New Technologies to the Study the Golf Putting

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Keywords: Golf Putting Analysis, Performance, Sports Technologies.

Abstract: The technology evolution, especially in the last decade, allowed major advances in golf putting analysis. This scientific advance benefited from multidisciplinary interaction, such as: biomechanics, engineering, mathematics and motor control. Utilizing digital cameras along automatic tracking techniques, it is possible to obtain valuable and accurate information about the putting gesture and/or ball trajectory. This can provide relevant feedback to athletes and coaches. In this work, we present several golf putting studies conducted on laboratorial environment. We believe that this work may contribute to a deeper analysis of human motor behavior and performance, not limited to this motor skill, but that can be applied to other motor patterns.

1 INTRODUCTION

The last decades have seen technological advances and new analyses tools that allowed for comprehensive interpretation of human movement patterns. As movement patterns are being analysed with improved technology, subtle individualities have been identified or signature patterns of movement. Moreover, it is suggested that technological advances and new tools that recently emerged to human movement pattern analysis can have a strong contribution to the study of golf putting (Couceiro et al., 2013).

These technological advances open a window of opportunity to further study and augment the understanding in several fields, such as sports sciences that benefit from a multidisciplinary approach (e.g., biomechanics, engineering, mathematics, motor control) (Dias et al.,2013).

By taking advantage of the available technology, researchers have focused their attention on the development of new methodologies for the study of human movement (Dias et al., *in press*). In this particular case, golf putting analysis can be improved with the use of automatic tracking and 2D analysis techniques, allowing to assess valuable and more accurate information about the putting gesture or ball trajectory; this can provide important feedback to athletes and coaches (Neal & Wilson, 1985; Couceiro, Dias, Martins, & Luz, 2012). These innovations also allowed an important breakthrough in golf putting study, by providing new information about gesture patterns and how these evolved over time (Neal et al., 2007; Dias et al., 2013).

Based on the coordination dynamics of each individual (cf. Kelso 1995), a deeper analysis of new coordinative patterns should be conducted. The goal behind it is to study human performance as an individual, unique, creative process wherein each athlete discovers new and different solutions to problems in motor movement.

In spite of this, new methods for the performance analysis of the golf putting have been suggested to focus on individual kinematic analysis, rather than the traditional pooling of group data in Couceiro et al. (2013); Dias et al., (2013b). The purpose was to understand the relevant changes resulting from the interaction between the athlete's characteristics and the surrounding context by analysing the motor behaviour profiles as measured by the individual kinematic strategies.

It is also important to mention that the use of new technologies involves a profounder approach than the traditional linear techniques, which mainly considers product measurements/variables that are a result of the movement (through average, standard deviation or variation coefficient), something that does not allow an analysis of the movement itself (Harbourne and Stergiou, 2009). In that sense, this work presents several studies from RoboCorp research group around the golf putting in laboratorial context.

Detection Methods in the Performance of Golf Putting

In a recent study of Couceiro and colleagues (2013) the performance of golf players was analysed. The sample comprised of expert players with a handicap between 5 and 15, with more than 10 years of golf practice, and inclusion in the golf national tournament. The players were 32 ± 10 years old, volunteers, male, and right-handed.

The adopted was the golf putting which can be simply described as the strike of a ball (Titleist; model Pro V1, Fairhaven, MA) with a putter (Putter Jumbo Black Beauty; size 35; standard, United Kingdom). The setup was developed in laboratorial context and consisted on a horizontal and still surface, placed on the ground over a ramp. The players performed 30 trials at 2 m from the hole.

A simple detection algorithm was used to detect the putter's head. To that end, a red marker identified the putter's head accordingly with the red–green–blue (RGB) range values defined. The detection algorithm was considered to be computationally efficient since it relied on simple image processing techniques, thus ensuring satisfactory results on both true positives (TP) detection and reduced processing time.

A Casio Exilim/High Speed EX-FH25 with 210 *Hz*, at a resolution of 480 × 360 pixels with a focal length of lens of 26 mm was used to detect the movement. To map the gestures into real coordinates (i.e., pixel.frame^{-1} to m.s^{-1}), a reference was created in the same plane of the player. This reference was the putt's metallic part with a length of 585 *mm*.

The graph presented in Figure 1 shows an example of the detected position of a golf club, in the horizontal plane, during the putting execution of an expert player.

This work was particularly important with regard to the study of movements and motor skills training, going beyond the analysis of the measures associated with the product and the response magnitude (e.g., position of the ball over the hole or quantification of the final result). Experimental results of the putting performance of different expert golf players clearly suggest that each one of them had a typical and distinct game style, i.e., a signature, confirming our hypothesis. However, the mathematical modelling of the putting gesture is required to accurately quantify the differences between players.

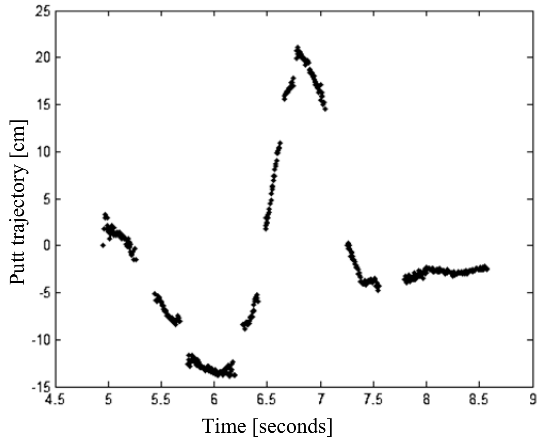

Figure 1: Example of a point cloud obtained with the detection algorithm (adapted from Couceiro et al., 2013).

Kinematical Model of the Putting

In the study of Luz and associates (2013), collected kinematic measures by filming the putter's trajectory to determine the putting phases. The following phases were identified: the backswing, downswing, ball impact and follow-through, as well as to analyze the amplitude, velocity, acceleration and overall duration of the movement.

The putting movement was analyzed using auto-tracking methodologies by autonomously comparing the current frame with the previous frame through a MatLab script. The kinematic model of the golf putting consists of two different stages that must be accomplished for each trial: 1) the detection of the putter using a simple color-based detection algorithm, and 2) the estimation of the kinematical model.

By analyzing the shape of various point clouds given by the detection algorithm (Figure 1), it was found that to model the putter's horizontal position in time a sinusoidal-like function had to be used. However, a function composed by only one or even two sinusoids was not precise enough to describe the gesture, as it is clear in f_1 and f_2 (Figure 2), which resulted in a mean squared error (MSE) of 2.6568 and 0.7124 units, respectively. This happens because the amplitude, angular frequency and phase of the

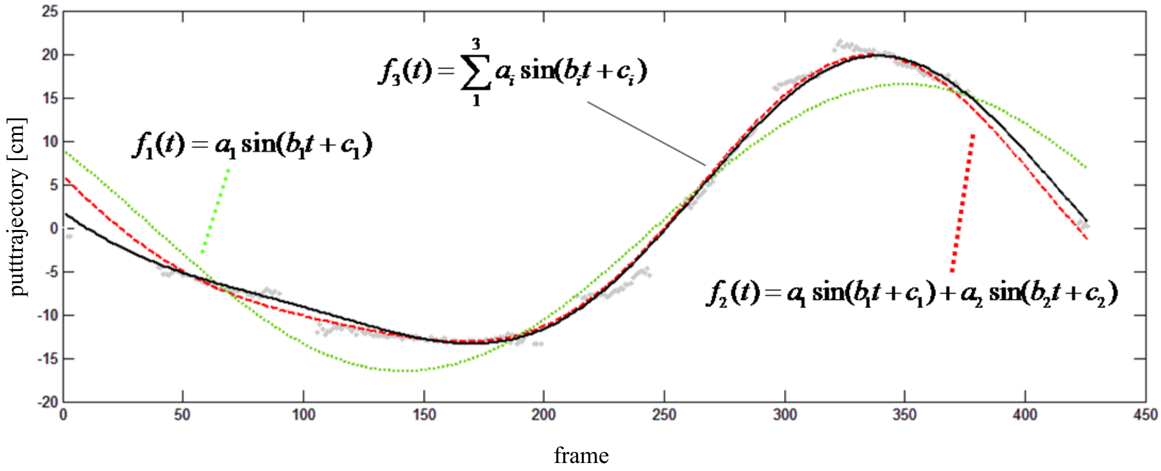

Figure 2: Fitting sinusoidal functions to a point cloud, representing the position of a golf club during putting execution (adapted from Couceiro et al., 2013).

descending half-wave, which corresponds to the player's backswing and downswing is usually different than the ascending half-wave, which corresponds to the ball's impact and follow-through.

Hence, a compromise between precision and complexity of the problem had to be assumed, because each sinusoid adds three more dimensions to the estimation problem (i.e., amplitude, angular frequency and phase of the corresponding sine wave).

In order to avoid the complexity of the problem to grow inappropriately, a function composed by the sum of three sinusoids was used, due to its precision, with a MSE of 0.6926 (cf., f_3, Figure 2). Thus, having the estimation function defined as a sum of three sine waves (equation 1), the estimation process results in a nine dimension mathematical (i.e., sinusoidal) function for each of the 30 trials of the 10 players in study.

$$f(t) = a_1 sin(b_1 t + c_1) + a_2 sin(b_2 t + c_2) + a_3 sin(b_3 t + c_3) \qquad (1)$$

In that sense, it was possible to represent that function by compiling the 9 parameters in six two-dimensional groups (*i.e.*, a_1 *vs* b_1, a_1 *vs* c_1, a_2 *vs* b_2, a_2 *vs* c_2, a_3 *vs* b_3, a_3 *vs* c_3). Nonetheless, such representation would result in a complex scatter chart with a large number of data points plotted within the same space making it difficult to distinguish individual data sub-sets and challenging to associate each data sub-set to a specific player.

By creating a mathematical model that represents the general trajectory of the putter, one can apply classifications methods in which the inputs of the system are comprised of the aforementioned 9 parameters. Nevertheless, an estimation technique is necessary to first retrieve the parameters from equation 1.

A Methodology for Detection and Estimation in the Analysis of Golf Putting

The contribution on Couceiro et al., (2012) was divided in two distinct steps: *i*) estimation of the parameters from equation 1; and *ii*) classification of each player based on the output of the estimation technique.

After detecting the dynamic position of the golf club during the execution of the putt and developing the mathematical model (equation 1), estimation techniques were compared in terms of computational complexity and memory toward the optimization of the most adequate parameters. Five different estimation techniques were studied, applied and compared in this work: *i*) gradient descent; *ii*) pattern search; *iii*) downhill simplex; *iv*) the particle swarm optimization (PSO); and *v*) Darwinian particle swarm optimization (DPSO). Results confirmed the superior performance of the Darwinian Particle Swam Optimization (DPSO) method.

Afterwards, five classification algorithms were used to extract the unique signatures of each player: *i*) linear discriminant analysis (LDA); *ii*) quadratic discriminant analysis (QDA); *iii*) naïve bayes with gaussian distribution (NV); *iv*) naïve bayes with kernel smoothing density estimate (NVK); and *v*) least squares support vector machines (SVM). The five classification methods were compared through the analysis of the confusion matrix and the area under the receiver operating characteristic curve (AUC), and the *SVM* was the one that presented an overall better performance.

Experimental results clearly show, when considering different trials of every player, that each of them as a typical and distinct style of play. Properties of the average putter trajectory of each player, like the parameters of the estimation model, the timing of the play and its different stages, as well as the amplitude of the strokes fully characterize the putting gesture of every player, equivalently to a signature pattern. To completely study the golf putting, the analysis of the product variables followed the analysis of the process variables.

A Fractional Calculus approach for the evaluation of the Golf Lip-out

The main goal of this work (cf. Couceiro et al., 2012) was to differentiate trials where lip-out occurred with the ones where it did not, in order to provide a fair performance analysis. In this article, we propose a new metric based on fractional calculus, in order to differentiate trials where lip-out did not occurred from the ones where it did.

Little attention has been given to the use of performance metrics in golf's product variables in a way beyond error quantification (i.e., if the ball enters or not the hole and in case it does not, the radial distance from the ball position to the hole). Thus, the lip-out is not typically taken in consideration.

Despite the fact the ball goes in the hole direction and only then deviates from it, going to what turns to be its final position, this trial is usually considered as another one that would be a complete miss, as it is only measured the final ball position to the hole.

The state-of-the-art shows that fractional calculus has been drawing researchers' attention for the last two decade, being rediscovered and applied in an increasing number of fields. Mathematicians have done the theoretical development of FC, but many researchers have been using fractional derivatives as a tool and a way to further explore their applications (Dias and Mendes, 2010; Dias et al., 2013).

The major research about putting is focused on the performance analysis confirming the relevance of it in order to evaluate a complex motor ability, which is different from player to player. Furthermore, the radial error is an important quantitative evaluation form of player's error during practice in laboratory or field (i.e., self-learning situations).

In that sense, recent works (Dias and Mendes, 2010; Dias et al., 2013) about golf putting adopt product variables (e.g., radial error evaluation) as part of the player's performance analysis. The radial error (ε_i) can be easily obtained using the Pythagoras theorem, as it is the hypotenuse of the right triangle relating both legs defined by lateral error ε_i^x and longitudinal error ε_i^y (Figure 3).

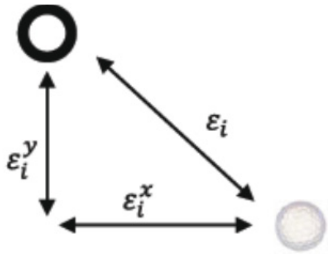

Figure 3: Representation of the three measured errors (adapted from Couceiro et al., 2012).

Considering this information, in our study we could use FC in two different approaches: (i) as a correction metric in order to calculate a new virtual position for ball's final position based on its trajectory induced by the putting lip-out, thus "softening" the radial error in these trials; (ii) as performance metric for golf putting. In this article, we focus on the first approach.

In this study, further information about fractional calculus and the developed metric will be given. When analyzing temporal signals, non-temporal measures such as arithmetic mean value, standard deviation, variation coefficient or, in this case, the radial error do not provide all the needed information in order to evaluate a signal and can, sometimes, lead to deceiving data analysis.

Although the radial error can give valuable information as a product variable, it is not always representative of ball's trajectory through the green. Hence, the computational requirements of the herein proposed algorithm linearly depend on the duration of the trajectory, as the number of terms r requires storing the last r balls position. Bearing this idea in mind, it can be considered a lip-out trial, where the player accurately performs the putt, but the ball goes around the hole ending up in a position with a high lateral error ε_i^y value.

In this case, the radial error has a high value, and using only this measure, it would be considered the same as if the putt resulted in a linear trajectory between the putting spot and the ball final position.

Considering the golf putting, coaches and players can benefit by having more knowledge about the lip-out effect and perhaps can find strategies to deal with it in training or competition. Nevertheless, there is no effective or "magical" way to avoid this phenomenon from happening. Being the *handicap* away to classify the player's performance, there is a void in current research, establishing a possible link between the *handicap* and the number of strikes with lip-out. For the purpose of our study, we are not going that far, but instead propose a new metric, based on *fractional calculus* (and its inherent "memory" feature), allowing the ball repositioning in trials where lip-out occurred, giving more credit to the fact the ball was hit toward the hole.

Design of a Golf Putting Pneumatic Mechanism Integer *vs.* Fractional Order *PID*

In this study (cf. Couceiro et al., 2012), a novel test bed for evaluation of the golf putting is proposed. The developed putting mechanism consisted on a pneumatic system that emulates the golf putting based on real reference data of expert golf players previously studied in (Dias et al., 2010). The idea was to deeply understand the mechanics of the golf putting. For instance, most golf experts consider that the key to a successful putt is in the power of the follow-through (Pelz, 2000).

However, the importance of the follow-through may be only an indication that the first part of the stroke (i.e., backswing and downswing) was well played. Also, it is not clear if the vertical trajectory of the putter is relevant for the success of the putting. Being a pendulum-like movement it is known that when the putter reaches the ball (i.e., angle of inclination of the putter near 90 degrees) the vertical velocity is zero or near zero. Instead of using a regular putter, can we say that we could obtain the same performance if applying the exact same force on the golf ball using, for instance, a snooker cue?

We emphasize that the focus of this work will not be directly related with the analysis of the phases of the putting motion presented in Figure 4 (e.g., backswing, downswing, ball impact and follow-through) (Pelz, 2000). However, the proposed mechanism will allow reproducing the phases of the horizontal component of the putting.

The putting mechanism consists on a pneumatic actuator CE1B32-200 equipped with a putter's head from a Putter Jumbo Black Beauty (Figure 4). As an important driving element, the pneumatic cylinder is widely used in industrial applications for many automation purposes thanks to their variety of advantages.

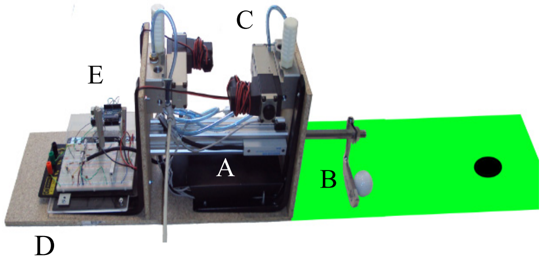

Figure 4: Putting Mechanism: A - High Precision Scale Cylinder CE1B32-200 with encoder; B - Putter's head from Putter Jumbo Black Beauty; C - Electro-Valves VER2000-03F; D - Interface Board; E - *Arduino* Control Board (adapted from Couceiro et al., 2012).

Given the above, the proposed mechanism respects the ecological validity of putting performance with regards to the ball impact velocity, which is very accurate. This mechanism also allows executing consistent replications of the movement (Pelz, 2000). Therefore, this novel approach suggests that it would be possible to study the golf putting, thus revealing the mechanics of this gesture in field and laboratory context.

In order to analyze the controllers' performances, a real-time data acquisition program was designed in *MatLab* to capture the system output data through the communication interface between the *PC* and the *Arduino* controller. Experimental results were divided in two stages: *i*) Optimization and comparison of the integer and fractional order *PID* controllers; and *ii*) Evaluation of the proposed Putting Mechanism while simulating, under the same conditions, a set of 30 trials previously performed by an expert subject when facing a ramp constraint. In all experimental results the system pressure was set to 6 bar and the controllers were updated each time the external interrupts were activated, thus computing the time between pulses.

In that sense, we presented the accuracy of the putting device comparing it with real data obtained from 30 trials performed by an expert golf player with a handicap of 5. In order to allow a straightforward comparison with the golf player, the mechanism was deployed in an artificial green to hit the ball two meters away from the hole. The reference trajectory performed by the golf player at each trial was sent to the microcontroller through serial communication.

The analysis of a set of trials is not directly accessible and need a graphical or geometrical representation. To analyze the radial error that is calculated using the lateral error (x-axis) and longitudinal error (y-axis) within the sport context, one of the most common representations is the error ellipse. The error ellipse allows a two-dimensional graphical analysis representing the influence of the lateral and longitudinal error (i.e., accuracy) and the variability (i.e., precision) of a given player. By observing the shape, size and orientation of the ellipse, one can easily compare different players or, as it is presented in this study, compare a man with a machine.

As it can be observed, there is a high similarity in the shape of both ellipses. It is noteworthy that the golf player was more accurate than the developed mechanism since it only missed 4 trials (accuracy of 86.67%) against 11 missed trials from the machine (accuracy of 63.33%) – see Figure 5.

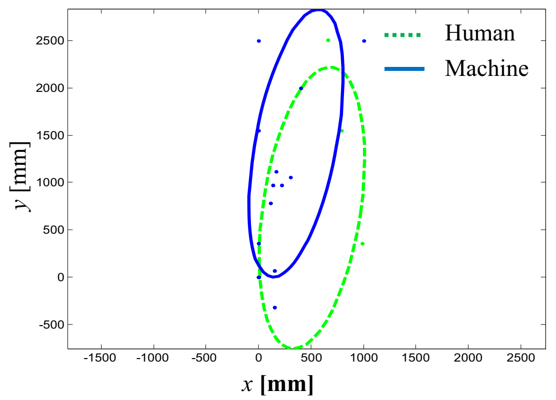

Figure 5: Error ellipse of 30 trials performed by an expert golf player and subsequently emulated by the putting mechanism (adapted from Couceiro et al., 2012).

However, the area of the ellipse for both the player and the putting mechanism was 2.20 m^2 and 1.75 m^2, respectively. This means that, despite the higher accuracy of the human player, the device was more precise thus presenting a lower variability.

Despite these results, like other experimental devices, this mechanism can simulate the putting, but it can hardly represent unequivocally the motor performance of a human being. This is because each individual has different characteristics and profiles that represent a "putting signature" that is distinctive from subject to subject, which may not be fully replicated by a machine.

Hence, man versus machine analogy is inevitable and strides for the multi and interdisciplinary research that crosses knowledge of several research fields (e.g., engineering, sport science and biomechanics) to meet the challenges in science.

This work, more than just presenting a mechanism or experimental device that can replicate the putting gesture, it is worth proposing a novel creative process that can serve as support for future researchers who wish to further study this movement.

A Non-linear Analysis of Golf Putting

The aim of this study (Dias et al., *in press*) was to investigate both golf putting precision and accuracy. In that sense, the authors proposed a new approach using error ellipses and Fourier series to analyse product variable tendencies in golf putting performance. The sample consisted of 10 male golfers, adults (33.80 ± 11.89 years), volunteers, right handed and experts (10.82 ± 5.40 handicap).

In this study, the radial error was obtained using Pythagoras' theorem, as it is the hypotenuse of the right triangle relating both legs defined by lateral error and longitudinal error, and the ellipse centre was calculated using the radial error of the 30 trials with MatLab (Couceiro et al., 2012). Afterwards, by analyzing the ellipse's size and area one can quantify the accuracy and precision of the golf putting considering the hole (Dias et al., 2013).

Using the Fourier series it was possible to analyze the putting performance tendencies of the players. Any tendency may be approximated by a truncated Fourier series for the *n* degree, adjusted to the nonlinear least squares (equation 2).In this work, the Fourier series were used to analyze both the maximal velocity of the putting performance and the radial error over the 30 trials for each practice condition (Maor, 2002; Ardito et al., 2008):

$$q_n(t) = a_0 + \sum_{j=1}^{n} \left(a_j \cos\left(\frac{j2\pi}{T} t \right) + b_j \sin\left(\frac{j2\pi}{T} t \right) \right) \quad (2)$$

(adapted from Dias et al., *in press*)

Thus, T (Trials) = 30, the coefficients a_0, …, a_n e b_1,…,b_n are obtained using the *Trust Region* method, solving the problems of the non-linear least squares (Maor, 2002; Ardito et al., 2008).

The choice regarding the process variable selected was supported by Pelz (2000), that consider the maximal velocity as one of the most important variables on the golf putting.

The maximal velocity was retrieved directly from the direct acquisition of the golf club using two digital cameras (Casio Exilim/High Speed EX-FH25). Images were processed at 210 Hz (camera one) which allowed a detailed analysis of the putter movement, and 30 Hz (camera two) to analyze the ball's trajectory (Dias et al., 2013). For that purpose, an auto-tracking methodology that autonomously compares the current frame with the previous one was developed using MatLab (Dias et al., 2013). Based on Couceiro et al. (2013), we converted the pixel/frame value of the putter movement in metric units (e.g., m/s).

The results indicate that both precision and accuracy of putting performance were adjusted based on the variability conditions and task constraints. It is also noteworthy that the higher ranked players were very regular and stable in their performance even under different practice conditions of variability.

2 PRACTICAL APPLICATIONS

The evolution in technology, especially in the last decade, allowed major advances in golf putting analysis. The feedback from this analysis, being it qualitative or quantitative, is of high importance, providing a deeper knowledge around the putt. For instance, in a laboratorial context, real and practical situations for learning can be provided.

Although the presented information revolves around the golf putting, it can be optimized and used for analysis in other sports. Actually, the higher specialization of the performance analysis allows further understanding about the unique properties of each player.

Using this pertinent information one can adjust the training programs to the player specificities. Furthermore, the analysis about this kind of human movement during the golf putting performance can help to develop recovery programs in cases of injuries in some muscular groups. Therefore, the technological approaches about golf putting analysis help towards an improved performance of athletes.

ACKNOWLEDGEMENTS

The authors would like to thank to the Professor João Barreiros (FMH.UTL) for his technical support.

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