposed in these references and is the one we choose to use here in all the performed nonlinear simulations.

3 Numerical simulations and discussion of results

In the following, and using the tools described in section 2, namely the linear model and the symbolic-numerical formulation, using a spectral/Galerkin approach, we present an analysis of the linear rotordynamics and some nonlinear simulations.

To do so, theoretical stability domains were computed from the eigenvalues of the linearized model and stability charts were built. Such stability charts were defined as a function of the spinning velocity and the asymmetry-ratio $\beta = \frac{X_0}{X}$ (where $X_0$ and $X$ stand for the structural stiffnesses) and of the rotor reduced eccentricity $\epsilon = \frac{X}{X_0}$ (were $X_0$ is the structural static eccentricity and $X$ is the average gap). That is, we computed stability charts $rpm \times \beta$ (or $rpm \times 1/\beta$) and $rpm \times \epsilon$. In these charts, the white color represent the region of predicted linear stability. A grey scale translate the instability domain which is qualitatively characterized by the different types of instabilities predicted by the linear model. The qualitatively different types of instabilities are described by vertical grey bars near each figure: one can find the occurrence of divergence or flutter associated with the different rotodynamic modes (zero frequency, forward, backward whirl). Note that the stiffness-asymmetry $\beta$ is defined here as $\beta = \frac{K_x}{K_y}$. This mean that $0 < \beta < \infty$ theoretically. However, in presenting our results, the stability charts were computed for $0 < \beta \leq 1$ whenever $K_x < K_y$ using $K_x = K_{ref}^{stiff}$ and $0 < 1/\beta \leq 1$ if $K_y < K_x$ using $K_y = K_{ref}^{stiff}$. Note that whenever $\epsilon \neq 0$, because the reduced eccentricity is defined and implemented (along the direction $X$) as $\epsilon = \frac{X}{X_0}$, the stability charts for $0 < \beta \leq 1$ and $0 < 1/\beta \leq 1$ are not similar.

Campbell diagrams to illustrate the modal behavior of some chosen configurations are also presented. Note that the modal curves show and additional zero frequency mode, discussed in Moreira et al. [14]. In those diagrams, zero frequency modes are represented by a thin line, the backward whirl mode by a thick size line and the forward whirl mode by a medium size line.

Finally, we present time-domain numerical simulations of some stable solutions and nonlinear limit-cycles which stem from linearly-unstable solutions.

The main geometric, physical and modal parameters used on numerical simulations can be consulted in Table 1.

In Figure 2 the stability charts $rpm \times \beta$ and $rpm \times 1/\beta$ for an eccentricity $\epsilon = 0$ are presented. Note that in this case ($\epsilon = 0$) these charts must be similar. Observe that, interestingly, the stability range increases as $\beta$ decrease for $0.75 \leq \beta \leq 1$. For this range one can find the occurrence of flutter type instability at each corresponding regime. When $\beta = 0.75$ one verifies that the range of stability abruptly decreases and for $\beta \leq 0.75$ one can now observe the occurrence of a divergence type instability at about 750 rpm and later the occurrence of a flutter type instability after a restabilization range. Note that for $\beta \leq 0.75$ (and for centered rotors) this parameter (the stiffness-asymmetry) seems not to have influence on the rotodynamic stability range.

In Figures 3–5, for reduced eccentricities $\epsilon = 0$, $\epsilon = 0.25$ and $\epsilon = 0.75$, we display the corresponding $rpm \times \beta$ and $rpm \times 1/\beta$ stability charts. Observe that as the reduced eccentricity increases, the differences between $rpm \times \beta$ and $rpm \times 1/\beta$ stability charts get more and more significant showing that the rotodynamic stability depend on the directions along which the eccentricity and/or the stiffness asymmetry are applied. Clearly, for larger reduced eccentricities ($\epsilon = 0.75$) one can find larger stability ranges for $0 < 1/\beta < 1$ (that is, assuming $K_x > K_y$) than for $0 < \beta \leq 1$ (that is, assuming $K_y > K_x$). This fact suggest that for a given reduced eccentricity it is possible to improve the stability range by implementing an appropriate asymmetry-ratio. One can conclude that for asymmetrically supported rotors the stability range decreases if eccentricity is applied along the axis of the lower support stiffness value.
Figure 4: Stability charts rpm×β and rpm×1/β for ε = 0.50.

Figure 5: Stability charts rpm×β and rpm×1/β for ε = 0.75.

From Figures 2–6 one can also confirm (observing the line β = 1/β = 1) that for symmetrically supported rotors the stability range always decreases when the eccentricity increases.

Figure 6: Stability charts rpm×ε for β = 0.4 and 1/β = 0.4.

Finally, in Figure 6, stability charts rpm×ε for β = 0.4 and 1/β = 0.4 are displayed. One observe that typically in both charts a maximum stability range is verified for about ε = 0.2 (chart for β = 0.4) and ε = 0.7 (chart for 1/β = 0.4), respectively. This fact tends to confirm that it is possible to improve the stability range choosing appropriate eccentricities/stiffness asymmetry-ratios. Note that for large eccentricities Figure 6 must be interpreted with some reserve. Indeed for these range of eccentricities the linearizing assumptions are not verified.

Figure 7: Campbell diagrams for β = 1.0 and β = 0.4 (ε = 0).

In Figure 7 we present Campbell diagrams for chosen configurations. In these diagrams one can observe the predicted modal frequencies and damping as a function of the spinning velocity.

In Figure 7, and for a reduced eccentricity of ε = 0, we display the Campbell diagrams for β = 1 and β = 0.4. One observe that for β = 1 our system is unstable (by futter in the forward whirl mode) above the spinning velocity at about 1100 rpm. For β = 0.4 our system become unstable by divergence between 750 and 1200 rpm. The predicted restabilization regimes between 1200 and 1500 rpm can be observed in this system. The described dynamics can be also deduced by observing Figure 2. Note the agreement between the information reported by the Campbell diagrams and the stability charts.

Finally, in Figures 8–10, time-domain numerical simulations (representing 10 seconds of free response) of stable solutions and nonlinear limit-cycles which stem from linearly-unstable solutions are displayed.

Figure 8: Nonlinear simulations: β = 0.4 and ε = 0.

In Figure 8 we observe a backward whirling high amplitude limit-cycle at 800 rpm. Note that linear theory predicts instability by divergence between 700 and 1150 rpm (see Figure 7, right). Interestingly, one verifies the occurrence of a restabilization, between 1150 and 1500 rpm, observing the stable behavior at the spinning velocity of 1300 rpm. This phenomenon is also predicted by linear theory.
on the rotor configuration. For $0 < \beta < \beta_{\text{im}}$ this parameter does not have significant influence on the stability range. (ii) For symmetrically supported rotors configuration the stability range always decreases when the eccentricity increases. (iii) For asymmetrically supported rotors the stability range decreases if eccentricity is applied along the axis of the lower support stiffness value. Conversely the stability range increases when eccentricity is applied along the axis of the upper stiffness value.

These results may be of industrial relevance and confirm that it is possible to improve the stability range choosing appropriate eccentricities/stiffness asymmetry-ratios. We note that this fact was stressed before by Antunes et al. [1].

Despite using recently developed theoretical tools, the reported linearized predictions do not account for the rotor drift (and so the dynamic eccentricity) which strongly govern the system dynamics.

Experimental work is currently being done to further assess the validity of the reported theoretical results.

5 Acknowledgments

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References


Section 3:

Vibration analysis and control