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OSCILLATION CRITERIA FOR THE FIRST-ORDER LINEAR DIFFERENCE EQUATIONS WITH SEVERAL DELAY ARGUMENTS

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We consider a difference equation with delayed arguments

$$\Delta u(k) + \sum_{i=1}^{m} p_i(k)\, u(\tau_i(k)) = 0,$$

where $\Delta u(k) = u(k+1) - u(k)$, $p_i : N \to R$, $\tau_i : N \to N$, $\lim_{k\to+\infty} \tau_i(k) = +\infty$, $i = 1, \ldots, m$, and establish sufficient conditions for all proper solutions of this equation to be oscillatory.

1. Introduction

The aim of the present work is to study a difference equation

$$\Delta u(k) + \sum_{i=1}^{m} p_i(k)\, u(\tau_i(k)) = 0, \tag{1.1}$$

where $\Delta u(k) = u(k+1) - u(k)$ and, for $1 \le i \le m$,

$$p_i : N \to R^+, \quad \tau_i : N \to N, \tag{1.2}$$

$$\tau_i(k) \le k - 1 \quad \text{for} \quad k \in N, \quad \text{and} \quad \lim_{k\to+\infty} \tau_i(k) = +\infty. \tag{1.3}$$

Define

$$\tau_*(\cdot) = \min\{\tau_i(\cdot) : i = 1, \ldots, m\}.$$

Definition 1.1. *Let $N_n = \{n, n+1, \ldots\}$ and let $n_0 = \min\{\tau_*(k) : k \in N_n\}$. We say that a function $u : N_{n_0} \to R$ is a proper solution of Eq. (1.1) if it satisfies (1.1) on N_n and*

$$\sup\{|u(i)| : i \ge k\} > 0 \quad \textit{for any} \quad k \in N_{n_0}.$$

Definition 1.2. *We say that a solution $u : N_{n_0} \to R$ of (1.1) is oscillatory if, for any $k \in N_{n_0}$, there exist $n_1, n_2 \in N_k$ such that $u(n_1) \cdot u(n_2) \le 0$. Otherwise, the solution is called nonoscillatory.*

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