LIMITING PERFORMANCE ANALYSIS OF A VEHICLE RESTRAINT SYSTEM

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SUMMARY: In this paper the limiting performance analysis of a vehicle restraint system is studied. A discrete model of the human thorax is used. The driver displacement relative to the dashboard is minimized and several injury criteria are required to remain bellow a safety threshold value. The optimal control force acting on the driver is found. A design and control sensitivity analysis formulation is derived for mechanical systems. The equations of motion and the sensitivity equations are integrated at-once as it is typical for the static response. Mathematical programming is used for the optimal control process.

KEYWORDS: Optimum control, Sensitivity analysis, Injury prevention, Impact, Space-time finite elements.

1. Introduction

The seat belt is widely regarded as one, if not the most, important piece of safety equipment in a vehicle. Seat belt system performance is being continually improved with the use of such devices as pre-tensioners and belt load limiters which are becoming common features in today vehicles [1]. Further improvement can be obtained by implementing active control as an integral part of the restraint system, for example with dual stage belt pre-tensioners which modulate their response with the severity of the vehicle deceleration. As such, the usefulness of a benchmark performance to evaluate the quality of the restraint system is obvious.

The limiting performance analysis of a restraint system can be obtained by finding the solution of an optimum control problem in which the control variable is the force acting on the vehicle occupant, in order to minimise its risk of injury. Since the control force is generic, not representing any predetermined design, then the limiting performance analysis measures the limits on the improvements of the restraint system with respect to the prescribed performance criteria. The performance of such a system can be seen as a benchmark to which the performance of a real restraint system can be compared.

The fundamentals of shock and impact isolation may be studied in [2]. The theory of optimal shock and impact isolation is described in [3]. A survey of problems related to the limiting performance analysis of injury prevention systems is given in [4]. The problem may be formulated as a control optimization problem. If we have more than a performance objective, then the problem is a multicriteria optimization problem.

In this work this methodology is applied to a published discrete nonlinear thoracic model having multiple degrees of freedom. A set of functionals are defined as constraints corresponding to injury criteria. The dynamic response is modelled via time finite elements, and is integrated by at-once integration. The optimal control force is obtained by nonlinear programming.

2. Response Analysis

The virtual work dynamic equilibrium equation of the system at the time *t* is given as

$$\delta W = \int (\rho\, {}^t\ddot{\boldsymbol{u}}\bullet\delta\, {}^t\boldsymbol{u} + {}^t\boldsymbol{S}\bullet\delta\, {}^t\boldsymbol{\varepsilon} - {}^t\boldsymbol{f}\bullet\delta\, {}^t\boldsymbol{u})\, {}^0dV - \int {}^t\boldsymbol{T}\bullet\delta\, {}^t\boldsymbol{u}\, {}^0d\Gamma \tag{1}$$

where all the quantities are referred to the undeformed configuration, δ represents the variation of the state fields, '•' refers to the standard tensor product, upper dot '.' refers to the material time derivative, ρ is the mass density at time $t = 0$, ${}^t\boldsymbol{u}$ is the displacement, and ${}^t\boldsymbol{S}$ is the second-Piola stress measure. After space discretization, we've got the equation of motion

$$\boldsymbol{M}\,{}^t\ddot{\boldsymbol{U}} + {}^t\boldsymbol{C}_S\,{}^t\dot{\boldsymbol{U}} + {}^t\boldsymbol{K}_S\,{}^t\boldsymbol{U} = {}^t\boldsymbol{P} \tag{2}$$

where $\boldsymbol{M}$ is the mass matrix, ${}^t\boldsymbol{C}_S \equiv {}^t\boldsymbol{C}_S({}^t\dot{\boldsymbol{U}})$, ${}^t\boldsymbol{K}_S \equiv {}^t\boldsymbol{K}_S({}^t\boldsymbol{U})$ are respectively the damping and stiffness matrices, ${}^t\boldsymbol{P}$ is the loading vector and ${}^t\boldsymbol{U}, {}^t\dot{\boldsymbol{U}}, {}^t\ddot{\boldsymbol{U}}$ are respectively the displacement, velocity and acceleration vectors, all the quantities defined at time t.

For temporal modeling, finite elements of dimension Δt were considered, selecting hermitean cubic elements to model the displacements and quadratic lagrangean elements to model the loading. By taking the time derivative of the Eq. (2) on one hand, and on the other hand its integration once and then twice with average values of stiffness and damping in Δt given as

$$\bar{\boldsymbol{K}} \equiv {}^{t+\Delta t/2}\boldsymbol{K}_S({}^t\boldsymbol{U} + {}^t\dot{\boldsymbol{U}}\,\Delta t/2), \qquad \bar{\boldsymbol{C}} \equiv {}^{t+\Delta t/2}\boldsymbol{C}_S({}^t\dot{\boldsymbol{U}} + {}^t\ddot{\boldsymbol{U}}\,\Delta t/2) \tag{3}$$

one obtains four equations that combine to give the dynamic finite time-element equation as

$$\boldsymbol{D}_S^e\,\boldsymbol{z}^e = \boldsymbol{P}^e \tag{4}$$

where

$$\boldsymbol{D}_S^e = \left[\begin{array}{ccc|ccc} {}^t\boldsymbol{K}_S & {}^t\boldsymbol{C}_S & \boldsymbol{M} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} \\ \hline {}^t\boldsymbol{D}_{S_{11}} & {}^t\boldsymbol{D}_{S_{12}} & \boldsymbol{0}_{nxn} & {}^{t+\Delta t}\boldsymbol{D}_{S_{11}} & {}^{t+\Delta t}\boldsymbol{D}_{S_{12}} & \boldsymbol{0}_{nxn} \\ {}^t\boldsymbol{D}_{S_{21}} & {}^t\boldsymbol{D}_{S_{22}} & \boldsymbol{0}_{nxn} & {}^{t+\Delta t}\boldsymbol{D}_{S_{21}} & {}^{t+\Delta t}\boldsymbol{D}_{S_{22}} & \boldsymbol{0}_{nxn} \end{array}\right], \qquad \boldsymbol{P}^e = \begin{Bmatrix} {}^t\boldsymbol{P} \\ \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \end{Bmatrix} \tag{5}$$

and

$$\boldsymbol{z}^e = ({}^t\boldsymbol{z}, {}^{t+\Delta t}\boldsymbol{z}), \qquad {}^t\boldsymbol{z} = ({}^t\boldsymbol{U}, {}^t\dot{\boldsymbol{U}}, {}^t\ddot{\boldsymbol{U}}) \tag{6}$$

In Eq. (5) n stands for number of space degrees-of-freedom and ${}^t\boldsymbol{D}_{S_{jk}}$ are functions of $\bar{\boldsymbol{K}}, \bar{\boldsymbol{C}}, \boldsymbol{M}$. The Eq. (4) may be solved step-by-step, i.e., element-by-element in time, or assembled to be solved at-once. In this case, we have to assemble for a total time interval T discretized in N time nodes, resulting in the dynamic equation

$$\boldsymbol{D}_S\,\boldsymbol{z} = \boldsymbol{P} \tag{7}$$

where $2n$ time boundary conditions are imposed by transferring the corresponding columns of the assembled matrix $\boldsymbol{D}_S$ to the right-hand side of Eq. (7) after multiplying by the vector U_c of those conditions, resulting the equation

$$\hat{\boldsymbol{K}}_S\hat{\boldsymbol{U}} = \hat{\boldsymbol{P}}, \qquad \hat{\boldsymbol{P}} = \boldsymbol{P} - \boldsymbol{D}_c\boldsymbol{U}_c \tag{8}$$

The Eq. (8) is a nonlinear equation where $\hat{\boldsymbol{K}}_S$ is a nonsymmetrical matrix dependent on the response. Therefore, the Eq. (8) has to be solved iteratively.

3. Design Sensitivity Analysis

Consider now a general performance measure defined in the time interval $[0,T]$ as

$$\Psi = \int G({}^t\boldsymbol{z}, {}^t\boldsymbol{b}, t)\,dt \tag{9}$$

where ${}^t\boldsymbol{b}$ is the vector of design and control variables and the other quantities were defined in Eqs. (1) and (6). It contains the external forces ${}^t\boldsymbol{P}$. The design sensitivity analysis problem is to derive the total design variation of the measure in Eq. (9) with respect to the design ${}^t\boldsymbol{b}$, for the system represented by the equation of motion, Eq. (8)
.

3.1. The Adjoint Method of Design Sensitivity Analysis

The total design variation for the performance measure of Eq. (9), for preview control problems, is

$$\bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \tilde{\delta}\Psi \tag{10}$$

where $\bar{\bar{\delta}}\Psi$ and $\tilde{\delta}\Psi$ represent respectively the explicit and implicit (state dependent) design variations. In order to formulate the adjoint method of design sensitivity analysis, replace the arbitrary variation of state fields by adjoint fields into the virtual work equation as

$$W^a = (\hat{\boldsymbol{K}}_S\, \hat{\boldsymbol{U}} - \hat{\boldsymbol{P}}) \bullet \hat{\boldsymbol{U}}^a = 0 \tag{11}$$

and define an extended 'action' function

$$A = \Psi - W^a \tag{12}$$

The basic idea of introducing an adjoint system is to replace the implicit design variations of the state fields by explicit design variations and adjoint fields, then determining these adjoint fields by vanishing the implicit design variation of the 'action' function A [8] as

$$\tilde{\delta}A = 0 \tag{13}$$

Therefore, the total design variation of Ψ can be written as

$$\bar{\delta}\Psi = \bar{\bar{\delta}}A \tag{14}$$

3.2. Design Sensitivity Analysis Modeling

In order to solve the design sensitivity analysis problem, the sensitivities are firstly performed at the element level and then the sensitivity equations are assembled. The explicit design variation of the vector of element forces of Eq. (4) is

$$\bar{\bar{\delta}}\boldsymbol{R}^e = \bar{\bar{\delta}}\boldsymbol{P}^e - \bar{\bar{\delta}}\boldsymbol{F}^e, \qquad \boldsymbol{F}^e = \boldsymbol{D}_S^e\, \boldsymbol{z}^e \tag{15}$$

and the implicit design variation of the internal forces gives

$$\tilde{\delta}\boldsymbol{F}^e = \tilde{\delta}\boldsymbol{D}_S^e\, \boldsymbol{z}^e + \boldsymbol{D}_S^e\, \tilde{\delta}\boldsymbol{z}^e = \boldsymbol{D}^e\, \tilde{\delta}\boldsymbol{z}^e \tag{16}$$

where

$$\boldsymbol{D}^e = \left[\begin{array}{ccc:ccc} {}^t\boldsymbol{K} & {}^t\boldsymbol{C} & \boldsymbol{M} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} \\ \hdashline {}^t\boldsymbol{D}_{11} & {}^t\boldsymbol{D}_{12} & {}^t\boldsymbol{D}_{13} & {}^{t+\Delta t}\boldsymbol{D}_{11} & {}^{t+\Delta t}\boldsymbol{D}_{12} & \boldsymbol{0}_{nxn} \\ {}^t\boldsymbol{D}_{21} & {}^t\boldsymbol{D}_{22} & {}^t\boldsymbol{D}_{23} & {}^{t+\Delta t}\boldsymbol{D}_{21} & {}^{t+\Delta t}\boldsymbol{D}_{22} & \boldsymbol{0}_{nxn} \end{array}\right] \tag{17}$$

with

$${}^t\boldsymbol{K} = {}^t\boldsymbol{K}_S + {}^t\boldsymbol{K}_{S,\boldsymbol{U}}\, {}^t\boldsymbol{U}, \qquad {}^t\boldsymbol{C} = {}^t\boldsymbol{C}_S + {}^t\boldsymbol{C}_{S,\dot{\boldsymbol{U}}}\, {}^t\dot{\boldsymbol{U}} \tag{18}$$

Sensitivities of Eq. (15) and the element dynamic matrix of Eq. (17) are assembled and again imposed the time boundary conditions resulting respectively $\bar{\bar{\delta}}\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{K}}$.

Now, the application of the Eq. (13) to the Eq. (12) gives the adjoint system equilibrium

$$\hat{\boldsymbol{K}}^T \hat{\boldsymbol{U}}^a = (\Psi_{,\hat{\boldsymbol{U}}})^T \tag{19}$$

Thus, the total design variation of Eq. (14) is

$$\bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \hat{\boldsymbol{U}}^a \bullet \bar{\bar{\delta}}\hat{\boldsymbol{R}} \tag{20}$$

4. Thoracic Model and Injury Criteria

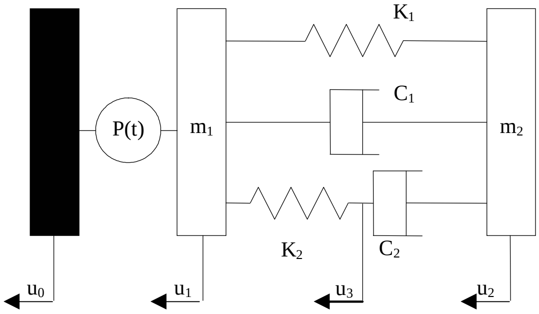

Figure 1 – Model of the human thorax

The model shown in Fig.1 represents the human thorax [5, 7]. Mass m_1 represents the sternum, rib structure and thoracic contents and m_2 represents the remaining portion of the thorax and the part of the total body mass that is coupled to the thorax by the vertebral column. Masses m_1=0.3 *kg* and m_2=18 *kg* are interconnected by a nonlinear spring K_1 that represents the elasticity of the rib cage and directly coupled viscera. The nonlinear damper C_1 connecting m_1 and m_2 represents thoracic damping derived, for example, from air in the lungs and blood in the thoracic vasculature displaced during an impact. The linear spring K_2 (13153 *N/m*) and damper C_2 (175.4 *Ns/m*) combined in series model the thoracic muscular tissue behaviour. The action of a restraint system on the thorax is represented by a control force *P(t)* between mass m_1 and the vehicle.
The variables u_0, u_1, u_2 and u_3 represent respectively the displacements of the vehicle and masses m_1, m_2 and m_3 referred to an inertial frame. Mass m_3 is considered zero.
The force exerted by the spring K_1 is given by:

$$\begin{cases} K_{1_1}\left(u_2 - u_1\right) & if \quad 0 \le u_2 - u_1 \le \delta_0 \\ K_{1_2}\left(u_2 - u_1\right) - (K_{1_1} - K_{1_2}) & if \quad u_2 - u_1 \ge \delta_0 \end{cases} \tag{21}$$

where $\delta_0 = 0.03m$ is a critical value of the chest compression at which the thorax changes stiffness. The value of K_{1_1} is 10522 *N/m* and the value of K_{1_2} is 71901 *N/m*.
The force exerted by the damper C_1 is given by:

$$\begin{cases} C_{1_1}\left(\dot{u}_2 - \dot{u}_1\right) & if \quad \dot{u}_2 - \dot{u}_1 \ge \delta_0 \\ C_{1_2}\left(\dot{u}_2 - \dot{u}_1\right) & if \quad \dot{u}_2 - \dot{u}_1 < \delta_0 \end{cases} \tag{22}$$

with $C_{1_1} = 403.3Ns/m$ and $C_{1_2} = 2192.1Ns/m$.
In order to simulate an impact at the instant t=0 with the vehicle traveling at the speed v_0=48km/h, one considers the following initial conditions:

$$\begin{gathered} u_0(0) = 0; u_1(0) = 0; u_2(0) = 0; u_3(0) = 0 \\ \dot{u}_0(0) = v_0; \dot{u}_1(0) = v_0; \dot{u}_2(0) = v_0; \dot{u}_3(0) = v_0 \end{gathered} \tag{23}$$

Also it is assumed the vehicle undergoes a half-sine wave deceleration pulse of duration $T_a = 0.1s$:

$$\ddot{u}_0 = -A\sin\left(\frac{\pi t}{T_a}\right) \tag{24}$$

The amplitude A of the pulse is calculated as

$$A = \frac{\pi v_0}{2T_a} \tag{25}$$

such that the vehicle decelerates to a full stop at the end of the duration of the pulse $t = T_a$:

$$\dot{u}_0\left(T_a\right) = 0 \tag{26}$$

To measure thoracic injuries, the following injury criteria are defined [5]:
Maximum chest compression:

$$\Psi_1 \equiv \max_{t \in [0,\infty[} \left|u_2 - u_1\right| \tag{27}$$

Maximum chest acceleration:

$$\Psi_2 \equiv \max_{t \in [0,\infty[} \left|\ddot{u}_2\right| \tag{28}$$

Maximum rate of chest compression:

$$\Psi_3 \equiv \max_{t \in [0,\infty[} \left|\dot{u}_2 - \dot{u}_1\right| \tag{29}$$

Maximum chest viscous response:

$$\Psi_4 \equiv \max_{t \in [0,\infty[} \left|\left(\dot{u}_2 - \dot{u}_1\right)\left(u_2 - u_1\right)\right| \tag{30}$$

The maximum chest viscous response is the instantaneous product of the chest compression and the rate of chest compression. It characterizes the rate sensitivity of injury threshold.

5. Optimal Design

We whish to minimise the maximum excursion of the vehicle driver, defined as:

$$\Psi_0 \equiv \max_{t \in [0,\infty[} \left|u_1 - u_0\right| \tag{31}$$

which measures the safety space in front of the occupant, i.e., the distance between the occupant and the dashboard/steering wheel of the vehicle. The previously defined injury criteria will be required to remain below a prescribed value. We can now formulate the optimisation problem as follows:
Find the optimal control *P(t)* in order to minimise Ψ_0 such that

$$\begin{aligned} &\Psi_1 \le 0.046m \\ &\Psi_2 \le 80g \\ &\Psi_3 \le 6m/s \\ &\Psi_4 \le 0.229m^2/s \end{aligned} \tag{32}$$

6. Optimization Results

The total time of analysis was considered 0.2*s* and the time discretization was made considering 20 time intervals of 0.01*s* each, which resulted in 41 control variables. The optimal control was found by mathematical programming and it is presented in Fig. 2 below with other optimization results:

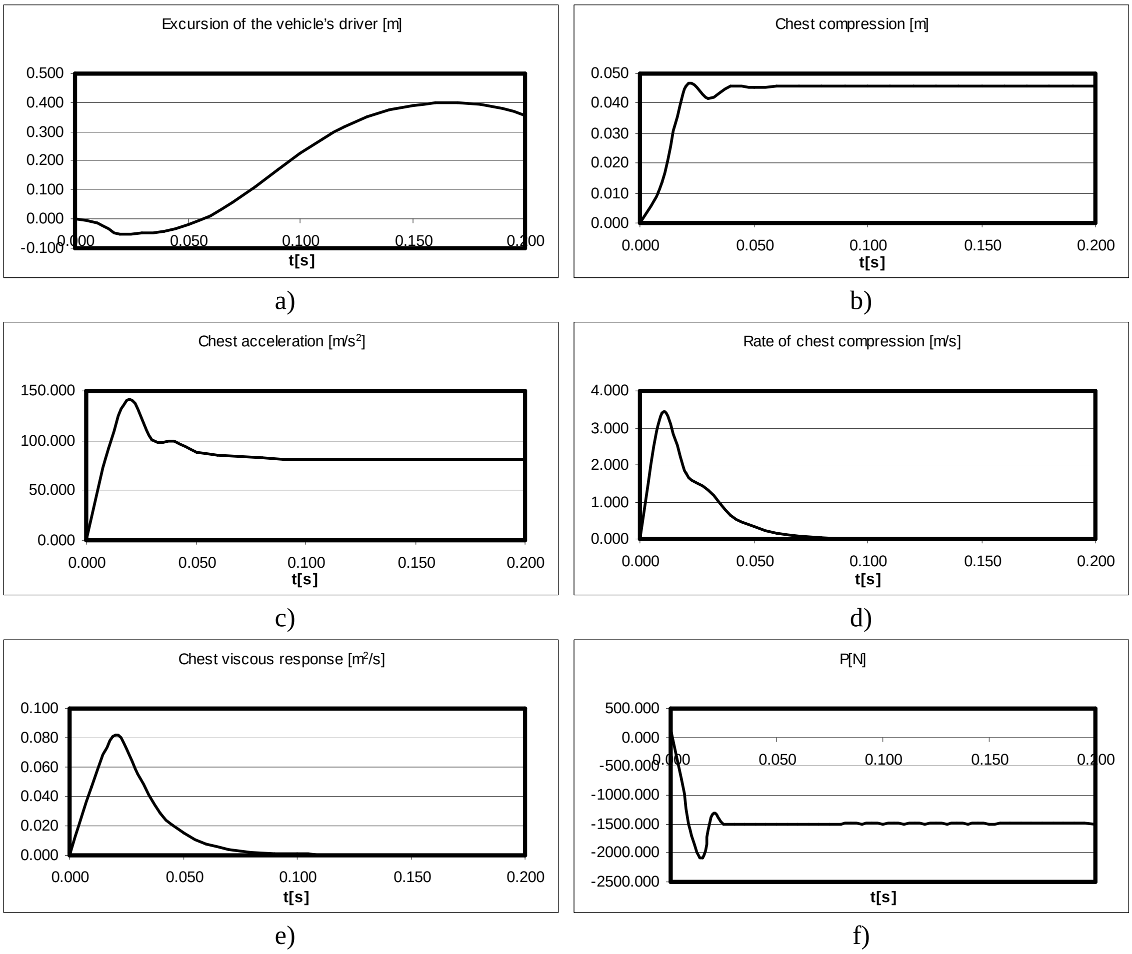

Figure 2 – Optimization results

The optimal control obtained is shown in Figure 2 f). One can see that the excursion of the vehicle driver does not exceed 0.4*m* and that all the injury criteria remained below the prescribed admissible values. Of those, only the chest compression reaches its maximum allowed value of 0.046*m*. Also, from 0.075*s* onwards the rate of chest compression remains equal to zero, which indicates that m_1 and m_2 travel at the same velocity, which results in a constant value of the chest compression, as seen in Figure 2 d) and Figure 2 b).

5. REFERENCES

[1] Viano, D., "Lap-shoulder Belts: Some Historical Aspects", Seat Belts: The Development of an Essential Safety Feature, 2003, Society of Automotive Engineers, Warrendale, PA.

[2] Harris, C.M., Crede, C.E., "Shock and Vibration Handbook", McGraw-Hill, 1996.

[3] Balandin, D.V., Bolotnik, N.N., Pilkey, W.D., "Optimal protection from impact and shock: theory and methods", Applied Mechanics Reviews, 53(9), 2000, p.p. 237-264.

[4] Balandin, D.V., Bolotnik, N.N., Pilkey, W.D., "Control Science for Injury Prevention", Proceedings of the PhysCon 2003 conference, St. Petersburg, Russia, 2003, pp. 1195-1200.

[5] Kent, R., Bass, C., Woods, W., Sherwood, C., Madeley, N.-J., Salzar, R., and Kitagawa, Y., "Muscle Tetanus and Loading Effects on the Elastic and Viscous Characteristics of the Thorax," Traffic Injury Prevention, 4, 2003, pp. 297–314.

[6] Eppinger, R. et al., "Development of Improved Injury Criteria for the Assessment of Advanced Automotive Restraint Systems," II. National High- way Traffic Safety Administration, U.S. Department of Transportation, Washington, DC., 1999.

[7] Lobdell, T. E., Kroell, C. K., Schneider, D. C., Hering, W. E., and Nahum, A.M., "Impact Response of the Human Thorax", Human Impact Response Measurement and Simulation, Plenum Press, New York-London, 1973, pp. 201–245.

[8] Cardoso J.B., and Arora J.S.," Design Sensitivity Analysis of Nonlinear Dynamic Response of Structural and Mechanical Systems", Structural Optimization, 4, 1992, p.p. 37-46.