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ON THE OSCILLATIONS OF FOURTH ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT. We establish some sufficient conditions for the oscillations of all solutions of fourth order functional differential equations

$$\frac{d}{dt}\left(a(t)\left(\frac{d^3}{dt^3}x(t)\right)^{\alpha}\right)+q(t)\,f(x[g(t)])=0$$

and

$$\frac{d}{dt}\left(a(t)\left(\frac{d^3}{dt^3}x(t)\right)^{\alpha}\right)=q(t)\,f(x[g(t)])+p(t)\,h(x[\sigma(t)])$$

when $\int^{\infty} a^{-1/\alpha}(s)\,ds<\infty$. The case when $\int^{\infty} a^{-1/\alpha}(s)\,ds=\infty$ is also included.

1. Introduction

This paper deals with the oscillatory behavior of solutions of fourth order functional differential equations

$$\frac{d}{dt}\left(a(t)\left(\frac{d^3}{dt^3}x(t)\right)^{\alpha}\right)+q(t)\,f(x[g(t)])=0 \tag{1}$$

and

$$\frac{d}{dt}\left(a(t)\left(\frac{d^3}{dt^3}x(t)\right)^{\alpha}\right)=q(t)\,f(x[g(t)])+p(t)\,h(x[\sigma(t)]) \tag{2}$$

where the following conditions are assumed to hold:

(i) α is the ratio of two positive odd integers;
(ii) $a(t)$, $p(t)$ and $q(t)\in C([t_0,\infty),(0,\infty))$;
(iii) $g(t)$ and $\sigma(t)\in C^1([t_0,\infty),\mathbb{R})$, $g(t)<t$, $\sigma(t)>t$, $g'(t)\geqslant 0$ and $\sigma'(t)\geqslant 0$ for $t\geqslant t_0$ and $\lim_{t\to\infty} g(t)=\infty$;

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