Instituto Superior
de Engenharia
Politécnico de Coimbra

DEPARTMENT OF CHEMICAL AND BIOLOGICAL ENGINEERING

Life Cycle Analysis – A Global Approach

Dissertation to fulfill the Master's degree in Engineering and Management of Physical Assets

Author

Paulo Alexandre Lucas Figueiredo

Supervisors

Hugo David Nogueira Raposo

José Manuel Torres Farinha

Coimbra, June 2024

INSTITUTO POLITÉCNICO
DE COIMBRA

INSTITUTO SUPERIOR
DE ENGENHARIA
DE COIMBRA

ACKNOWLEDGEMENTS

The accomplishment of this thesis is not only due to me, as the author, but to a group of people who have had a direct and/or indirect influence on the work carried out over the last few years. I would like to start by expressing my gratitude, in no particular order, to Professor José Manuel Torres Farinha and Professor Hugo David Nogueira Raposo. Not only were my supervisors during the writing of this thesis, but my thanks go much further than that. Because of their influence on my degree in Mechanical Engineering, I chose this Master's in Engineering and Management of Physical Assets as the next step in my academic career. This choice has so far fulfilled me. Furthermore, I would also like to thank them not only for all their theoretical and academic teachings, but also for their tireless efforts to support me continuously and prepare me for "real life". Undoubtedly, I have grown a lot and I owe a huge amount of it to these two professors, who, regardless of what was going on in their personal lives, always made themselves available.

I would like to thank Engineer Pedro Fernandes and Engineer Ricardo Alves from *Celbi*, who worked and co-operated tirelessly during the case study on heavy industry equipment. They were always available to help with all the needs of the study and, above all, in collecting all the data we required and were able to provide.

I would also like to thank all the teachers on this master's programme who have prepared me for the different areas that include engineering and asset management itself. In essence, this thesis is not just about the chosen theme, but the culmination of the varied knowledge gathered through all the teachers.

Undoubtedly, another indispensable foundation is my family and friends. They have made me what I am today and have always made sure that I never lacked anything. I am grateful for the fact that my main concern throughout this journey has been to do my best in my academic career. If there is anyone to whom I owe this privilege, it is my family and friends.

Finally, I would like to thank God for giving me health and always opening doors and opportunities for me every day.

RESUMO

A engenharia e gestão de ativos tem-se revelado cada vez mais decisiva para a eficiência de qualquer organização, bem como para o aumento da fiabilidade dos próprios ativos físicos.

A verdade é que este tipo de abordagem não permite apenas encarar um ativo de forma individual e detalhada durante todo o seu ciclo de vida. Permite também encarar de um ponto de vista abrangente, não só considerando o tempo de vida do ativo, mas também todo o histórico da sua tipologia, tal como permite o estudo de continuidade após a sua substituição.

Foram realizados dois estudos de caso em que se aplicaram os modelos de Ciclo de Vida Útil e Económica a duas situações distintas: aos ativos de um parque infantil e a ativos específicos de uma indústria pesada. Em ambos os casos, o objetivo foi estimar o tempo ideal de substituição ou de renovação desses ativos.

O primeiro, trata-se de um problema em que os dados são muito escassos e em que foi necessário recorrer a várias considerações iniciais arbitrárias. Este estudo demonstrou parte da versatilidade dos modelos aplicados, bem como a sua enorme mais-valia para a engenharia e gestão deste tipo de ativos.

Na sequência da apresentação deste caso de estudo no 17º Congresso Nacional de Manutenção e 10º Encontro de Manutenção dos Países de Língua Portuguesa, a atenção centrou-se na aplicação de Modelos de Ciclo de Vida ainda mais rigorosos, mas desta vez na área da indústria pesada.

Em cooperação com uma empresa vocacionada para a produção de pasta de papel (*Celbi*), foi possível obter alguns dados reais de funcionamento da linha de descasque e corte de madeira instalada em meados de 2017.

Ao contrário do estudo do ciclo de vida dos parques infantis, foi possível recolher dados reais sobre os custos de exploração. Além disso, trata-se de um ativo que produz um retorno financeiro e, por conseguinte, a área de negócio é diferente.

Neste último caso, foram aplicados seis modelos de ciclo de vida - o modelo de Ciclo de Vida Útil e cinco modelos de Ciclo de Vida Económica diferentes. Por este motivo, foram consideradas novas variáveis e diferentes abordagens.

Palavras-chave: Análise do ciclo de vida; Modelos de ciclo de vida; Ativos físicos; Parques infantis; Ativos da indústria pesada.

ABSTRACT

Asset engineering and management has increasingly shown itself to be crucial to any organisation's efficiency, as well as to increasing the reliability of the physical assets themselves.

The truth is that this type of approach not only makes possible to look at an asset per si and in detail for all of their life cycles. It also makes it possible to look from a much broader viewpoint, not only considering the asset's lifetime, but also the entire history, as well as making it possible to study its continuity after it has been replaced.

Two case studies were carried out in which Lifespan and Economic Replacement Models were applied to two very different situations: to the assets of a children's playground and specific assets of a heavy industry. In both cases, the aim was to estimate the ideal replacement or refurbishment time for those assets.

The first one, is a problem with very poor data, where it was necessary to take several arbitrary initial considerations. This demonstrated part of the versatility of these models, together with their enormous added value for the engineering and management of these types of assets.

Following the presentation of this case study at the 17th National Maintenance Congress and 10th Maintenance Meeting of Portuguese-speaking Countries, attention was focussed on the application of even more rigorous Life Cycle Models, but this time in the area of heavy industry.

In cooperation with a company focussed on paper pulp production (*Celbi*), it was possible to obtain some real operating data for the Wood Debarking and Chipping Line installed in mid-2017.

Unlike the study of the life cycle of playgrounds, it was possible to collect real data on exploitation costs. Furthermore, it is an asset that produces a financial return and therefore the business environment is different.

In the latter case, six life cycle models were applied - the Lifespan and five different Economic models. For this reason, new variables and different approaches were considered.

Keywords: Life Cycle Analysis; Life Cycle Models; Physical Assets; Playgrounds; Heavy Industry Assets.

APPENDED PAPERS

The following papers are appended to this thesis:

Paper I: P. Figueiredo, J. T. Farinha, and H. D. N. Raposo, "Análise de Ciclo de Vida dos Ativos Físicos de Parques infantis – Caso de Estudo," Coimbra: APMI, 2023. – Paper to be published in next issue of Maintenance Magazine.

Paper II: P. Figueiredo, J. T. Farinha, and H. D. N. Raposo, "Otimização e Gestão do Tempo de Manutenção," Coimbra: ENEGI, 12th-13th September 2024. – Paper to be published in the proceedings of ENEGI 2024.

Paper III: P. Figueiredo, J. T. Farinha, and H. D. N. Raposo, "Modelos conceptuais de Ciclo de Vida para Equipamentos Hospitalares," Coimbra: ATEHP, 26th-27th September 2024. – Paper to be published in the proceedings of ATEHP 2024.

Paper IV: P. Figueiredo, J. T. Farinha, and H. D. N. Raposo, "Life Cycle Assessment of an Equipment in a Paper Pulp Industry" – Paper to be submitted to an international journal.

INDEX

List of Figures

LIST OF TABLES

ACRONYMS AND ABBREVIATIONS

A	Availability
ALCM	Asset Life Cycle Management
AM	Asset Management
AMMF	Asset Management Modelling Framework
APMI	Associação Portuguesa de Manutenção Industrial
BS EN	British Standard European Norm
CAPEx	Capital Expenditure
CBM	Condition-Based Maintenance
CM	Condition Monitoring
DL	Decreto-Lei
DoD	United Sates Department of Defence
EAM	Engineering Asset Management
EN	European Norm
IPDSS	Intelligent Predictive Decision Support System
ISO	International Organization for Standardization
KPI	Key Performance Indicators
LCA	Life Cycle Assessment
LCC	Life Cycle Cost
LCCA	Life Cycle Cost Assessment
LCI	Life Cycle Investment
LCP	Life Cycle Profit
LCSA	Life Cycle Sustainability Assessment
LCR	Life Cycle of Physical Assets with Recovery
MTAC	Minimization of Total Average Cost
MTAC-RPV	Minimizing Total Average Cost Reduced to Present Value
MTBF	Mean Time Between Failure
MTTR	Mean Time To Repair
NP	Norma Portuguesa
OEE	Overall Equipment Efficiency
OPEx	Operational Expenditure
PAM	Physical Asset Management
PAS	Publicly Available Specification
ROI	Return On Investment
RQ	Research Question
SLCA	Social Life Cycle Assessment

SMIT	Sistema Modular Integrado de Terologia
TCO	Total Cost of Ownership
TPM	Total Productive Maintenance
UAI	Uniform Annual Income
UK	United Kingdom
US	United Sates

LIST OF SYMBOLS

Latin Alphabet

A	Availability
A_t	Smoothing Parameter
B	Value of Benefits
C^t_{0D}	Value of the Asset
C^t_{0N}	Value of the New Asset
CE	Exploration Cost
CM	Maintenance Cost
CO	Operating Cost
dl	Annual Rate of Depreciation
€	Euro
E_t	Absolute Error
Er%	Relative Error
F	Operational Costs
GR	Global Result
h	Inflation Rate
i_A	Apparent Rate
j	Period
ipaf	Total Maintenance Cost for Daily Tasks per Intervention
ipbf	Total Maintenance Cost for Quarterly Tasks per Intervention
m	Meters
m^2	Square Meters
m^3	Cubic Meters
mdo	Hourly Labor Price
M	Maintenance Costs
M_t	Smoothing Parameter
NP	Non-production Costs
NPV_{TD}	Total Net Present Value considering Devaluation of Assets
NPV_{FpD}	Net Present Value of Financial Production Movement
NPV_{Fe}	Net Present Value of Financial Expenses Movement
NPV_{NT}	Total Net Present Value considering a New Assets
NPV_{FpN}	Net Present Value of the Financial Production Movement
p	Profit Rate
P	Present Value

P’	Present value of cession
paf	Total Maintenance Time for Daily Tasks per Intervention
pbf	Total Maintenance Time for Quarterly Tasks per Intervention
N	Lifetime corresponding to Cessation Value
r	Interest Rate
R	Risk Rate
RSME	Root Mean Square Error
S	Predicted Value
SD	Sum of the Digits
t	Period
T	Exponential Rate of Depreciation
T’	Number of Period Considered
ton	Tonne
VC	Cessation Value
VCn	Residual Value of the Equipment
Vn	Value of the Equipment in a given Period
x	Actual Value

Greek alphabet

α	Smoothing Parameter
β	Smoothing Parameter

1 INTRODUCTION

This chapter contextualizes the subject of this thesis, presents the reasons behind the research, the questions it aims to answer, the limitations found during the research and explains its structure.

1.1 Background

With the growing exigency of the economic and industrial world, the number of challenges and opportunities that arise is exponential. The truth is that in order to keep up with technological developments, whether in products or even machinery, all the processes, services and systems involved need to keep pace with this evolution.

The interest in research and development in the areas of maintenance and asset management emerged as a consequence of this necessity to keep up with the increased demands on the quality of services and products. Nowadays, however, it has also become a cause as well as a reason to raise companies' quality standards.

1.1.1 Maintenance and Management of Physical Assets

The field of maintenance has proved its worth from an engineering point of view over the last few decades. What's more, it has cemented its importance in extending the life of the countless assets that an organisation can own. In fact, we have moved from a general and secondary approach in terms of the organisation's hierarchy to creating maintenance and asset management departments.

Despite the structuring, it is inconceivable for an organisation to operate efficiently and effectively without the various departments being well connected and working as a whole. In fact, maintenance and physical asset management hold hands in order to get the best out of both areas, but it is necessary to have an in-depth knowledge of both. Only this way it is possible to combine all the factors that influence the Life Cycles of Physical Assets and thus maximise all the inherent investments.

Although Physical Asset Management is an evolution of Maintenance Management, looking at the definitions of these terms in the NP EN 13306:2021 standard, it is easy to understand the relationship between the two and the importance of synchronising these areas for an organisation: Maintenance is the combination of all the technical, administrative and management actions, during the Life Cycle of an asset, aimed at maintaining or restoring it to a state in which it can fulfil its required function. Maintenance management considers all the management activities that determine maintenance requirements, objectives, strategies and responsibilities, and the implementation of these by various means, such as planning, controlling and improving maintenance activities and economic aspects.

Physical Asset Management is based on a holistic approach to Maintenance Management. According to ISO 55000, asset management is a set of interrelated or interacting elements of an organisation to establish policies and objectives and processes to achieve those objectives.

An asset management system, on the other hand, is a management system for asset management whose function is to establish asset management policy and asset management objectives (Figure 1).

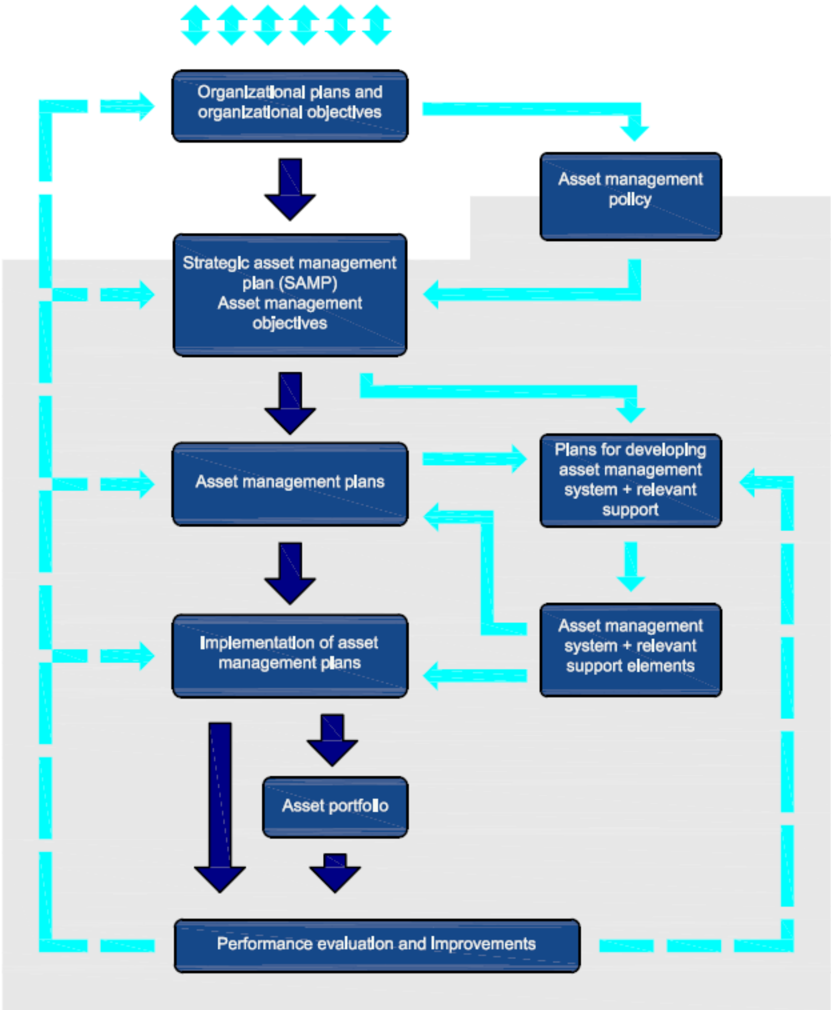

Figure 1 - The relationship between key elements of an asset management system [Source: ISO 55000]

1.1.2 Life Cycle Analysis

Within the field of asset management, there is a very important area where the focus of this thesis is centred, which is the Life Cycle Analysis of Physical Assets. The importance of this analysis is high because both facilitates and improves the management of the assets in question.

According to the EN 13306:2021 standard, the lifecycle is the series of stages through which an item goes, from its conception to disposal. Where a typical system lifecycle consists of acquisition, operation, maintenance, modernisation, decommissioning and/or disposal.

Life cycle analysis not only allows managers to better manage assets while they are still able to play their intended role for the organisation, it also allows them to make predictions for assets that will be the result of future investments. Just as it allows for this prediction, if the analysis is well executed and recurrent in the organisation, it also allows the manager to draw on asset histories for both present and future assets.

When the history exists and is well-structured it helps a lot in numerous fields of management, whether from the point of view of the analysis itself, or from countless other aspects, such as maintenance, recurring failures and the reasons for them, how the asset reacts under certain operating conditions and so on.

If the execution of an organisation's asset lifecycle analysis is implemented as an integral part of the company's working culture, the company can evolve and reach other levels of management quality. For example, according to ISO 55000, the creation of an asset management system is usually cross-functional and based on life cycle considerations; this can provide a focal point for addressing the issues of functional integration of the organisation and life cycle planning.

1.1.3 Life Cycle Models

The analysis of an asset's life cycle can be based on various methods. In this thesis, the aim has been to analyse and relate (where possible) the methods of replacing physical assets by studying useful life cycle replacement models.

According to the EN 13306:2021 standard, the useful life of an asset is defined as the "time interval from first use until the instant when a limiting state is reached". The first use does not include testing activities before the asset is handed over to the end user.

What essentially varies in the consideration of the useful life of the asset from case to case is the characteristic that defines the limit state of use. Unlike economic replacement models, what defines the limit state of use of an asset is the characteristic that is relevant to its function, which can vary, for example, among the following:

- Failure rate;
- Maintenance logistics requirement;
- Physical condition;
- Economic factors;
- Age;
- Obsolescence;

- Changes in user requirements.

Basically, the manager who analyses and pretends to estimate the useful life cycle of an asset considers which characteristic is decisive, based on the purpose for which the asset was designed and acquired. A bridge can also be established to the economic side, where the relationship between maintenance costs and the capital amortisation costs of equivalent new equipment is considered, but this method will be dealt with in greater detail in the course of this thesis.

Another model that will be discussed in the course of this thesis is the Economic Life Cycle Model. As this is a model that can be calculated in various ways, including or not including various quantitative parameters that allow the problem to be contextualised and made more precise, the main objective is to rationalize the expenses associated to the operation of the asset.

[1] considers that economic replacement models make it possible to determine the most appropriate time for decommissioning or renewing equipment, which minimises the average total cost of operation, maintenance and capital tie-up.

Therefore, these models help the manager to create action plans and outline maintenance and exploration strategies to maximise all the company's investments in physical assets.

1.2 Purpose and objectives

The main purpose of this thesis is to be able to determine the best model for life cycle modelling, depending on the type of asset being studied. Choosing the right life cycle model for the asset under study will allow to establish the right time to withdraw or to renew the equipment. Each case is different, so it is also important to treat the initial data and considerations. Such as analysing the investment risk from an operational point of view and possible algorithmic and statistical analyses that enable missing data to be arbitrated.

1.3 Research Questions

RQ1 – What is the most adequate Physical Asset Life Cycle that best supports children's playground?

RQ2 – What is the most adequate Physical Asset Life Cycle that best supports a specific asset of a heavy industry?

RQ3 - What is the right time to withdraw or to renew a children's playground to minimize the waste and increase sustainability?

RQ4 - What is the right time to withdraw or to renew a specific asset of a heavy industry to minimize the waste and increase sustainability?

1.4 Thesis Structure

This thesis is organised as follows:

- Chapter 1. Introduction: This chapter presents the theme of the thesis, the objectives, the research questions to be answered and the structure of the work.
- Chapter 2. State of the Art: This section presents the state of the art of the various key points on the theme of the thesis and fundamental aspects for the possible answer given to the research questions.
- Chapter 3. Case Studies: All the work carried out in this thesis is presented in this chapter. The work is divided into two distinct case studies, but where it is possible to find similarities and enriching comparisons for understanding the thesis theme.
- Chapter 4. Conclusion and further research: The conclusions of the whole thesis are drawn in this chapter. In addition, suggestions are made for the continuation of the work carried out through future research.

2 STATE OF THE ART

This section presents the state of the art of the life cycle models applied and analysed in depth during the writing of this thesis. In this way, the aim is to make it easier to understand the current state of scientific knowledge in this area of study, allowing the work carried out to be perceived from the same starting point.

2.1 Engineering and Physical Assets Management

The engineering and management of physical assets is a growing area, both in terms of methodologies and research and in terms of its importance to organisations. It is very broad and operates in various sectors, from financial management to data analysis, logistics, sustainability and metrology.

Engineering Asset Management (EAM) as a discipline addresses the value contribution of AM to an organization's success. The EAM system can be roughly defined as: "The system that plans and controls the asset-related activities and their relationships to ensure the asset performance that meets the intended competitive strategy of the organization". This system has significant potential to influence all aspects of asset's life cycle activities from concept design to disposal. The AM activities focus on controlling the life cycle activities of assets, but their nature is both interdisciplinary and collaborative [2], [3].

Specifically, for EAM, the objective can be considered from various points of views. [2] asks the following question: "How can an intervention in the processes that relate capacity and financial values be effective in achieving a given objective, such as increasing the level of capacity or reducing costs?".

Management takes place at different levels of an organization, and therefore inevitably has an influence at various points, such as on the ability to provide services and the value profile that asset managers are concerned. [2] says that "an engineer involved in monitoring conditions very close to the basic engineering process may look at a specific point in a complex process, which may have an impact on the connected values in only a small part. An information manager may be concerned with providing the data to support measurements of relationships. An accountant may be interested in how costs are caused by the operation of assets and a human resources manager may be concerned about the health and safety issues arising from the process. Senior management may be concerned with overall profitability, long-term strategies, the life cycle of the asset and its relationship to the organization's policy. The different points of view of managing the capability-value profile, relating to the different levels of the organization, are thus governed by the fundamental decision-making categories in the respective strategic, tactical and operational contexts." The author stated that the EAM needs an information system to be able to gather data that can be used to support decision-making [2].

[4] stated that EAM is a multidisciplinary approach that includes management, economics and information technology. Like Amadi, the author agrees that EAM decisions cover a wide range of disciplines, from operational to strategic aspects.

According to [5], "EAM is concerned with the life cycle management of "engineered" physical assets in order to achieve the business objectives for an organization that may own or manage an asset. EAM becomes a vital part of business management for many organizations especially when capital investment in plant equipment or infrastructure is significant and the productivity/sustainability of the asset is crucial to the competitive capability of the business.

Most of the literature of EAM is focused on two important aspects: the technology and communication technology required in the management of data relating to assets and the decision-making techniques in the management of the engineered assets. EAM concerns the productive use of those assets that provide the value supporting all assets in the economy. It is thus essential to all that it is carried through as effectively as possible" [2], [4], [5].

In fact, holistic knowledge of these areas promotes integrated strategic asset management, seeking to answer the various challenges that arise in the daily lives of organisations and society.

[6] agree that the evolution from maintenance management to physical asset management promotes a holistic view of managing the life of physical assets. And that this management of physical assets is fundamental to the correct development and implementation of asset management in ISO 5500x standards.

[5] claim that the concept of Physical Asset Management (PAM) has gained considerable prominence in various industries around the world. One of the very important reasons behind this is that many organizations and industries found that to achieve the organization goals; one must manage the physical assets effectively and efficiently. The authors believe that this is because companies are beginning to realise how important efficient and effective management of their physical assets can be in achieving their objectives. In fact, greater efficiency in physical assets leads to a higher level of production and revenue. This consequently minimises operating and capital costs.

PAM has become an integral part of the maintenance areas of organisations, and the aim of maintenance is to keep the physical asset within the operating conditions for which it was conceived. [5], [7], [8], [9].

In conformity with the observations made by Farinha *et al.* on the article by Katičić *et al.* "Asset Management is a relatively new discipline that provides methods and tools for effective management of Physical Assets to maximize their utilization during entire Life Cycle. Asset Management evolved from Maintenance Management to provide a holistic approach to manage the life of a physical asset. This management is important for the performance of any organization, particularly Physical Asset intensive organizations. Therefore, it is no wonder that asset management professionals around the world work on the development of new models and concepts such as engineering asset management, integrated strategic asset management or asset governance, in order to find the best solutions to meet the changing nature and challenges of organizations and society. Today, we recognize that asset governance is a key point for leading role in the development and implementation of asset management in the company and it is evidence in PAS 55 and the ISO 55000 standards" [6], [10].

2.1.1 Life cycle study of physical assets

In the field of physical asset life cycles models, the progress made in recent years has been remarkable. As well as constantly evolving calculation methodologies, there has also been an evolution in the accuracy of terminology in the subject. Various authors have used terms such as Life Cycle Cost (LCC), Life Cycle Assessment (LCA) or Life Cycle Cost Analysis (LCCA), among others.

[11] studied "the dimensions of sustainability that addressed the most within international standards and thereby present a life cycle perspective. The intent is to support Life Cycle Assessment (LCA) practitioners in identifying case by case the standards to be used and the methods of considering life cycle concepts beyond environmental aspects". The authors found that the "results showed that the dimension of sustainability addressed most frequently is the environmental dimension, and it is associated with LCAs and ecological labels. The second dimension of sustainability is the economic dimension, and it is associated with technology development and the adoption of life cycle costing". And conclude that their study "shows that (i) the concept of life cycle is present in several standards with different meanings and applications, (ii) all the dimensions of sustainability are considered by the totality of the analysed standards, and (iii) the environment is the dimension addressed the most".

In accordance with the findings of [12],"the history of the application of Life Cycle Costing (LCC) began in the UK in the late 1950's." Although the document is aimed at LCC applied to buildings, it is possible to draw cross-cutting conclusions on other types of physical assets. Goh & Sun said that "major improvements are necessary to make LCC comparable with common economic evaluation methods (e.g. benefit-to-cost ratio, net benefits and savings-to-investment ratio, for capital investment analysis related to buildings)".

[13] suggest that "to improve the design of products and reduce design changes, cost, and time to market, concurrent engineering or life cycle engineering has emerged as an effective approach to addressing these issues in today's competitive global market. As over 70% of the total life cycle cost of a product is committed at the early design stage, designers are in a position to substantially reduce the life cycle cost of the products they design, by giving due consideration to life cycle cost implications of their design decisions." Then the authors add that "methodologies and tools are needed to directly provide cost information to designers. Life Cycle

Cost (LCC) analysis provides a framework for specifying the estimated total incremental cost of developing, producing, using, and retiring a particular item. This paper looks at the issues of LCC analysis and the tools that have been developed to provide engineers with cost information to guide them in design".

According to [14], "Life Cycle Costing (LCC) was initially used by US Department of Defence (DoD) to seek optimal costs for acquiring, owing and operating an equipment during its useful life (also including any disposal costs)". The authors add that "these costs calculation methods usually do not include the three performance parameters (quality, productivity and availability) of the Overall Equipment Efficiency (OEE) measure, or lost profit, although Life Cycle Profit (LCP) were introduced already 1983 in literature".

[15] emphasises the importance of analysing the life cycle of physical assets: "Investing in hospital infrastructure is not just a financing activity. It is important to consider the whole life cycle of an asset. For example, it is necessary to think about the operating life of an asset before building it, because this can influence investment costs and follow-up costs substantially". Of the 12 investment models presented by Eicher, although applied to hospitals, some can be extrapolated to other areas and organisations. This document also discusses maintenance models, such as insourcing versus outsourcing.

[16] claim "The subject of Life Cycle Cost (LCC) has a lack of research along time. Instead of LCC it also may managed as Life Cycle Assessment (LCA). Rarely it is treated like Life Cycle Investment (LCI). The problem of Life Cycle Cost (LCC) has received very little attention along time. LCC is also known as Life Cycle Assessment (LCA), or more rarely it is called Life Cycle Investment (LCI). By consequence, the perspectives and activity sectors from which the Physical Assets are seen may provide a specific approach".

The truth is that from the point of view of the organisation's management and growth, as well as the way it looks at these purchases, it is important to refer to the expenditure associated with physical assets as investments and not as costs. Hence the term Life Cycle Investment (LCI). Although it is still a relatively unused term compared to others, it has been increasingly recognised over time. Because when it comes to the reality of this type of problem, the point of view adopted in the management approach is crucial to the path to be followed, the way to grow and, consequently, the results to be obtained. Nevertheless, despite the terminology, the study of the life cycles of physical assets has been deepened over the last few decades.

In assessing the life cycle sustainability of products, [17] highlights the Life Cycle Sustainability Assessment (LCSA) model, which is the sum of the Life Cycle Assessment (LCA), plus the Life Cycle Cost (LCC), plus the Social Life Cycle Assessment (SLCA).

According to [18], "from the European Union Journal 'life-cycle' means all consecutive and/or interlinked stages, including research and development to be carried out, production, trading and its conditions, transport, use and maintenance, throughout the existence of the product or the works or the provision of the service, from raw material acquisition or generation of resources to disposal, clearance and end of service or utilisation.

According the same journal, there is a concern on the life cycles when contracting; it states the following:

1. Life-cycle costing shall to the extent relevant cover parts or all of the following costs over the life cycle of a product, service or works:

a) costs, borne by the contracting authority or other users, such as:

i) costs relating to acquisition;

ii) costs of use, such as consumption of energy and other resources;

iii) maintenance costs;

iv) end of life costs, such as collection and recycling costs.

b) costs imputed to environmental externalities linked to the product, service or works during its life cycle, provided their monetary value can be determined and verified; such costs may include the cost of emissions of greenhouse gases and of other pollutant emissions and other climate change mitigation costs.

2. Where contracting authorities assess the costs using a life-cycle costing approach, they shall indicate in the procurement documents the data to be provided by the tenderers and the method which the contracting authority will use to determine the life-cycle costs on the basis of those data.

The method used for the assessment of costs imputed to environmental externalities shall fulfil all of the following conditions:

a) it is based on objectively verifiable and non-discriminatory criteria. In particular, where it has not been established for repeated or continuous application, it shall not unduly favour or disadvantage certain economic operators;

b) it is accessible to all interested parties;

c) the data required can be provided with reasonable effort by normally diligent economic operators, including economic operators from third countries party to the GPA or other international agreements by which the Union is bound.

3. Whenever a common method for the calculation of life-cycle costs has been made mandatory by a legislative act of the Union, that common method shall be applied for the assessment of life-cycle costs".

Beginning with the creation of Total Productive Maintenance by [19] in Japan, where the author already referred to LCC as an important parameter for the implementation of this system, as well as the need to study it further. According to Nakajima, the Life Cycle Cost (LCC) (the cost incurred during the equipment's lifespan) required to maintain equipment at its optimal level is limited. TPM strives to achieve overall equipment effectiveness by maximizing output while minimizing input, i.e., LCC.

Moving on to the definition of Terotechnology, according to [20], it is "the technology of installation, commissioning, maintenance, replacement, and removal of plant machinery and equipment, with feedback on the operation and design thereof and on related subjects and practices. Terotechnology is the maintenance of assets in an optimal manner. It is the combination of management, financial, engineering, and other practices applied to physical assets such as plants, machinery, equipment, buildings, and structures in pursuit of economic life cycle costs. It is related to the reliability and maintainability of physical assets and also takes into account the processes of installation, commissioning, operation, maintenance, modification, and replacement. Decisions are influenced by feedback on design, performance, and cost information throughout the life cycle of a project. It can equally be applied to products, as the product of one organization often becomes the asset of another".

According to [21], it is possible to find some similarities between the Terotechnology described by Husband and the Total Productive Maintenance (TPM) explained years earlier by Seiichi Nakagima; although the TPM also focuses on motivating the people who make up the organisation.

Inspired by these concepts, [22] developed a new approach, extracting the technological component from Husband's Terotechnology, creating the new concept of Therology which, according to Farinha, "is defined as the combined utilization of operational research techniques, information management, and engineering, with the objective of accompanying the life cycle of facilities and equipment; it includes the definition of specifications of purchase, installation, and reception, and also the management and control of its maintenance, modification, and replacement and its accompanying in service, too".

Farinha states that "from the point of view of implementation, Therology must be analysed using its strategic, tactical, and operational aspects:

- Strategic - Is related to new procurement policies, definitions of methodologies of management, control of the life cycle of physical assets, the necessary human and material resources, and their replacement.

- Tactical - Is related to the standardization problems of the physical asset, its maintainability and reliability, cost control of the several resources involved, and personnel training, among others.

- Operational - Is related to maintenance itself, that is, to guarantee the normal operation of physical assets by planning and controlling interventions. These, in turn, provide technical and economic data, which provide several indicators necessary to evaluate the performance of the defined strategy."

[1] assumes that the broad view of physical asset management that underpins the overall approach of his book is based on maintenance management itself, but with a broad view of its Life Cycle.

[10] state that "It is necessary to determine the best maintenance strategy for each physical asset to reach the desired Return on Investment (ROI) and the corresponding availability. Maintenance can be systematic, condition monitoring, predictive and so on. The documents accompanying new equipment usually recommend scheduled maintenance, based on time or some functioning variable; maintenance following this type of recommendation usually has a well-defined budget along the equipment's life cycle. However, availability can be increased using other types of maintenance, for example, condition monitoring".

[23] refer that "the high costs in maintaining today's complex and sophisticated equipment make it necessary to enhance modern maintenance management systems". In order to support conventional Condition-Based Maintenance (CBM), the authors implemented the Intelligent Predictive Decision Support System (IPDSS), which "add the capability of intelligent condition-based fault diagnosis and the power of predicting the trend of equipment deterioration". The authors conclude that the "results showed that the IPDSS model provided reliable fault diagnosis and strong predictive power for the trend of equipment deterioration. These valuable results could be used as input to an integrated maintenance management system to pre-plan and pre-schedule maintenance work, to reduce inventory costs for spare parts, to cut down unplanned forced outage and to minimise the risk of catastrophic failure".

[24] emphasize the relevance of LCC evaluation in all areas where physical assets are key. [25], in their turn, said: "Cost-competent maintenance and management of civil infrastructure requires balanced consideration of both the structure performance and the total cost accrued over the entire life-cycle. Most existing maintenance and management systems are developed on the basis of life-cycle cost minimization only. The single maintenance and management solution thus obtained, however, does not necessarily result in satisfactory long-term structure performance". So, the authors analysed "the recent development of life-cycle maintenance and management planning for deteriorating civil infrastructure with emphasis on bridges using optimization techniques and considering simultaneously multiple and often competing criteria in terms of condition, safety and life-cycle cost. This multiple-objective approach leads to a large pool of alternative maintenance and management solutions that helps active decision-making by choosing a compromise solution of preferably balancing structure performance and life-cycle cost".

In line with the conclusions drawn by [26], "Life Cycle Costing (LCC) is a way of thinking where attention is paid to the total costs that occur during a product's entire life cycle". The authors also said that "in addition to the estimation of future costs, an essential feature of LCC is cost monitoring during a product's life cycle," and "the sum of life cycle costs of many products often substantially exceeds the initial purchase price". Likewise, [27] state that "Life cycle costing is a powerful technique that supports the analytical processes by which managers can make the most cost-effective decisions on options presented to them at differing life cycle stages and at different levels of the life cycle cost estimate".

[28] discuss "the relations among some technical maintenance Key Performance Indicators (KPI's), specifically Mean Time to Repair (MTTR), Mean Time Between Failure (MTBF), Availability (A), and the dimension of the reserve fleet. The Return On Investment (ROI), a financial indicator to assess the equipment's financial performance, is used as a "bridge" between the maintenance and the economic fields, showing us when the equipment starts creating profit or loss to the company. It looks at models to determine the influence of these variables on the withdrawal time and size of the reserve fleet and uses oil analysis as an example of how condition monitoring may influence the availability of the whole bus fleet and the size of the reserve fleet".

To quantify ownership costs and ongoing operating costs, Malano *et al.* [18], [29] use an asset management modelling framework (AMMF). A Life Cycle Cost (LCC) model for irrigation and drainage assets.

"An asset management modelling framework consists of two main components:

1) A database of assets consisting of geographical location of assets, design features, maintenance records and asset condition and performance;
2) An analysis module which enables the modelling of future asset strategies including the calculation of future liabilities and life cycle asset costing associated with alternative courses of action".

[30] relate the Life Cycle to products, "relevant information which can be used to assess the subsequent maintenance costs are requirements for product life cycle, e.g. service costs according to Total Cost of Ownership (TCO) as well as serviceability and maintainability of the product. Furthermore, organizational framework conditions for the service are defined in the planning phase." The authors state that that methodology "will help to integrate the perspective of service and lifecycle costs more into product development. By analysing the costs of maintenance services throughout the entire development process, it is possible to identify possible effects and the top causes of costs at an early stage".

Farinha *et al.* briefly analyse some of the statements made by Bengtsson & Kurdve: "Bengtsson & Kurdve present an LCC or TCO analysis of machining equipment in a Swedish company and discuss life cycle profit (LCP). A company with a low LCC does not necessarily have a high Life Cycle Profit (LCP). LCC is centred on the costs. However, if the asset's owner (LCC/TCO) is a company, the LCC must be analysed simultaneously with the benefits, i.e., the physical assets's production results, suggesting that the asset's life cycle must be seen from an investment point of view. The authors also present theory on LCC and life cycle profit (LCP). They note the confusion around the LCC concept: "LCC and TCO have often been defined in similar ways and both may include not only cost aspects but also LCP, performance and profit aspects." The authors add, "There are a number of different options in working to achieve a high LCP". Furthermore, "reducing LCC can be one option; however, sometimes it might be of value to increase LCC in order to reduce or eliminate losses that will increase LCP more than the increases in LCC". This comment can be directly related to such maintenance policies as condition monitoring, including predictive maintenance, among others. Based on these authors' comments, it seems necessary to clarify the differences between LCC and the investment cycle" [10], [31].

Therefore, throughout the history and evolution of maintenance and asset management, the need to maximise investments in physical assets and the study of their life cycles has been considered. Terms that gained even more prominence when the ISO 55000 standard was approved in 2014, which refers to the overview, principles and terminology of asset management, as well as ISO 55001 and 55002, which refer to its requirements and implementation (respectively).

Several authors have tackled the subject of physical asset management and, consequently, Life Cycle Assessment (LCA) on the basis of the aforementioned standard. The principles of physical asset management, covering all stages of the asset life cycle, in the words of [32], "it starts with initial business appraisal and proceeds through identification of fixed asset needs, capability gap analysis, financial evaluation, logistic support analysis, life cycle costing, management of in-service assets, maintenance strategy, outsourcing, budgeting, cost-benefit analysis, disposal, and renewal. Industries to which this is applicable include: manufacturing; distribution; mining; electricity generation and supply; oil and gas; water; roads; railways; aviation; shipping; hospitals; retail centres; defence material; recreation and sporting facilities and local government".

This view reinforces the need for holistic knowledge in asset management. But it's not just Hastings who considers it; [2] says that the "first impetus for holistic approach arises from renewed emphasis on value-based performance measures. This makes it imperative for practitioners to demonstrate that physical asset management provides value to the associated business venture. Many decisions at the strategic level as to whether to acquire, buy, or sell a business venture depends on how effective the ownership, management and utilisation processes are at creating, or at least, sustaining the value profile as defined by the stakeholders. The primary objective for relevant due diligence studies is to establish to what extent prior or concurrent ownership, management and utilisation efforts have added, sustained or destroyed value to the physical asset base".

[33] also points out, with regard to the approach to asset life cycles, that "the view here is that it should be about creating and sustaining value, during each respective stage, and through all the life-cycle stages of an asset. Physical asset management involves a wide range of disciplines and processes covering the life-cycle stages of creating, establishing, exploiting and divesting a physical asset in a balanced manner

to satisfy the continuum of constraints imposed by business strategy, economy, ergonomics, technical and operational integrity, and regulatory compliance. Maintenance and Terotechnology are among the core competencies that form the necessary capabilities for physical asset management".

[34] show that investment in condition monitoring maintenance leads to greater availability and increased electricity production from wind energy systems. In addition, there is an increase in the quality of maintenance planning.

[35] realised that the "asset management is often one of the last options to maximise cost savings in a competitive global economy due to its intrinsic complexity, especially in many developing countries. Asset management in the process industry must consider the commissioning, operational and end-of-life phases of physical assets when commencing a design and implementation project. However, current asset management models show inefficiencies in terms of addressing life cycle costs comprehensively, as well as other aspects of sustainable development. An Asset Life Cycle Management (ALCM) model is subsequently proposed for assets in the process industry, which integrates the concepts of generic project management frameworks and systems engineering with operational reliability in order to address these inefficiencies."

Therefore, Schuman proposed a "holistic asset life cycle management (ALCM) model for physical assets in the process industry by aligning and integrating the relevant elements of project management, logistics engineering, systems engineering, maintenance management and life cycle costing. In its present form the ALCM model optimises the maintenance prevention process during the acquisition phase, thereby reducing maintenance costs during the utilisation phase".

In truth, the main objective of any asset manager is to increase the life cycle of their equipment. Managers can and should do this by investing in maintenance. According to [36], this can be done through condition monitoring techniques and tools, which also allow to increase availability. [37], in addition to condition monitoring techniques and tools, also considers the analysis of economic advantages. [38], in addition to addressing condition monitoring techniques and tools and analysing the economic advantages, also discusses the variable investment in condition monitoring throughout the life of the asset.

2.1.2 Life Cycle Models

Despite the constant and abrupt evolution of technologies and the way we look at physical assets, it is necessary to understand these different approaches and concepts before we can understand the holistic vision. As all the parts together make up the whole, it is necessary to understand their importance and connection.

[39], [40], [41], using a methodology based on the ant algorithm, demonstrated that in maintenance management, maintenance policy and logistics are fundamental to maximise investment in the asset's life cycle. In this paper, the authors also present

the maintenance software SMIT (Sistema Modular Integrado de Terologia). This system includes the traditional maintenance management modules, such as Work Orders, Maintenance Plans, Spare Parts, Technicians, Tools, Maintenance Objects, Fault Diagnosis, among other modules. In fact, some of the most important data for optimising asset lifecycles [42].

From the point of view of life cycle models, it is possible to assess the life of a physical asset from the point of view of useful life and from the point of view of economic life. All these models promote the possibility of determining an optimal period for replacing or decommissioning assets.

Before the publication of EN 13306:2021, where the definition of the limit state of useful life is explicit, it was usual to mix up the terms "useful life" and "economic life", almost as if they were seen from the same point of view and with transversal criteria.

According to [43], to find the most rational time to replace an equipment, according to the criterion of economic life, four situations could be considered:

i. When the asset is already unsuitable for work;
ii. When the asset has reached its lifespan;
iii. When the asset is already obsolete due to technological advances;
iv. When there are more efficient and economical solutions.

[44], refer that the following situations could be considered:

- Availability of new technologies;
- Compliance with safety standards or other mandatory requirements;
- Availability of spare parts;
- Obsolescence that may limit the asset's use.

As mentioned earlier in this thesis, all these terms are considered to be situations that support the criteria of useful life analysis models. [28] refer that "the evaluation of the equipment economic lifespan is another common method to estimate the withdrawal time: i.e., when the equipment maintenance costs exceed the cost of maintenance plus the capital amortization of new similar equipment".

[45] said that "The main goal of the Life Cycle Costing approach is to optimize life cycle costs of the assets or investment project without loss their performance", and that the main categories of LCC are the following:

- investment (acquisition) costs;
- operation costs;
- maintenance costs;
- renewal costs;

- disposal (retirement) costs.

[10], [46] says that "the life cycle cost of an item is the sum of all funds expended in support of the item from its conception and fabrication through its operation to the end of its useful life". To this the author adds, "Life cycle costing is concerned with optimizing value for money in the ownership of physical assets by taking into consideration all the cost factors relating to the asset during its operational life". The author considers and divides the LCC variables as follows: initial capital costs; useful life of the asset; discount rate; operating and maintenance costs; disposal costs; information and feedback; uncertainty and sensitivity analysis. All these variables represent costs.

According to [47], companies want to keep their standards above those of their competitors, always focusing on achieving a leading position. However, in order to maintain these standards, a culture of continuous improvement is needed in all processes.

Therefore, being able to find the optimum replacement time for an asset can set the competitive advantage between different organisations by reducing the costs associated with the maintenance policy used to extend the asset's useful life. Companies' commitment to rationalising costs, including maintenance costs, is decisive, particularly in the field of energy efficiency.

"Raposo *et al.* present a importance of the investment analysis in life cycle cost using econometric models, to support this choice the author stats that equipment replacement is a broad concept that ranges from the selection of similar assets, to replace existing ones, to the evaluation of assets that act in completely different ways in the performance of the same function, such as more energy efficient equipment" [18], [47].

[28], [48], [49], [50], [51], [52], [53], [54] presented an integrated econometric model for bus replacement and determination of the size of the reserve fleet based on predictive maintenance. The author quoted that "Condition Monitoring (CM) represents an approach to preventive maintenance based on knowledge of the equipment's condition determined by monitoring one or several parameters that permit us to evaluate the equipment's health. The maintenance of a passenger bus is a strategic activity to maximize its life cycle, involving a combination of management, technical and economic actions to achieve high availability at reasonable costs".

[28], [48], [49], [50], [51], [52], [53], [54] also said that "the Life Cycle Cost (LCC) of an asset represents the sum of all capital spent to support it from design and manufacturing, through its operation until the end of its life, (CAPEx+OPEx – Capital Expenditure+Operational Expenditure). The LCC of an asset can be significantly higher than the value of the initial investment and is usually defined at the design phase. Bescherer [49], [50], [51], [52] says as much as 70-90% of the total LCC costs are defined at this stage. To this, Aoudia *et al.* (2008) add that poor maintenance management contributes to a significant increase in LCC".

Often the predominant decision-making criterion when acquiring a physical asset is the initial investment. Not forgetting the likely absence of reliable data from the past. The truth is that there are few cross-case studies, and those that do exist are aimed at one single industry [28], [54]. [53] provide an overview of the use of LCC and the viability of its implementation, by summarising work on its application.

[28], [55] said that new management models are needed to improve equipment productivity and quality of service, with aspects like environmental sustainability, quality management standards, security, maintenance and energy included in the models. Many companies keep equipment in operation, even when it is no longer economically viable, simply because it isn't considered over its entire economic cycle.

[28], [56] say that traditional production systems are built on the principle of the economy of scale. The authors illustrate an equipment replacement problem in the context of Lean Thinking, showing the relevance of econometric models.

[55] presents some econometric models to improve the analysis of the LCC, considering the withdrawal time for medical equipment.

By combining known models of equipment replacement in continuous and discrete time, [57] found that the optimum equipment lifetime is shorter when the technological change incorporated is more intense. Which leads us to believe that the exponential technological advancement of recent decades requires greater care with the maintenance and analysis of asset life cycles.

[58] considers that the value of an asset is established by the expected cash flows of future benefits referred to the present value through a discount rate that reflects the risk of the decision. Taking this into account, [28] states that "methods considering the value of money over time are the most suitable to use in replacement decisions". [28], [59] considers that when analysing a company's operating activities with repeatable investments, the Uniform Equivalent Annual Cost approach is appropriate. Since the main objective is to determine the period with the lowest equivalent annual cost, i.e. the best technical replacement period according to the method, standardising the investment based on equivalent annual values simplifies the analysis when deciding. The capital recovery factor is the focus of the equivalent annual cost method. One of the great advantages of this type of methodology is the possibility of being able to compare multiple investment options in order to improve decision-making when replacing or not replacing a physical asset. To do this, it is necessary to collect certain information, such as [28], [60]:

- the acquisition value;
- the maintenance cost;
- the resale value or residual value at the end of each year;
- operating costs;
- the capital cost or minimum attractive rate.

According to [28] "When the equipment enters the final phase of its LCC, it is important to determine the most rational time to withdrawal it. Several variables are important in such evaluation, including:

- Purchase price of new equipment;
- Withdrawal value;
- Operating costs;
- Maintenance costs;
- Inflation and capitalization rates."

Sometimes, it is possible to obtain historical data that provides the manager with information on how the physical asset depreciates over time. However, it is often not possible to access this history. Therefore, according to [28], [61], it is possible to simulate this depreciation using the following methods, applying the most appropriate to each asset:

- Linear method - devaluation is constant over the years;
- Exponential method - devaluation decreases exponentially over the years;
- Sum of digits method - devaluation is not linear over the years, but is less than exponential.

[1] lists three common ways to determine the economic cycle for equipment replacement:

i. Income Annual Uniform Method;
ii. Minimizing Total Average Cost Method;
iii. Minimizing Total Average Cost Reduced to Present Value Method.

2.2 Problem Statement

One of the objections often raised when the subject of Life Cycle Models for replacing physical assets is introduced is the lack of comprehensiveness and methodologies that are transversal to any type of asset. This issue has been much debated in recent papers (2023).

[62] have endeavoured to "to determine whether the current studies have adequately addressed the gap in knowledge by effectively implementing Life Cycle Assessment (LCA) methodologies throughout the entire lifecycle of buildings while ensuring a sufficient level of detail and reliability in their findings". The authors conclude that, in relation to existing studies on LCA at the urban scale in the context of the built environment, there are still many advances to be made in order to improve the quality of the application of the models. Some of the paper's conclusions: "the majority of studies primarily focus on individual buildings, neglecting the evaluation of other urban elements such as transportation facilities, network distribution

systems, and green spaces"; "when employing bottom-up methodologies for a building-by-building approach, many studies encounter challenges related to data availability for all urban objects. Consequently, probabilistic methods have been employed to address this issue, leading to notable uncertainties"; "it is essential to acknowledge that the chosen district for LCA implementation may undergo significant changes over its lifespan".

[63] give a review of the literature covering the period 1994 to 2022, focussing on environmental life cycle assessment (LCA) applied to buildings and the rehabilitation of architectural structures. The authors address the following topics in their paper:

- Life Cycle Assessment;
- LCA and Life Cycle Cost Assessment;
- LCA and life cycle cost analysis;
- LCA and social life cycle assessment.

[64] wrote a paper on "the possibilities of using life cycle analysis to design specific integrated systems", with the aim of "summarize the life cycle assessment studies of stormwater management facilities so that they can be applied in the future as a basis for the design of efficient and sustainable drainage systems in urban areas".

[65] discuss "the role of LCA in evaluating and shaping strategies on the decarbonisation of energy systems, circular economy, sustainable consumption and sustainable finance". The authors also explore "how emerging LCA-based approaches make use of the planetary boundaries framework and other environmental assessment tools to support decisions", and state that "cross-comparisons between LCA applications for various mitigation strategies reveal differences in maturity level, methodological choices and the way that environmental assessment tools have been combined with LCA". Although the authors consider that economic LCAs on the decarbonisation of energy systems and sustainable consumption are common, they also state that economic applications to the circular economy and prospective LCAs for sustainable finance are poorly used. The authors believe that future research "should develop systematic classification of decision-support problems, harmonized data and comprehensive guidance to improve robustness and credibility of prospective economy-wide LCA".

In fact, analysing the state of the art on this subject, this gap can be seen, although some recent works have already tried to bridge it, and have made significant progress. This is the case with the paper by [16] mention that "The models were developed and validated in several areas, namely in the public transport of passengers, water field, and so on.

It is based on the work developed along time that the next section presents a synthesis of those models and also proposes new models that have the potential to be applied to any type of Physical Assets, filling an existing gap in the state of the art". Therefore, in the same article, the authors develop the following methods in order to "maximize the use of Physical Assets with the objective to diminish waste and increase sustainability":

- Uniform Annual Income Method (UAI) - with and without reduction to Present Value;
- Minimization of Total Average Cost Method (MTAC), with and without reduction to Present Value;
- Life Cycle Investment (LCI);
- Life Cycle of Physical Assets with Recovery (LCR).

The UAI and MTAC methods are detailed in the first case study described in this thesis "3.1 Assets of a children's playground" and the LCI and LCR methods are explained in the second case study "3.2 Assets of a heavy industry (*Celbi*'s Wood Debarking and Chipping Line)".

3 CASE STUDIES

The work carried out on this thesis began with its preparation at the end of the previous academic year (2022/2023). This preparatory work, in addition to reviewing and deepening the knowledge acquired about physical asset management and the consequent life cycle analysis, led to the preparation of the first case study of this work "Análise de Ciclo de Vida dos Ativos Físicos de Parques infantis – Caso de Estudo". After its presentation at APMI's 17th National Maintenance Congress and 10th Maintenance Meeting of Portuguese-speaking Countries, the preparation and execution of the second case study "Life Cycle Analysis of *Celbi's* Wood Debarking and Chipping Line" began.

Both case studies are described in this chapter.

3.1 Assets of a children's playground

The initial preparatory work for this thesis ended up becoming part of it. In addition to the study and research inherent in any project of this scope, it was considered a good option to stimulate, from the beginning, not only the cognitive domain, but also that of know-how, which is characteristic of engineering areas. It was therefore decided to apply the knowledge acquired in academic terms to a real situation in the context of the acquisition of playground construction/maintenance contracts.

A task that was initially intended to be a consolidation of knowledge, relatively more challenging than what had been done so far in an academic context, turned out to be an important piece of work. Either for the engineering and physical asset management field, or for society itself, since the research focused on some aspects typical of these contracts, which, since they are the responsibility of the state, are relevant to all taxpayers. Therefore, this case study has not only made it possible to consolidate and expand upon knowledge at the time, but it has also developed methodologies that can be used and improved, becoming decision-making tools for political organisations. As well as so that taxpayers themselves can also be aware of the decisions taken and what they are based on.

This case study focused on analysing the life cycle of the playground on Avenida Padre Américo in the municipality of Miranda do Corvo, in the district of Coimbra, Portugal. Through the *Base.Gov* platform, it was possible to obtain some relevant information from the call for tenders for the "Requalificação do Parque Infantil da Avenida Padre Américo" which took place in 2022, such as the acquisition cost, which was considered to be the initial investment - this parameter is crucial for the application of life cycle models.

But more than the application of the methodologies of these models, the major contribution of this work lies in its contextualisation and the considerations taken, bearing in mind the shortage of information found for the calculation of the life cycles of these physical assets. These aspects will be explained below.

The main purpose of using these models is to maximise the investments that inevitably must be made. However, this maximisation is often considered mainly from the point of view of financial return. Especially considering that from 2020 to 2022 at least 690 000.00€ was invested in upgrading or building playgrounds. This figure is based on the data collected for this case study, where only 18 contracts were counted (one playground in each district of Continental Portugal). The total value will be immeasurably higher if it is considered that there are more than 28 playgrounds in the Municipality of Miranda do Corvo alone, and all of them at national level require maintenance in order to be in compliance with Decree-Law 203/2015 and the European standards BS EN 1176:2017 ("Playground equipment and surfacing - Part 1: General safety requirements and test methods") and BS EN 1177:2018 ("Impact attenuating playground surfacing - Methods of test for determination of impact attenuation"). The content of these three documents was crucial to the development of the maintenance plan, which will be presented later in this chapter.

Nevertheless, many of the assets present in society's daily life, especially in the field of public responsibility, bring social returns. These returns, rather than being important from a political-electoral point of view, are reflected in the quality of life of the individuals who use these types of assets - in this particular case, children.

3.1.1 Return on investment from the playground

But how can the social return be translated when the investment is financial? To what extent is it worth building, requalifying and/or maintaining this type of asset? When and where should this type of intervention be carried out? These are answers that those responsible for making these kinds of decisions should be able to answer and, above all, justify.

It would be possible to quantify the level of satisfaction through surveys. However, since they would be voluntary, it could lead to false overall evaluations of the park if based on very specific events. In any case, it would be possible to survey the number of users of the playground, which would already be excellent data to cross-check with the evaluations in the surveys.

So, the major difference seen in this case study, compared to other applications of life cycle modelling for physical assets that have a directly measurable financial return, is the need to take some initial considerations and ponder future methodologies that can respond to these difficulties.

The great advantage of this case study is that it concluded that despite all the difficulties inherent to the typology and context of these physical assets, it is possible to apply these models in a consistent manner.

3.1.2 Life Cycle Models applied to the Padre Américo Avenue Playground

Methodologies from two types of life cycle models were used, which will be explained in detail in this sub-chapter:

- Useful life cycle model;
- Economic life cycle model.

The considerations taken into account for the calculations were the value of the investment for the installation of a new park and preventive maintenance over the life of the equipment. In addition to these specific values, the same calculations were carried out again, but this time considering the average of 18 random interventions (construction, installation or requalification), where the only selection criterion used was to analyse data from one playground per district in Continental Portugal. Once the calculations had been made, the results were compared to see if the investment in the Avenida Padre Américo playground was in line with the average obtained for the other 18 playgrounds, thus creating another criterion for the estimation of the quality of the investment, considering the national panorama.

3.1.2.1 Early considerations

In order to start the life cycle analysis of these physical assets, initial considerations were taken that correspond to a set of crucial data for the study:

- Acquisition cost (*CA*);
- Cessation value (*VC*);
- Lifetime corresponding to *VCn* (*N*);
- Exploration value (*CE*);
    - Operating Cost (*CO*);
    - Maintenance Cost (*CM*).
- Apparent rate (i_A).

As mentioned above, the *CA* was obtained through the preliminary consultation "Requalificação do Parque Infantil da Avenida Padre Américo" on the *Base.Gov* platform. This value is 56 745.00€.

The residual value of the equipment would be considered null, since in this type of physical asset there is no reuse of the equipment if it is no longer fit for purpose, given that the users are children. The value of components such as screws could have been taken into account, but this value would have had little relevance to the study. Apart from the fact that it would have taken up work time and caused some completely avoidable uncertainties. However, to ensure that null values were not included in the calculations, a symbolic value of 1.00€ was determined for the equipment's cessation value (*VC*) at 15 years of use. For the annual operating value of this type of asset, only maintenance costs were calculated since operating costs are not recognised.

According to the Portuguese Decree-Law no. 203/2015, maintenance actions must be carried out by the organisation responsible for the play area. Thus:

- Routine checks must be carried out covering the entire area occupied by the play and recreation space, including, in particular, the fences, gates, street furniture and support facilities;
- The entity responsible for the play area must ensure that it is repaired immediately or, if this is not feasible, that the damaged element is immobilised or removed, whenever there is damage to the play area, its equipment and impact surfaces, which is likely to put the safety of users at risk;
- Whenever the impact surface is made up of sand, wood chips or other loose material, the height of the layer of material suitable for absorbing the impact must be ensured, in accordance with the applicable standards;
- The entity responsible for the play area must keep the area permanently clean, including the equipment, impact surfaces, street furniture and support facilities;
- Whenever the impact surface consists of sand, wood shavings or other similar material, it must be regularly disinfected, maintained and cleaned using sieves or fine rakes, or completely renewed if necessary;
- The equipment and impact surfaces installed in the play areas must be subject to a "routine visual" inspection by the person in charge of the play area, carried out daily, and an "operational" inspection, carried out monthly. These inspections are carried out in accordance with the instructions provided by the equipment manufacturer, the provisions of the applicable standards and the general and specific recommendations set out in these;
- The responsible for the play area must establish an inspection plan for each type of equipment and keep the documentation relating to the inspections in an organised file in compliance with the applicable standards;
- When, as result of an inspection, defects of conformity or deterioration are detected that could endanger the safety of users, the entity responsible for the play area must immediately repair them or, if this is not possible, immobilise or remove the equipment;
- In cases where part of the equipment has to be dismantled or removed, the entity responsible for the play area must, in compliance with the rules, adopt the necessary procedures to protect or dismantle the fixings or foundations of the equipment;
- When an inspection reveals that the play area does not comply with the general safety requirement, the entity responsible must close it until the safety conditions are restored.

According to these points, the preventive maintenance plan was therefore developed with tasks at two intervals: daily and quarterly. The quarterly tasks and the respective labour times per intervention considered are shown in Table 1.

Table 1 - Quarterly tasks and respective labour times in minutes

General Safety Checks	Labour time in minutes
1. The readability and state of repair of warning signs	0.50
2. Deformation and cracks in equipment	3.33
3. Equipment protrusions	3.33
4. Sharp surfaces on equipment	3.33
5. Checking the screws (ends with less than two threads, rounded and smooth)	7.50
6. Check that there are no changes in the openings that could cause entrapment	7.50
7. Checking for possible animal infestations	2.00
Checks on the condition of materials and finishes	
1. Existence of rust and loose paint chips on metal surfaces	2.14
2. Clean surfaces and no vandalism	2.14
3. Existence of splinters in the wood and possible rotting or warping	2.14
4. Excessive wear and tear	2.14
5. Cracks in plastic components	2.14
6. Welds intact and without cracks	2.14
7. PVC coatings in good condition	2.14
Checks on fastening components	
1. Equipment available, safe and fully fitted	2.00
2. Pipe caps on the ends of the pipes	2.00
3. The connections/rollers are functional, lubricated and noise-free	2.00
4. The cables are secure and properly adjusted	2.00
5. Cables and strings in good condition	2.00
Structure checks	
1. Secure and stable footings/anchoring devices	3.33
2. Structural elements are fixed and secure	3.33
3. Springs/swings are in good condition	3.33
Checks on the floor and grip components	
1. Fixed and secure hand grip components	1.67
2. Level, stable and clean floor	1.67
3. Steps are tight and not excessively worn	1.67
Slide checks	
1. Polished, friction-free slide and rails	2.50
2. Clear slide	2.50
Checks on swings and moving parts	

1. Chains not twisted and free from excessive wear	3.13
2. S-hooks are unworn and closed with an accuracy of 0.04 inches	3.13
3. Swing hangers and swing bushes are in good condition	3.13
4. Swing seats are smooth and in good condition	3.13
5. Lightweight, smooth tyre seats in good condition	3.13
6. Tyre oscillating assemblies greased and in good condition	3.13
7. All moving parts are in good condition, secure and lubricated	3.13
Other	3.13
Checks on protective surfaces	
1. Loose filling material is levelled and at the right depth	3.13
2. Use areas free of obstacles and debris	3.13
3. Surface drainage in good condition and free of standing water	3.13
4. Wear mats correctly fixed and levelled	3.13
5. Intact unit surfaces, without depressions and grooves	3.13
6. Flat accessible surfaces (cross slope 1:48, running slope 1:16)	3.13
7. Accessible surfaces with no sudden changes in elevation (greater than 1/2 inch) and no gaps greater than 1/2 inch horizontally	3.13
8. Transfer platforms with a height above the surface of between 11 and 18 inches	3.13

Table 2 shows the labour times per intervention and the daily tasks considered.

Table 2 - Daily tasks and respective labour times in minutes

General Checks	Labour time in minutes
1. Park area free of rubbish, broken glass and weeds	3.00
2. Ice-free walkways and no tripping hazards	3.00
3. The occurrence of modifications (acts of vandalism) that could cause danger	3.00
4. Functional and well-maintained drains	3.00
5. Existence of risks from suspended materials that could fall	3.00
Checks on protective surfaces	
1. Surfaces free of foreign materials and debris	2.14
2. Level loose-fill surfaces, particularly under swings and slides	2.14
3. Loose fill surfaces are cleaned to an adequate depth and properly compacted	2.14
4. Intact unit surfaces and no tripping hazards	2.14
5. Exposed or loose bases	2.14
6. Functional drains and no standing water	2.14
7. Non-frozen protective surfaces	2.14

Checking playground equipment	
1. Damaged, loose, vandalised or missing parts	3.75
2. Stable and deformation-free equipment	3.75
3. Occurrence of modifications (e.g. ropes tied to parts)	3.75
4. All the moving parts, such as the swings and chains, are in good condition	3.75
Other	
1. Other	5.00

The total maintenance times per intervention for all tasks were thus obtained:

- Quarterly tasks (*pbf*) - 122.5 minutes or 2.04 hours;
- Daily tasks (*paf*) - 50 minutes or 0.83 hours.

In this way, the cost per intervention will be the product of a given hourly labour price (*mdo*) and the number of hours allocated to the maintenance plan. It is possible to discriminate the cost for daily and quarterly tasks as shown in equations 1.1 and 1.2, respectively:

$$ipbf = pbf \times mdo \tag{1.1}$$

$$ipaf = paf \times mdo \tag{1.2}$$

A value of 6€/hour was assumed for *mdo* and the following values were obtained:

- *ipbf* =12.25€;
- *ipaf* =5€.

In order to obtain an annual figure that can be included in life cycle model calculations, it is necessary to estimate the number of interventions in a year. Bearing in mind that the cost of maintenance is expected to be much higher and knowing that the costs of spare parts and travelling by technicians have not been included. It was estimated that the quarterly tasks would be part of four annual interventions and that the daily tasks would be part of 252 annual interventions (only on working days).

Obtaining the annual value (*CM*) using equation 1.3:

$$CM = (ipbf \times 4) + (ipaf \times 252) \tag{1.3}$$

It follows that *CE*=*CM*=1 309.00€. For the apparent rate i_A, an average and constant value of 8% was considered.

3.1.2.2 Life Cycle Models – Lifespan

As mentioned earlier in this thesis, useful life can have several criteria for its definition, depending on the point of view of those who want to estimate it. Since this study only worked with recognisable and primarily measurable criteria, the lifespan considered was when the equipment maintenance costs exceed the cost of maintenance plus the capital amortisation of new similar equipment, as shown in equation 1.4 where CM_j represents the cost of maintenance in each year j.

$$\sum_{j=1}^{n} \frac{CM_j}{(1+i_A)} > CA + \frac{CM_j}{(1+i_A)} \quad (1.4)$$

In order to be able to demonstrate the use of this method, and bearing in mind that the annual maintenance cost considered does not include all costs (such as corrective measures and travelling by the technician, for example) it was decided to add 1 309.00€ to the maintenance cost considered for each previous year. In conclusion, according to the useful life method, the equipment should be renewed or replaced in the 12th year of use when the maintenance costs exceed 57 957.04€ (maintenance costs plus the capital amortisation of the equivalent new equipment). As shown in Table 3 and Figure 2.

Table 3 - Lifespan method with apparent rate: annual values

Year	Σ CM_j (Apparent Rate)
0	- €
1	1 212.04 €
2	3 456.55 €
3	6 573.93 €
4	10 422.55 €
5	14 876.96 €
6	19 826.31 €
7	25 172.84 €
8	30 830.53 €
9	36 723.97 €
10	42 787.17 €
11	48 962.65 €
12	55 200.52 €
13	61 457.63 €

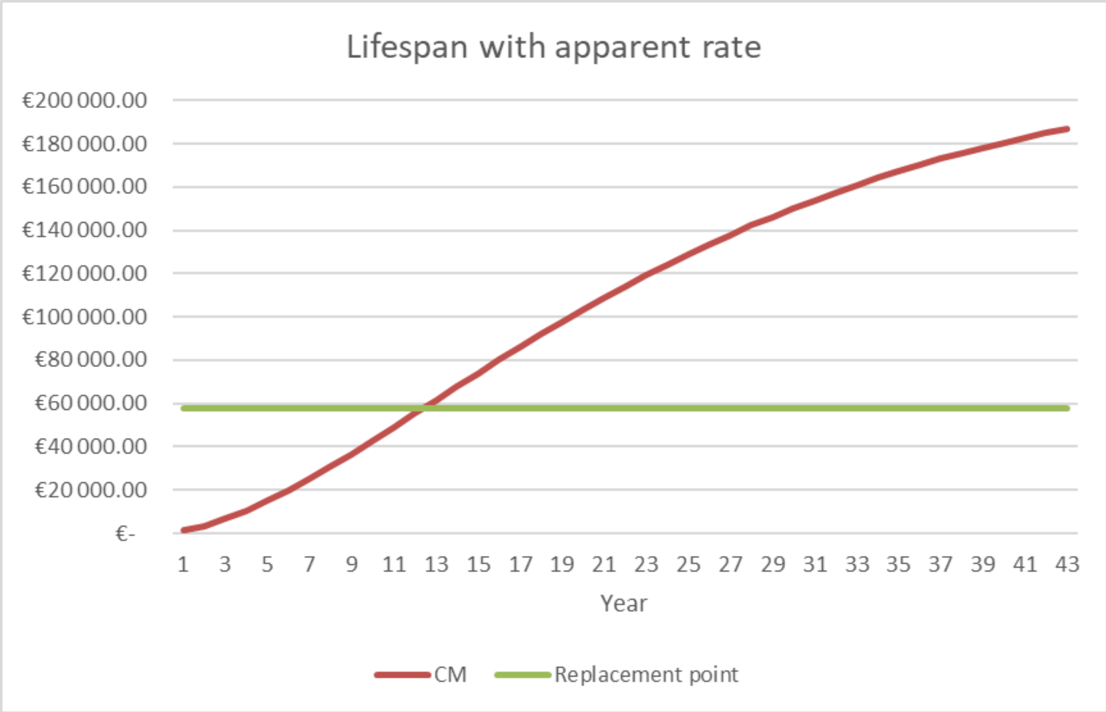

Figure 2 - Lifespan with apparent rate applied to the playground case study

3.1.2.3 Life Cycle Models – Economic

The following models were used to calculate the economic life cycle model:

- Income Annual Uniform Method (*UAI*);
- Minimizing Total Average Cost Method (*MTAC*);
- Minimizing Total Average Cost Reduced to Present Value Method (*MTAC-RPV*).

For each of them, the three depreciation methods mentioned above were applied, so that the differences could be visualised both in terms of calculation and graphically:

- Linear method;
- Exponential method;
- Sum of digits method.

The linear method of depreciation is calculated using equations 1.5 and 1.6. Where *dl* is the annual rate of depreciation; *CA* is the acquisition value of the equipment; *VCn* is the residual value of the equipment after *N* periods of time (in this case, years); *Vn* is the value of the equipment in a given period *n* (in this case, for purposes of consistency, it is also years).

$$dl = \frac{CA - VCn}{N} \quad (1.5)$$

$$Vn = CA - n \times dl \quad (1.6)$$

The results obtained are detailed in Table 4 and Figure 3.

Table 4 - Linear depreciation method: annual depreciation

Linear Depreciation Method		
dl	Annual Depreciation Rate	3 782.93 €
Linear Depreciation		
0	56 745.00 €	
1	52 962.07 €	
2	49 179.13 €	
3	45 396.20 €	
4	41 613.27 €	
5	37 830.33 €	
6	34 047.40 €	
7	30 264.47 €	
8	26 481.53 €	
9	22 698.60 €	
10	18 915.67 €	
11	15 132.73 €	
12	11 349.80 €	
13	7 566.87 €	
14	3 783.93 €	
15	1.00 €	

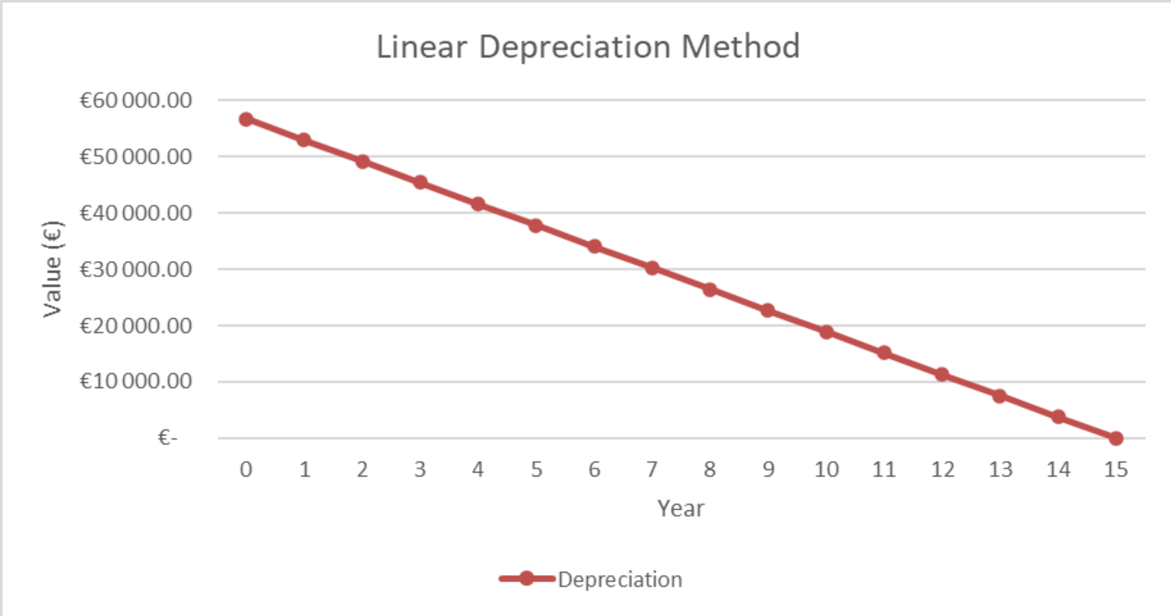

Figure 3 - Linear method of depreciation applied to the playground case study

The exponential method is applied using equations 1.7 and 1.8. Where T is the exponential rate of depreciation.

$$Vn = CA \times (1 - T)^n \tag{1.7}$$

$$T = (1 - \sqrt[N]{\frac{VC_n}{CA}}) \tag{1.8}$$

Using the exponential method, the results presented in Table 5 and Figure 4 were calculated.

Table 5 - Exponential depreciation method: annual depreciation

Exponential Method of Depreciation	
T	0.517972858
	Exponential depreciation
0	56 745.00 €
1	27 352.63 €
2	13 184.71 €
3	6 355.39 €
4	3 063.47 €
5	1 476.68 €
6	711.80 €
7	343.11 €
8	165.39 €
9	79.72 €
10	38.43 €

11	18.52 €
12	8.93 €
13	4.30 €
14	2.07 €
15	1.00 €

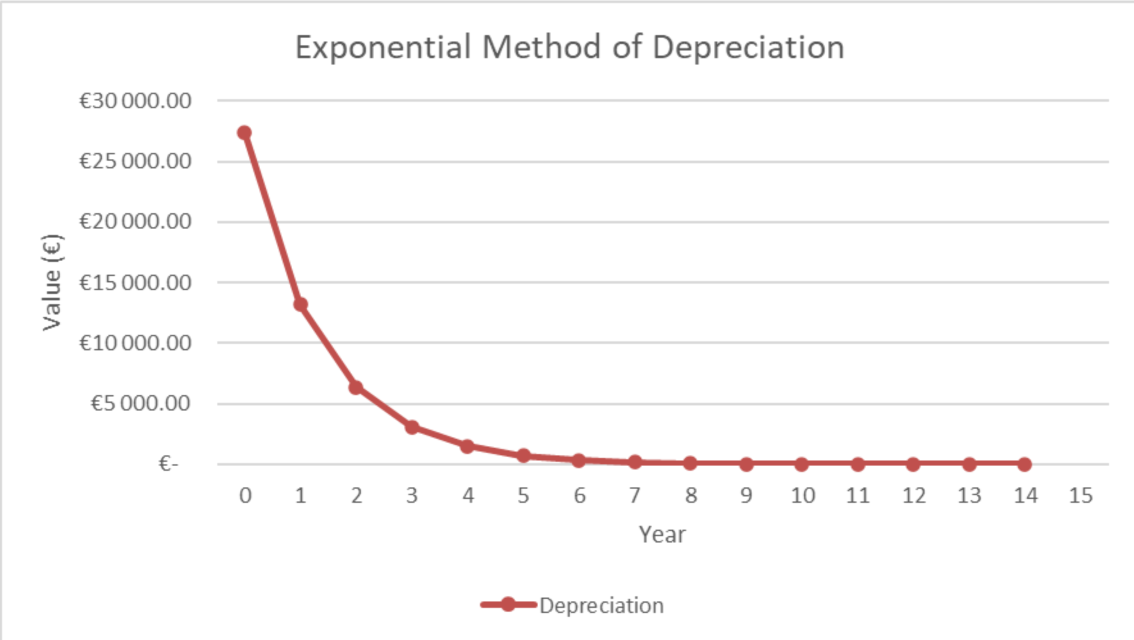

Figure 4 - Exponential method of depreciation applied to the playground case study

Ultimately, the equations that allow the sum of the digits method to be applied to the assets under study are 1.9, 1.10, 1.11 and 1.12, where *SD* represents the sum of the digits.

$$SD = \frac{N(N+1)}{2} \tag{1.9}$$

$$dl = \frac{N-(n-1)}{SD} \times (CA - VC_n) \tag{1.10}$$

$$Vn_1 = CA - dl \tag{1.11}$$

$$Vn = Vn - 1 - dl \tag{1.12}$$

The results obtained are presented in Table 6 and Figure 5.

Table 6 - Sum of the digits depreciation method: annual depreciation

Sum of Digits Depreciation Method		
dl	Annual Depreciation Rate for year n	7 565.87 €
Vn	Value of equipment in a given period n	56 745.00 €
SD	120	
Year	*dl*	Sum of the Digits Depreciation
0		56 745.00 €
1	7 093.00 €	49 652.00 €
2	6 620.13 €	43 031.87 €
3	6 147.27 €	36 884.60 €
4	5 674.40 €	31 210.20 €
5	5 201.53 €	26 008.67 €
6	4 728.67 €	21 280.00 €
7	4 255.80 €	17 024.20 €
8	3 782.93 €	13 241.27 €
9	3 310.07 €	9 931.20 €
10	2 837.20 €	7 094.00 €
11	2 364.33 €	4 729.67 €
12	1 891.47 €	2 838.20 €
13	1 418.60 €	1 419.60 €
14	945.73 €	473.87 €
15	472.87 €	1.00 €

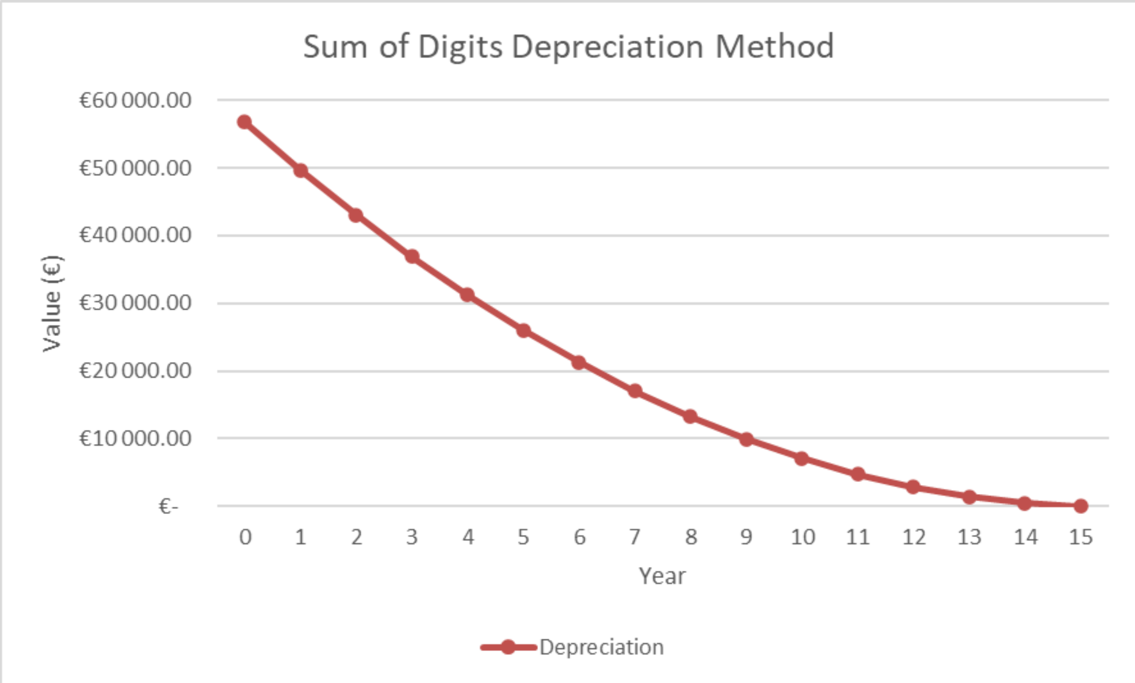

Figure 5 - Sum of the digits depreciation method applied to the playground case study

Once the curves for the three depreciation models adopted were obtained, they were applied to the economic models mentioned above.

Firstly, the calculation methodology for the uniform annual income (UAI_n) will be presented, which is based on the application of equation 1.13. As mentioned above, in this case the exploration values are summarised as the calculated maintenance costs, so the operating costs are 0€.

$$UAI_n = \frac{i_A \times (1 + i_A)^j}{(1 + i_A)^j - 1} \times (CA + \sum_{j=0}^{n} \frac{CM_j + CO_j}{(1 + i_A)^j} - \frac{V_n}{(1 + i_A)^j}) \quad (1.13)$$

Considering Vn as the value calculated differently using the three depreciation methods mentioned, the following values are obtained, as shown in Table 7.

Table 7 - Uniform annual income

i_A	8%	Linear Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	56 745.00 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		57 957.04 €	59 079.29 €	60 118.42 €	61 080.57 €	61 971.46 €	62 796.35 €	63 560.14 €	64 267.35 €	64 922.18 €	65 528.50 €	66 089.90 €	66 609.73 €	67 091.04 €	67 536.71 €	67 949.36 €
Assignment		52 962.07 €	49 179.13 €	45 396.20 €	41 613.27 €	37 830.33 €	34 047.40 €	30 264.47 €	26 481.53 €	22 698.60 €	18 915.67 €	15 132.73 €	11 349.80 €	7 566.87 €	3 783.93 €	1.00 €
Value P'.		49 038.95 €	42 163.18 €	36 036.97 €	30 586.99 €	25 746.69 €	21 455.64 €	17 659.03 €	14 307.15 €	11 354.95 €	8 761.61 €	6 490.17 €	4 507.16 €	2 782.32 €	1 288.28 €	0.32 €
P-P'		8 918.09 €	16 916.11 €	24 081.45 €	30 493.58 €	36 224.77 €	41 340.71 €	45 901.11 €	49 960.20 €	53 567.23 €	56 766.88 €	59 599.73 €	62 102.56 €	64 308.72 €	66 248.42 €	67 949.04 €
UAI		9 631.53 €	9 486.04 €	9 344.41 €	9 206.65 €	9 072.73 €	8 942.63 €	8 816.34 €	8 693.81 €	8 575.03 €	8 459.94 €	8 348.51 €	8 240.70 €	8 136.46 €	8 035.73 €	7 938.46 €
i_A	8%	Exponential Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	56 745.00 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		57 957.04 €	59 079.29 €	60 118.42 €	61 080.57 €	61 971.46 €	62 796.35 €	63 560.14 €	64 267.35 €	64 922.18 €	65 528.50 €	66 089.90 €	66 609.73 €	67 091.04 €	67 536.71 €	67 949.36 €
Assignment		27 352.63 €	13 184.71 €	6 355.39 €	3 063.47 €	1 476.68 €	711.80 €	343.11 €	165.39 €	79.72 €	38.43 €	18.52 €	8.93 €	4.30 €	2.07 €	1.00 €
Value P'.		25 326.51 €	11 303.76 €	5 045.11 €	2 251.74 €	1 005.00 €	448.55 €	200.20 €	89.35 €	39.88 €	17.80 €	7.94 €	3.55 €	1.58 €	0.71 €	0.32 €
P-P'		32 630.53 €	47 775.53 €	55 073.31 €	58 828.83 €	60 966.46 €	62 347.80 €	63 359.94 €	64 178.00 €	64 882.30 €	65 510.70 €	66 081.96 €	66 606.18 €	67 089.46 €	67 536.00 €	67 949.04 €
UAI		35 240.97 €	26 791.05 €	21 370.29 €	17 761.65 €	15 269.44 €	13 486.79 €	12 169.70 €	11 167.92 €	10 386.34 €	9 763.03 €	9 256.52 €	8 838.31 €	8 488.28 €	8 191.90 €	7 938.46 €
i_A	8%	Sum of Digits Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	56 745.00 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		57 957.04 €	59 079.29 €	60 118.42 €	61 080.57 €	61 971.46 €	62 796.35 €	63 560.14 €	64 267.35 €	64 922.18 €	65 528.50 €	66 089.90 €	66 609.73 €	67 091.04 €	67 536.71 €	67 949.36 €
Assignment		49 652.00 €	43 031.87 €	36 884.60 €	31 210.20 €	26 008.67 €	21 280.00 €	17 024.20 €	13 241.27 €	9 931.20 €	7 094.00 €	4 729.67 €	2 838.20 €	1 419.60 €	473.87 €	1.00 €
Value P'.		45 974.07 €	36 892.89 €	29 280.18 €	22 940.43 €	17 701.06 €	13 410.01 €	9 933.46 €	7 153.84 €	4 968.07 €	3 285.89 €	2 028.47 €	1 127.09 €	521.98 €	161.33 €	0.32 €
P-P'		11 982.96 €	22 186.40 €	30 838.24 €	38 140.15 €	44 270.40 €	49 386.34 €	53 626.68 €	57 113.51 €	59 954.10 €	62 242.60 €	64 061.43 €	65 482.64 €	66 569.06 €	67 375.37 €	67 949.04 €
UAI		12 941.60 €	12 441.45 €	11 966.27 €	11 515.30 €	11 087.81 €	10 683.03 €	10 300.21 €	9 938.59 €	9 597.44 €	9 275.98 €	8 973.49 €	8 689.22 €	8 422.44 €	8 172.42 €	7 938.46 €

The method for minimising the average total cost is obtained from equations 1.14, 1.15 and 1.16, where *Cn* is the average total cost.

$$Cn' = \frac{\sum_{j=0}^{n} CM_j + CO_j}{n} \tag{1.14}$$

$$Cn'' = \frac{CA - Vn}{n} \tag{1.15}$$

$$Cn(MTAC) = Cn' + Cn'' \tag{1.16}$$

The values obtained are illustrated in Table 8.

Table 8 - Minimizing Total Average Cost Method

Linear Depreciation Method																
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment (VN)		52 962.07 €	49 179.13 €	45 396.20 €	41 613.27 €	37 830.33 €	34 047.40 €	30 264.47 €	26 481.53 €	22 698.60 €	18 915.67 €	15 132.73 €	11 349.80 €	7 566.87 €	3 783.93 €	1.00 €
C''n		3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €	3 782.93 €
C'n+C''n		5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €	5 091.93 €

Exponential Depreciation Method																
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment		27 352.63 €	13 184.71 €	6 355.39 €	3 063.47 €	1 476.68 €	711.80 €	343.11 €	165.39 €	79.72 €	38.43 €	18.52 €	8.93 €	4.30 €	2.07 €	1.00 €
C''n		29 392.37 €	21 780.14 €	16 796.54 €	13 420.38 €	11 053.66 €	9 338.87 €	8 057.41 €	7 072.45 €	6 296.14 €	5 670.66 €	5 156.95 €	4 728.01 €	4 364.67 €	4 053.07 €	3 782.93 €
C'n+C''n		30 701.37 €	23 089.14 €	18 105.54 €	14 729.38 €	12 362.66 €	10 647.87 €	9 366.41 €	8 381.45 €	7 605.14 €	6 979.66 €	6 465.95 €	6 037.01 €	5 673.67 €	5 362.07 €	5 091.93 €

Sum of Digits Depreciation Method																
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment		49 652.00 €	43 031.87 €	36 884.60 €	31 210.20 €	26 008.67 €	21 280.00 €	17 024.20 €	13 241.27 €	9 931.20 €	7 094.00 €	4 729.67 €	2 838.20 €	1 419.60 €	473.87 €	1.00 €
C''n		7 093.00 €	6 856.57 €	6 620.13 €	6 383.70 €	6 147.27 €	5 910.83 €	5 674.40 €	5 437.97 €	5 201.53 €	4 965.10 €	4 728.67 €	4 492.23 €	4 255.80 €	4 019.37 €	3 782.93 €
C'n+C''n		8 402.00 €	8 165.57 €	7 929.13 €	7 692.70 €	7 456.27 €	7 219.83 €	6 983.40 €	6 746.97 €	6 510.53 €	6 274.10 €	6 037.67 €	5 801.23 €	5 564.80 €	5 328.37 €	5 091.93 €

Finally, for the *MTAC* method with reduction to present value, the values were calculated using equations 1.17, 1.18 and 1.19.

$$Cn' = \frac{1}{n} \times \frac{\sum_{j=1}^{n} CM_j + CO_j}{(1 + i_A)^j} \tag{1.17}$$

$$Cn'' = \frac{CA - \frac{V_n}{(1 + i_A)^j}}{n} \tag{1.18}$$

$$Cn(MTAC - RPV) = Cn' + Cn'' \tag{1.19}$$

The obtained results are displayed in Table 9. All the economic models ensure that the best year to replace the equipment is the 15^{th} year.

Table 9 - *MTAC* with reduction to present value

i_A	8%	Linear Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		52 962.07 €	49 179.13 €	45 396.20 €	41 613.27 €	37 830.33 €	34 047.40 €	30 264.47 €	26 481.53 €	22 698.60 €	18 915.67 €	15 132.73 €	11 349.80 €	7 566.87 €	3 783.93 €	1.00 €
Present Value (P')		49 038.95 €	42 163.18 €	36 036.97 €	30 586.99 €	25 746.69 €	21 455.64 €	17 659.03 €	14 307.15 €	11 354.95 €	8 761.61 €	6 490.17 €	4 507.16 €	2 782.32 €	1 288.28 €	0.32 €
C''n		7 706.05 €	7 290.91 €	6 902.68 €	6 539.50 €	6 199.66 €	5 881.56 €	5 583.71 €	5 304.73 €	5 043.34 €	4 798.34 €	4 568.62 €	4 353.15 €	4 150.98 €	3 961.19 €	3 782.98 €
C'n + C''n		8 918.09 €	8 458.06 €	8 027.15 €	7 623.40 €	7 244.95 €	6 890.12 €	6 557.30 €	6 245.03 €	5 951.91 €	5 676.69 €	5 418.16 €	5 175.21 €	4 946.82 €	4 732.03 €	4 529.94 €

i_A	8%	Exponential Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		27 352.63 €	13 184.71 €	6 355.39 €	3 063.47 €	1 476.68 €	711.80 €	343.11 €	165.39 €	79.72 €	38.43 €	18.52 €	8.93 €	4.30 €	2.07 €	1.00 €
Present Value (P')		25 326.51 €	11 303.76 €	5 045.11 €	2 251.74 €	1 005.00 €	448.55 €	200.20 €	89.35 €	39.88 €	17.80 €	7.94 €	3.55 €	1.58 €	0.71 €	0.32 €
C''n		31 418.49 €	22 720.62 €	17 233.30 €	13 623.31 €	11 148.00 €	9 382.74 €	8 077.83 €	7 081.96 €	6 300.57 €	5 672.72 €	5 157.91 €	4 728.45 €	4 364.88 €	4 053.16 €	3 782.98 €
C'n + C''n		32 630.53 €	23 887.76 €	18 357.77 €	14 707.21 €	12 193.29 €	10 391.30 €	9 051.42 €	8 022.25 €	7 209.14 €	6 551.07 €	6 007.45 €	5 550.52 €	5 160.73 €	4 824.00 €	4 529.94 €

i_A	8%	Sum of Digits Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	56 745.00 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		49 652.00 €	43 031.87 €	36 884.60 €	31 210.20 €	26 008.67 €	21 280.00 €	17 024.20 €	13 241.27 €	9 931.20 €	7 094.00 €	4 729.67 €	2 838.20 €	1 419.60 €	473.87 €	1.00 €
Present Value (P')		45 974.07 €	36 892.89 €	29 280.18 €	22 940.43 €	17 701.06 €	13 410.01 €	9 933.46 €	7 153.84 €	4 968.07 €	3 285.89 €	2 028.47 €	1 127.09 €	521.98 €	161.33 €	0.32 €
C''n		10 770.93 €	9 926.06 €	9 154.94 €	8 451.14 €	7 808.79 €	7 222.50 €	6 687.36 €	6 198.89 €	5 752.99 €	5 345.91 €	4 974.23 €	4 634.83 €	4 324.85 €	4 041.69 €	3 782.98 €
C'n + C''n		11 982.96 €	11 093.20 €	10 279.41 €	9 535.04 €	8 854.08 €	8 231.06 €	7 660.95 €	7 139.19 €	6 661.57 €	6 224.26 €	5 823.77 €	5 456.89 €	5 120.70 €	4 812.53 €	4 529.94 €

3.1.2.4 Data collection from 18 playgrounds in Continental Portugal

In order to provide a brief contextualisation of the data obtained for the playground from Avenida Padre Américo in Coimbra, similar public tenders were researched on the *Base.Gov* platform, but for other parks in all the districts of Continental Portugal.

The choice of only 18 districts in mainland Portugal is based on the fact that the budget for the islands has increased costs, both in terms of transport and travel, and due to the lack of resources, when compared to the mainland. For these reasons, the data could be of poor quality for this specific study.

Therefore, the data obtained is shown in Table 10. The average purchase price and price per m^2 of intervention per region (north, centre and south) are shown in Table 11. As there is not enough historical evidence to study the average purchase price for maintenance contracts, it was decided to apply the value calculated for the maintenance costs of the playground from Avenida Padre Américo.

Table 10 - Public tenders in Continental Portugal

Procedure Type	Date	Municipality	District	Access to procedural documents	Specified Work Plan	Park dimensions (m^2)	Hyperlink *Base.gov*	Base price	Adjudicated Value	Price per m^2
Prior Consultation	01/07/2022	Município de Miranda do Corvo	Coimbra	Yes	Yes	108	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9333379	66 035.73 €	56 745.00 €	525.42 €
Prior Consultation	09/08/2023	Município do Bombarral	Leiria	Yes	Yes	693	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=10182353	28 628.00 €	28 627.92 €	41.31 €
Prior Consultation	03/08/2023	Município de Sever do Vouga	Aveiro	Yes	Yes	Undefined	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=10173718	16 464.91 €	16 400.00 €	-
Public Tenders	13/12/2021	Município de Cascais	Lisboa	Yes	Yes	800	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=8490276	101 093.00 €	79 969.11 €	99.96 €
Direct Adjustment General Regime	19/04/2021	Município de Viana do Castelo	Viana do Castelo	Yes	No	238	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=7617929	22 500.00 €	22 265.00 €	93.55 €
Direct Adjustment General Regime	09/03/2023	Município de Miranda do Douro	Bragança	Yes	Yes	135	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9852131	- €	26 750.00 €	198.15 €
Prior Consultation	28/09/2020	Município de Alijó	Vila Real	Yes	No	319	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=7059009	52 000.00 €	51 772.90 €	162.30 €
Prior Consultation	18/03/2022	Município de Braga	Braga	Yes	No	179	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9125866	48 831.00 €	48 730.98 €	272.24 €

Prior Consultation	21/03/2022	Município de Amarante	Porto	Yes	No	Undefined	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9131086	25 000.00 €	24 996.90 €	-
Prior Consultation	28/06/2022	Município de Tondela	Viseu	Yes	No	276	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9327174	48 300.00 €	48 268.11 €	174.88 €
Prior Consultation	13/11/2020	Município de Aguiar da Beira	Guarda	No	No	319	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=7282508	48 614.00 €	48 529.00 €	152.13 €
Prior Consultation	10/08/2023	Município de Fundão	Castelo Branco	No	No	Undefined	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=10183689	- €	33 741.25 €	-
Prior Consultation	18/03/2021	Município da Chamusca	Santarém	Yes	No	289	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=7574082	39 778.20 €	39 511.82 €	136.72 €
Direct Adjustment General Regime	31/10/2022	Município de Alter do Chão	Portalegre	Yes	No	166	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9527074	9 030.00 €	9 030.00 €	54.40 €
Prior Consultation	27/07/2023	Município da Moita	Setúbal	Yes	Yes	218	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=10163266	11 997.32 €	11 480.00 €	52.66 €
Direct Adjustment General Regime	22/05/2023	Município de Ajustrel	Beja	Yes	No	153	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=9999832	22 961.80 €	22 961.80 €	150.08 €
Prior Consultation	31/08/2020	Município de Albufeira	Faro	Yes	Yes	172	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=6888028	103 456.84 €	103 400.00 €	601.16 €
Prior Consultation	19/07/2023	Município de Montemor-o-Novo	Évora	No	No	60	https://www.base.gov.pt/Base4/pt/detalhe/?type=contratos&id=7838018	- €	17 395.00 €	289.92 €

Table 11 - Average acquisition values

Average Acquisition Value (North)	Average Acquisition Value (Middle)	Average Acquisition Value (South)	Average Acquisition Value (Overall)
37 130.65 €	39 069.26 €	38 809.20 €	38 365.27 €
Price per m^2 (North)	Price per m^2 (Centre)	Price per m^2 (South)	Average Acquisition Value per m^2 (Overall)
180.22 €	168.32 €	273.45 €	200.32 €

3.1.2.5 Lifespan method using data from 18 playgrounds in Continental Portugal

Accordingly, using the national average value of acquisition for the construction and/or upgrading of playgrounds, the useful life cycle data shown in Table 12 and Figure 6 is obtained. This time, the average purchase price is 38 365.27€ and the annual maintenance expenses are still 1 309.00€. So, according to the lifespan replacement model, the average playground should be replaced or renovated during the 9th year of use, when maintenance costs exceed 39 577.30€ (maintenance costs plus the capital amortisation of equivalent new equipment).

Table 12 - Lifespan method with apparent rate - annual values (average for Continental Portugal)

Year	Σ *CMj* (Apparent Rate)
0	- €
1	1 212.04 €
2	3 456.55 €
3	6 573.93 €
4	10 422.55 €
5	14 876.96 €
6	19 826.31 €
7	25 172.84 €
8	30 830.53 €
9	36 723.97 €
10	42 787.17 €
11	48 962.65 €

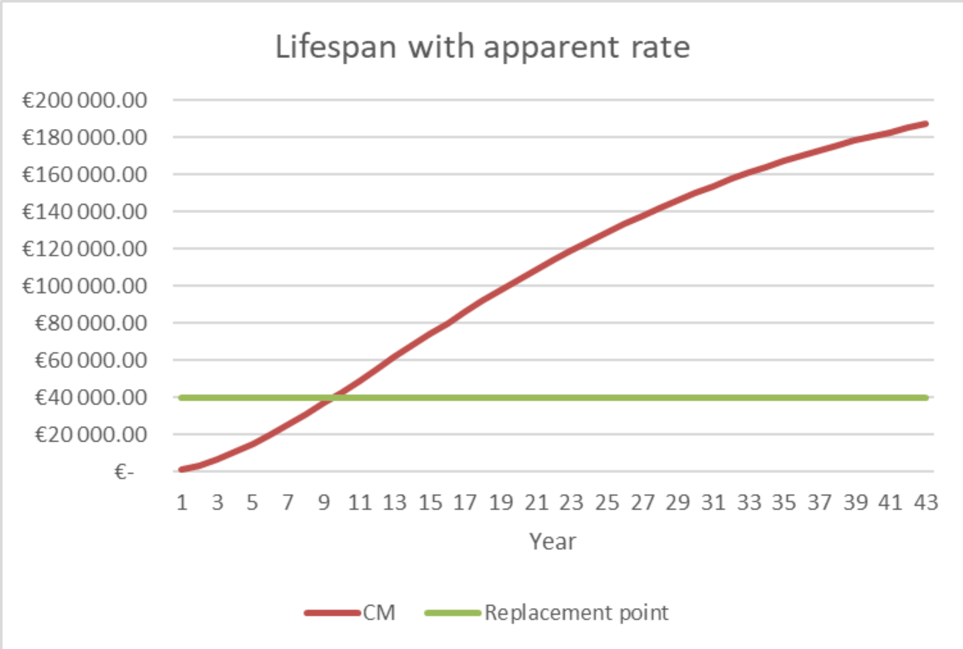

Figure 6 - Lifespan method with apparent rate applied to the playground case study (average Continental Portugal)

3.1.2.6 Economic method using data from 18 playgrounds in Continental Portugal

The results obtained with the new acquisition value for the depreciation methods were the following:

- The linear depreciation method shown in Table 13 and Figure 7.

Table 13 - Linear depreciation method: annual depreciation (average for Continental Portugal)

Linear Depreciation Method		
dl	Annual Depreciation Rate	2 557.62 €
Linear Depreciation		
0	38 365.27 €	
1	35 807.65 €	
2	33 250.03 €	
3	30 692.41 €	
4	28 134.80 €	
5	25 577.18 €	
6	23 019.56 €	
7	20 461.94 €	
8	17 904.32 €	
9	15 346.71 €	
10	12 789.09 €	
11	10 231.47 €	

12	7 673.85 €
13	5 116.24 €
14	2 558.62 €
15	1.00 €

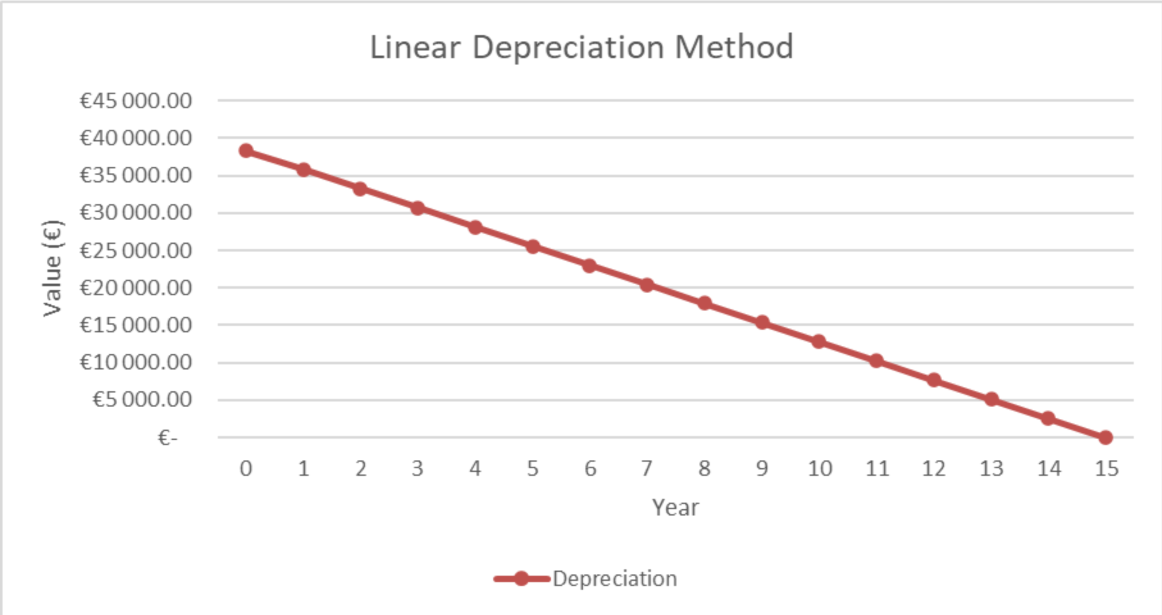

Figure 7 - Linear depreciation method applied to the playground case study (average for Continental Portugal)

- Exponential method as presented in Table 14 and Figure 8.

Table 14 - Exponential depreciation method: annual values (average for Continental Portugal)

Exponential method	
T	0.505229134
Exponential depreciation	
0	38 365.27 €
1	18 982.02 €
2	9 391.75 €
3	4 646.76 €
4	2 299.08 €
5	1 137.52 €
6	562.81 €
7	278.46 €
8	137.78 €
9	68.17 €
10	33.73 €
11	16.69 €

12	8.26 €
13	4.08 €
14	2.02 €
15	1.00 €

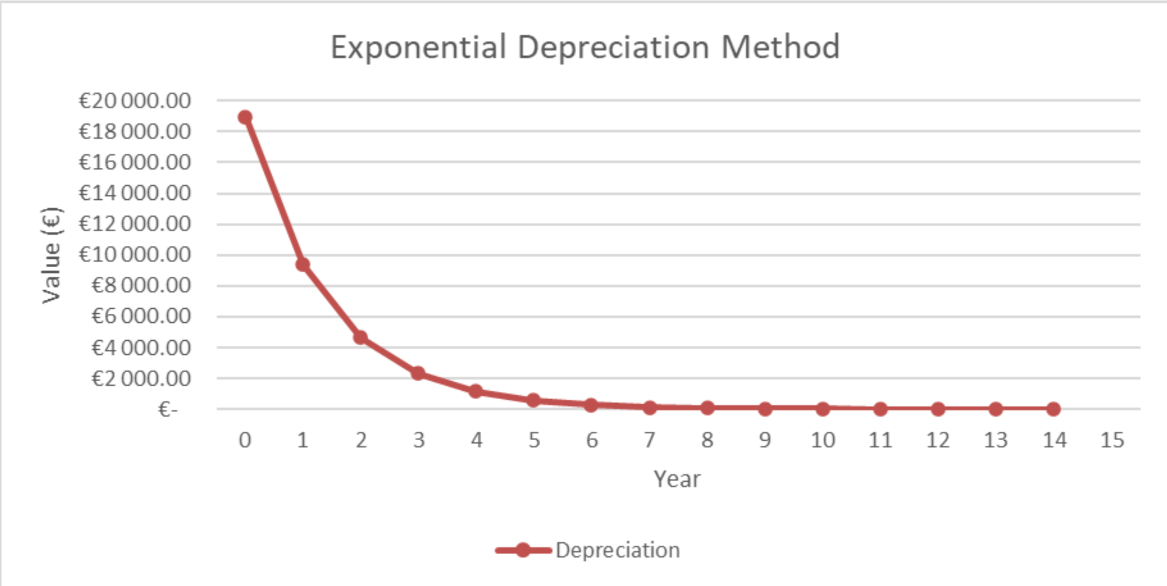

Figure 8 - Exponential depreciation method applied to the playground case study (average for Continental Portugal)

- Sum of the digits method illustrated in Table 15 and Figure 9.

Table 15 - Sum of the digits depreciation method: annual values (average for Continental Portugal)

Sum of Digits Depreciation Method		
SD	120	
Year	*dl*	Sum of the Digits Depreciation
0		38 365.27 €
1	4 795.53 €	33 569.73 €
2	4 475.83 €	29 093.90 €
3	4 156.13 €	24 937.77 €
4	3 836.43 €	21 101.35 €
5	3 516.72 €	17 584.62 €
6	3 197.02 €	14 387.60 €
7	2 877.32 €	11 510.28 €
8	2 557.62 €	8 952.66 €

9	2 237.92 €	6 714.75 €
10	1 918.21 €	4 796.53 €
11	1 598.51 €	3 198.02 €
12	1 278.81 €	1 919.21 €
13	959.11 €	960.11 €
14	639.40 €	320.70 €
15	319.70 €	1.00 €

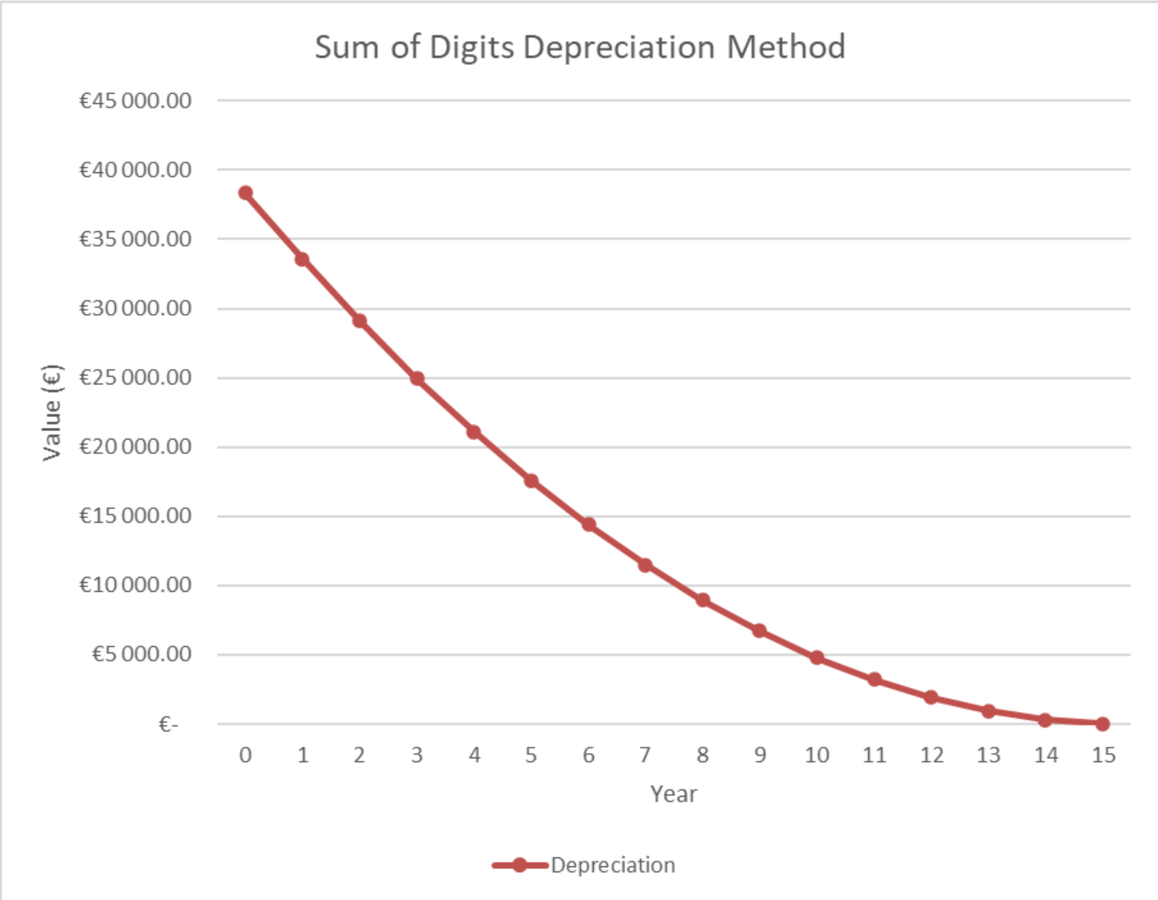

Figure 9 - Sum of the digits depreciation method applied to the playground case study (Continental Portugal average)

Applying the new values to the economic models explained previously:

- Income Annual Uniform Method (*UAI*), represented in Table 16;
- Minimizing Total Average Cost Method (*MTAC*), represented in Table 17;
- Minimizing Total Average Cost Reduced to Present Value Method (*MTAC-RPV*), represented in Table 18.

Table 16 - *UAI* (average for Continental Portugal)

i_A	8%	Linear Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	38 365.27 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		39 577.30 €	40 699.56 €	41 738.69 €	42 700.84 €	43 591.72 €	44 416.62 €	45 180.40 €	45 887.62 €	46 542.44 €	47 148.76 €	47 710.17 €	48 229.99 €	48 711.31 €	49 156.97 €	49 569.62 €
Assignment		35 807.65 €	33 250.03 €	30 692.41 €	28 134.80 €	25 577.18 €	23 019.56 €	20 461.94 €	17 904.32 €	15 346.71 €	12 789.09 €	10 231.47 €	7 673.85 €	5 116.24 €	2 558.62 €	1.00 €
Value P'.		33 155.23 €	28 506.54 €	24 364.63 €	20 679.91 €	17 407.40 €	14 506.23 €	11 939.35 €	9 673.15 €	7 677.17 €	5 923.82 €	4 388.10 €	3 047.39 €	1 881.23 €	871.11 €	0.32 €
P-P'		6 422.07 €	12 193.02 €	17 374.06 €	22 020.93 €	26 184.33 €	29 910.39 €	33 241.06 €	36 214.47 €	38 865.27 €	41 224.94 €	43 322.07 €	45 182.60 €	46 830.08 €	48 285.86 €	49 569.31 €
UAI		6 935.84 €	6 837.47 €	6 741.72 €	6 648.58 €	6 558.03 €	6 470.08 €	6 384.69 €	6 301.85 €	6 221.54 €	6 143.73 €	6 068.40 €	5 995.51 €	5 925.03 €	5 856.92 €	5 791.16 €
i_A	8%	Exponential Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	38 365.27 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		39 577.30 €	40 699.56 €	41 738.69 €	42 700.84 €	43 591.72 €	44 416.62 €	45 180.40 €	45 887.62 €	46 542.44 €	47 148.76 €	47 710.17 €	48 229.99 €	48 711.31 €	49 156.97 €	49 569.62 €
Assignment		18 982.02 €	9 391.75 €	4 646.76 €	2 299.08 €	1 137.52 €	562.81 €	278.46 €	137.78 €	68.17 €	33.73 €	16.69 €	8.26 €	4.08 €	2.02 €	1.00 €
Value P'.		17 575.94 €	8 051.91 €	3 688.75 €	1 689.89 €	774.18 €	354.67 €	162.48 €	74.44 €	34.10 €	15.62 €	7.16 €	3.28 €	1.50 €	0.69 €	0.32 €
P-P'		22 001.36 €	32 647.65 €	38 049.94 €	41 010.95 €	42 817.55 €	44 061.95 €	45 017.92 €	45 813.18 €	46 508.34 €	47 133.14 €	47 703.01 €	48 226.71 €	48 709.81 €	49 156.28 €	49 569.31 €
UAI		23 761.47 €	18 307.80 €	14 764.65 €	12 382.06 €	10 723.93 €	9 531.28 €	8 646.70 €	7 972.17 €	7 445.04 €	7 024.23 €	6 682.06 €	6 399.44 €	6 162.85 €	5 962.50 €	5 791.16 €
i_A	8%	Sum of Digits Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CA	38 365.27 €															
CE		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Value P.		39 577.30 €	40 699.56 €	41 738.69 €	42 700.84 €	43 591.72 €	44 416.62 €	45 180.40 €	45 887.62 €	46 542.44 €	47 148.76 €	47 710.17 €	48 229.99 €	48 711.31 €	49 156.97 €	49 569.62 €
Assignment		33 569.73 €	29 093.90 €	24 937.77 €	21 101.35 €	17 584.62 €	14 387.60 €	11 510.28 €	8 952.66 €	6 714.75 €	4 796.53 €	3 198.02 €	1 919.21 €	960.11 €	320.70 €	1.00 €
Value P'.		31 083.09 €	24 943.33 €	19 796.41 €	15 510.12 €	11 967.80 €	9 066.63 €	6 716.14 €	4 836.84 €	3 359.05 €	2 221.72 €	1 371.58 €	762.15 €	353.03 €	109.19 €	0.32 €
P-P'		8 494.22 €	15 756.23 €	21 942.28 €	27 190.72 €	31 623.93 €	35 349.99 €	38 464.27 €	41 050.77 €	43 183.40 €	44 927.04 €	46 338.59 €	47 467.85 €	48 358.28 €	49 047.79 €	49 569.31 €
UAI		9 173.75 €	8 835.61 €	8 514.34 €	8 209.44 €	7 920.42 €	7 646.75 €	7 387.92 €	7 143.44 €	6 912.79 €	6 695.45 €	6 490.94 €	6 298.75 €	6 118.38 €	5 949.34 €	5 791.16 €

Table 17 - *MTAC* (average for Continental Portugal)

Linear Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment		35 807.65 €	33 250.03 €	30 692.41 €	28 134.80 €	25 577.18 €	23 019.56 €	20 461.94 €	17 904.32 €	15 346.71 €	12 789.09 €	10 231.47 €	7 673.85 €	5 116.24 €	2 558.62 €	1.00 €
C''n		2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €	2 557.62 €
C'n+C''n		3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €	3 866.62 €

Exponential Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment		18 982.02 €	9 391.75 €	4 646.76 €	2 299.08 €	1 137.52 €	562.81 €	278.46 €	137.78 €	68.17 €	33.73 €	16.69 €	8.26 €	4.08 €	2.02 €	1.00 €
C''n		19 383.25 €	14 486.76 €	11 239.50 €	9 016.55 €	7 445.55 €	6 300.41 €	5 440.97 €	4 778.44 €	4 255.23 €	3 833.15 €	3 486.23 €	3 196.42 €	2 950.86 €	2 740.23 €	2 557.62 €
C'n+C''n		20 692.25 €	15 795.76 €	12 548.50 €	10 325.55 €	8 754.55 €	7 609.41 €	6 749.97 €	6 087.44 €	5 564.23 €	5 142.15 €	4 795.23 €	4 505.42 €	4 259.86 €	4 049.23 €	3 866.62 €

Sum of Digits Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
C'n		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Assignment		33 569.73 €	29 093.90 €	24 937.77 €	21 101.35 €	17 584.62 €	14 387.60 €	11 510.28 €	8 952.66 €	6 714.75 €	4 796.53 €	3 198.02 €	1 919.21 €	960.11 €	320.70 €	1.00 €
C''n		4 795.53 €	4 635.68 €	4 475.83 €	4 315.98 €	4 156.13 €	3 996.28 €	3 836.43 €	3 676.58 €	3 516.72 €	3 356.87 €	3 197.02 €	3 037.17 €	2 877.32 €	2 717.47 €	2 557.62 €
C'n+C''n		6 104.53 €	5 944.68 €	5 784.83 €	5 624.98 €	5 465.13 €	5 305.28 €	5 145.43 €	4 985.58 €	4 825.72 €	4 665.87 €	4 506.02 €	4 346.17 €	4 186.32 €	4 026.47 €	3 866.62 €

Table 18 - *MTAC-RPV* (average for Continental Portugal)

i_A	8%	Linear Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		35 807.65 €	33 250.03 €	30 692.41 €	28 134.80 €	25 577.18 €	23 019.56 €	20 461.94 €	17 904.32 €	15 346.71 €	12 789.09 €	10 231.47 €	7 673.85 €	5 116.24 €	2 558.62 €	1.00 €
Present Value (P')		33 155.23 €	28 506.54 €	24 364.63 €	20 679.91 €	17 407.40 €	14 506.23 €	11 939.35 €	9 673.15 €	7 677.17 €	5 923.82 €	4 388.10 €	3 047.39 €	1 881.23 €	871.11 €	0.32 €
C''n		5 210.04 €	4 929.36 €	4 666.88 €	4 421.34 €	4 191.57 €	3 976.51 €	3 775.13 €	3 586.51 €	3 409.79 €	3 244.14 €	3 088.83 €	2 943.16 €	2 806.46 €	2 678.15 €	2 557.66 €
C'n + C''n		6 422.07 €	6 096.51 €	5 791.35 €	5 505.23 €	5 236.87 €	4 985.06 €	4 748.72 €	4 526.81 €	4 318.36 €	4 122.49 €	3 938.37 €	3 765.22 €	3 602.31 €	3 448.99 €	3 304.62 €

i_A	8%	Exponential Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		18 982.02 €	9 391.75 €	4 646.76 €	2 299.08 €	1 137.52 €	562.81 €	278.46 €	137.78 €	68.17 €	33.73 €	16.69 €	8.26 €	4.08 €	2.02 €	1.00 €
Present Value (P')		17 575.94 €	8 051.91 €	3 688.75 €	1 689.89 €	774.18 €	354.67 €	162.48 €	74.44 €	34.10 €	15.62 €	7.16 €	3.28 €	1.50 €	0.69 €	0.32 €
C''n		20 789.33 €	15 156.68 €	11 558.84 €	9 168.84 €	7 518.22 €	6 335.10 €	5 457.54 €	4 786.35 €	4 259.02 €	3 834.96 €	3 487.10 €	3 196.83 €	2 951.06 €	2 740.33 €	2 557.66 €
C'n + C''n		22 001.36 €	16 323.82 €	12 683.31 €	10 252.74 €	8 563.51 €	7 343.66 €	6 431.13 €	5 726.65 €	5 167.59 €	4 713.31 €	4 336.64 €	4 018.89 €	3 746.91 €	3 511.16 €	3 304.62 €

i_A	8%	Sum of Digits Depreciation Method														
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acquisition (CA)	38 365.27 €															
Exploration (CE)		1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €	1 309.00 €
Present Value (P)		1 212.04 €	1 122.26 €	1 039.13 €	962.15 €	890.88 €	824.89 €	763.79 €	707.21 €	654.83 €	606.32 €	561.41 €	519.82 €	481.32 €	445.66 €	412.65 €
C'n		1 212.04 €	1 167.15 €	1 124.47 €	1 083.89 €	1 045.29 €	1 008.56 €	973.59 €	940.29 €	908.58 €	878.35 €	849.54 €	822.06 €	795.85 €	770.84 €	746.96 €
Assignment		33 569.73 €	29 093.90 €	24 937.77 €	21 101.35 €	17 584.62 €	14 387.60 €	11 510.28 €	8 952.66 €	6 714.75 €	4 796.53 €	3 198.02 €	1 919.21 €	960.11 €	320.70 €	1.00 €
Present Value (P')		31 083.09 €	24 943.33 €	19 796.41 €	15 510.12 €	11 967.80 €	9 066.63 €	6 716.14 €	4 836.84 €	3 359.05 €	2 221.72 €	1 371.58 €	762.15 €	353.03 €	109.19 €	0.32 €
C''n		7 282.18 €	6 710.97 €	6 189.62 €	5 713.79 €	5 279.49 €	4 883.11 €	4 521.30 €	4 191.05 €	3 889.58 €	3 614.35 €	3 363.06 €	3 133.59 €	2 924.02 €	2 732.58 €	2 557.66 €
C'n + C''n		8 494.22 €	7 878.11 €	7 314.09 €	6 797.68 €	6 324.79 €	5 891.66 €	5 494.90 €	5 131.35 €	4 798.16 €	4 492.70 €	4 212.60 €	3 955.65 €	3 719.87 €	3 503.41 €	3 304.62 €

3.1.2.7 Results comparison

This analysis revealed that the value of the acquisition of the Padre Américo Avenue Playground is slightly above the national average, both for tenders and for the price per m^2 of intervention. This finding can be interpreted in a few different ways, the most obvious being that this was a prior consultation tender, which usually results in higher values than those that end up being awarded.

Both for the national average and for the park under study, the lifespan is shorter than the economic one. This can be justified by the fact that a fixed exploration value is considered (which is unlikely to reflect reality).

In both cases, the economic models suggest replacement in the 15th year - which is considered the limit for replacing this type of asset due to the possible advanced state of degradation and obsolescence of the equipment.

Finally, it is also possible to compare the graphs of the economic models obtained for the specific case study park and for the data obtained with the national averages. Figures 10, 11 and 12 correspond to the Padre Américo Avenue Playground.

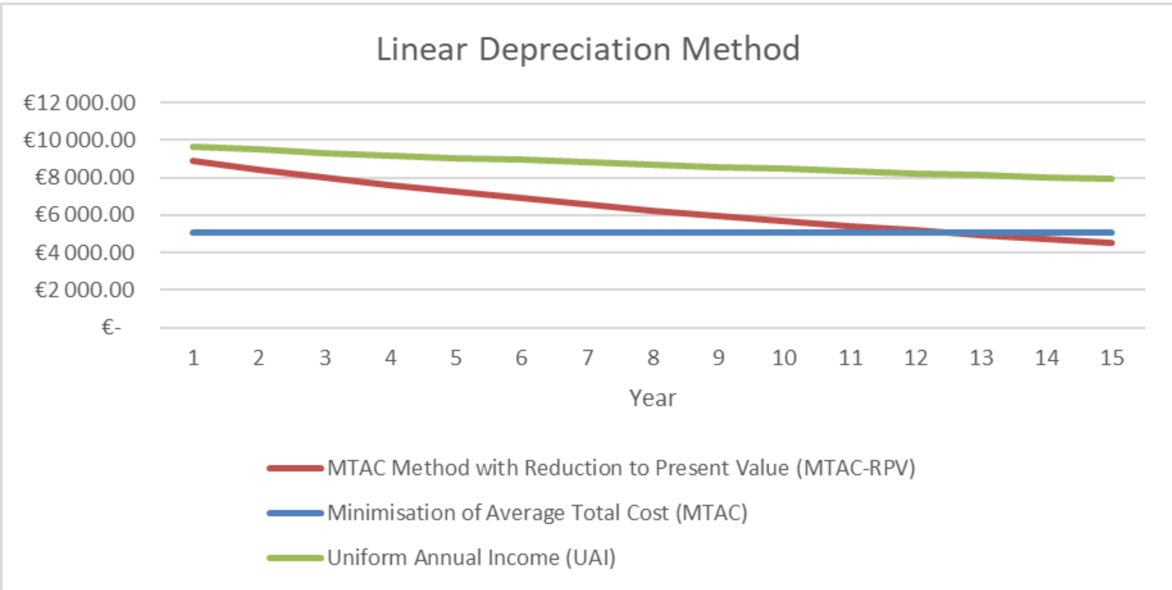

Figure 10 - Linear depreciation method applied to the playground case study: comparison between economic models

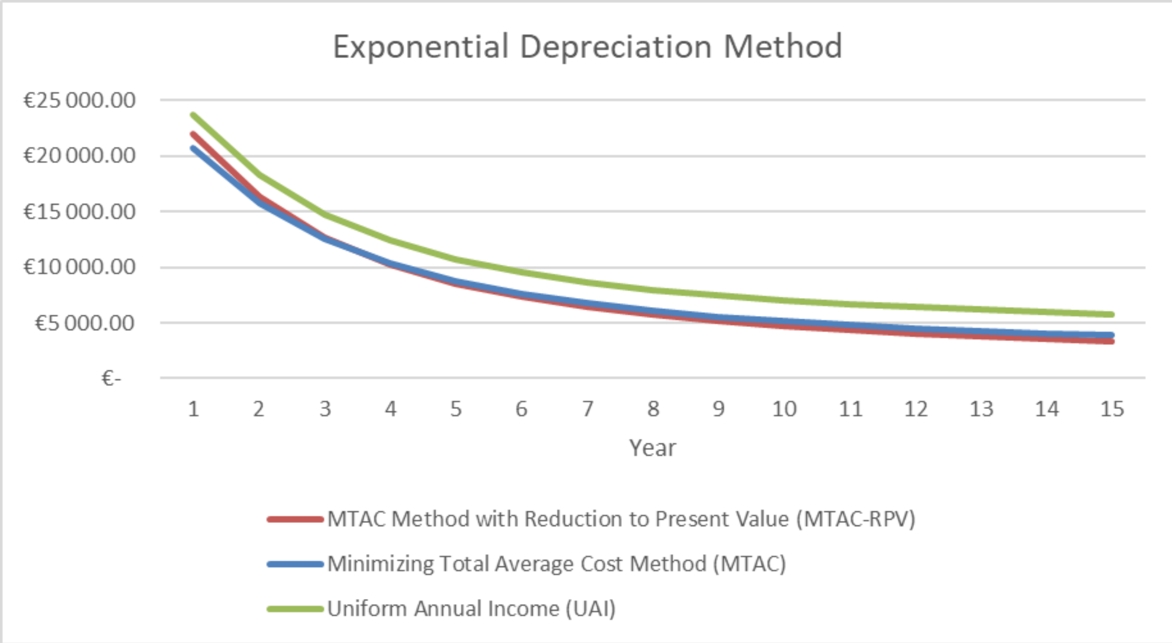

Figure 11 - Exponential method of depreciation applied to the playground case study: comparison between economic models

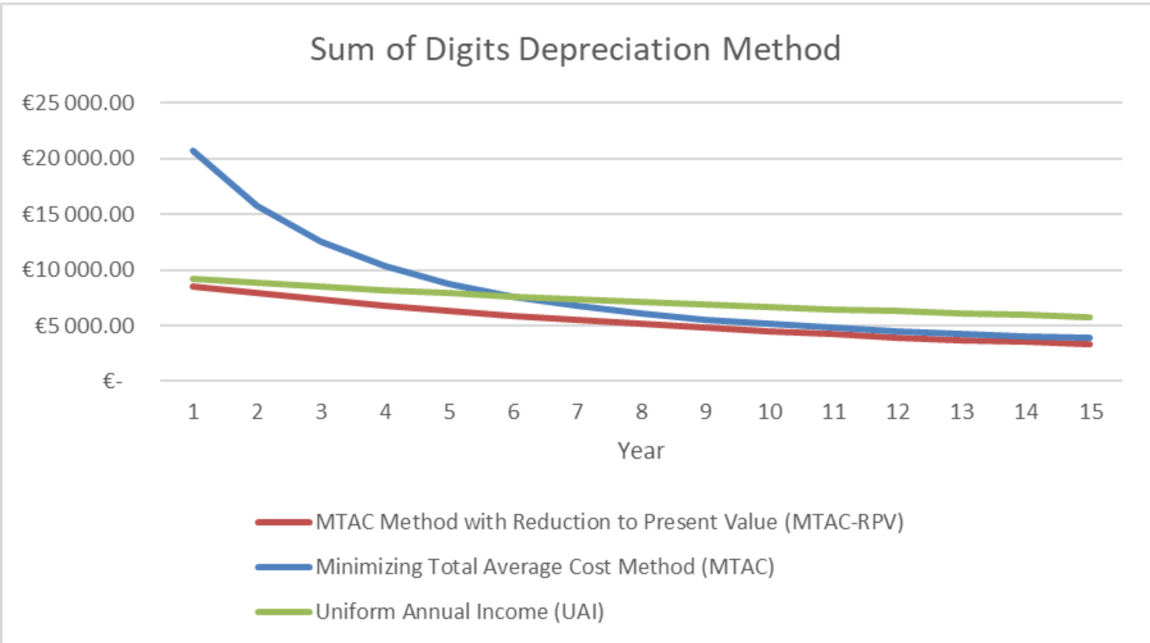

Figure 12 - Sum of the digits depreciation method applied to the playground case study: comparison of economic models

Figures 13, 14 and 15 correspond to the models calculated with the average values.

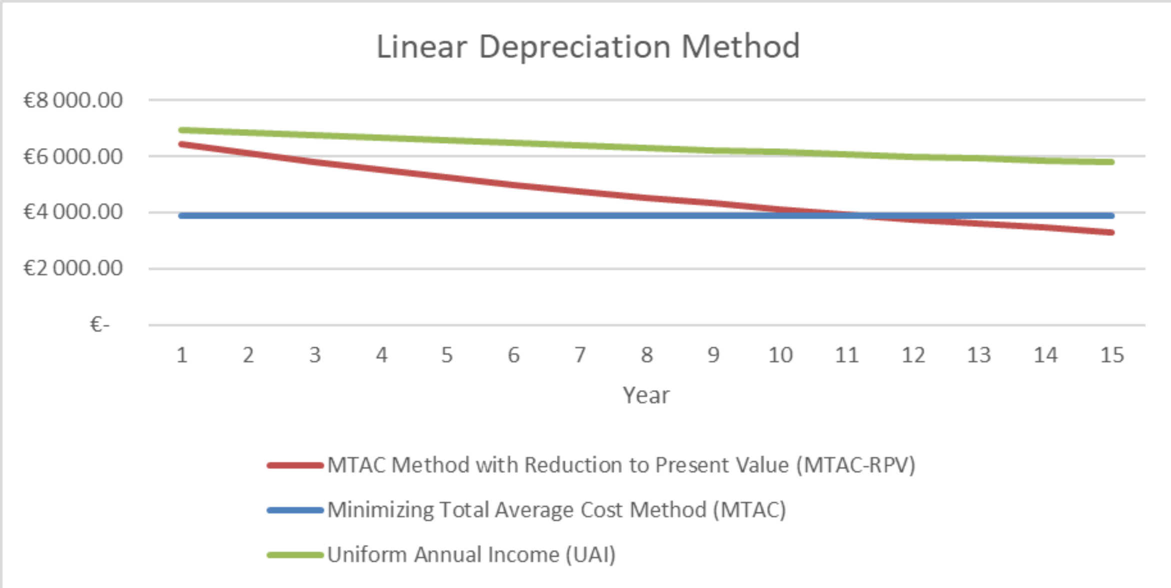

Figure 13 - Linear depreciation method applied to the playground case study: comparison between economic models (average for Continental Portugal)

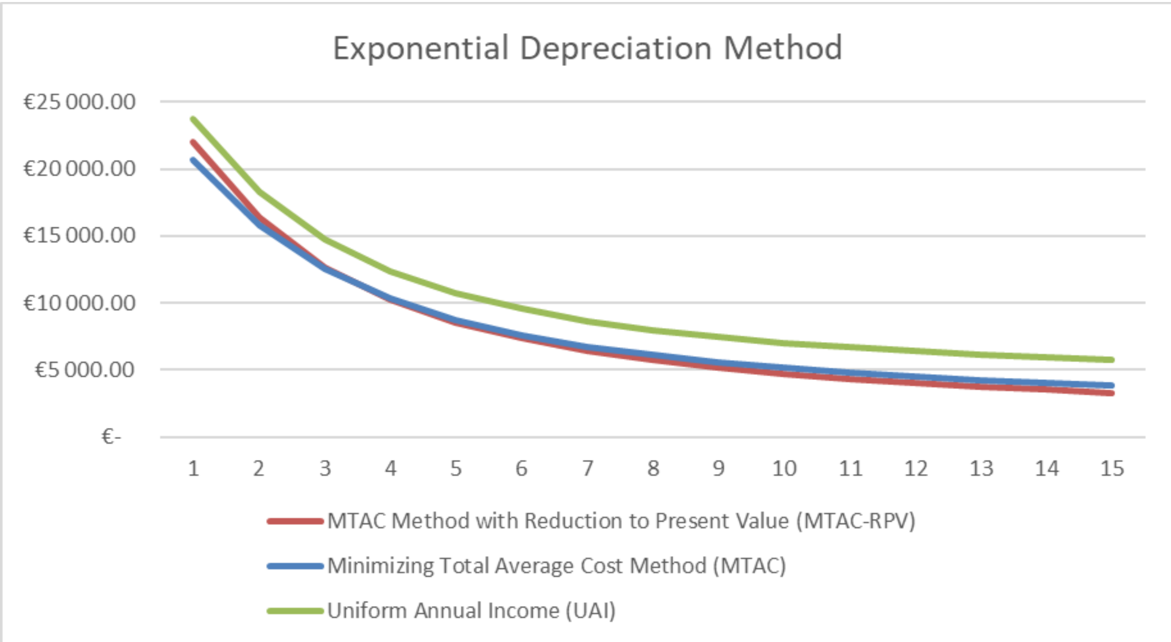

Figure 14 - Exponential depreciation method applied to the playground case study: comparison between economic models (average for Continental Portugal)

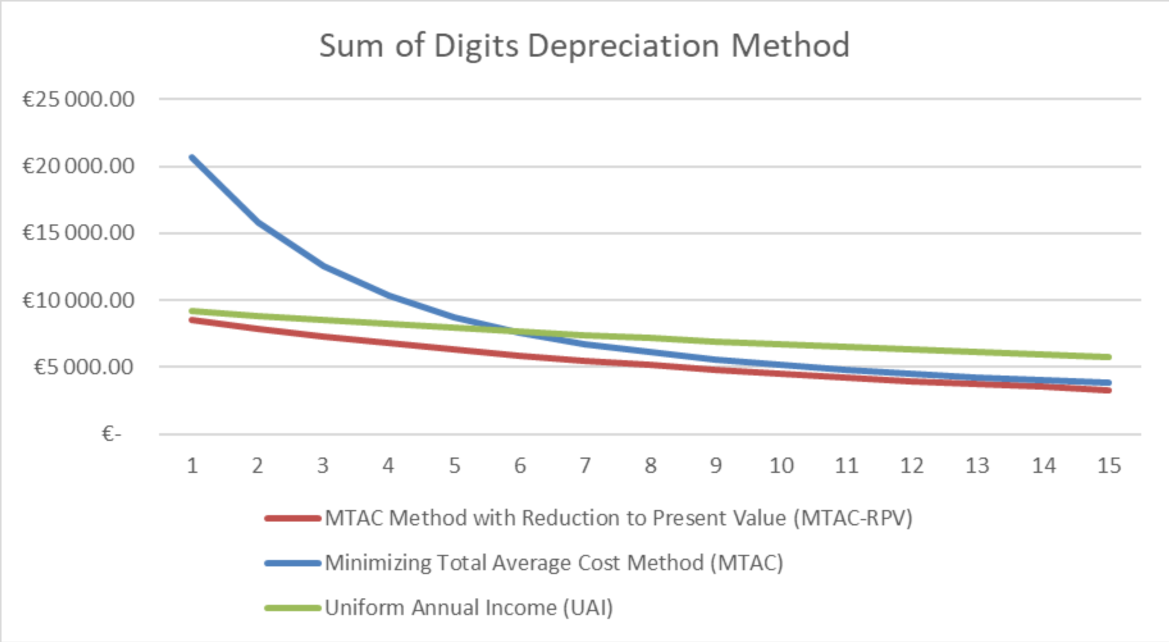

Figure 15 - Sum of the digits depreciation method applied to the playground case study: comparison between economic models (average for Continental Portugal)

3.1.2.8 Conclusions

This case study has shown that there is a lack of historical data on playgrounds in Portugal, despite the high level of investment in this type of state asset every year.

As playgrounds are subject to very strict rules when it comes to guaranteeing the safety of those who use them (mostly children), systematic preventive maintenance must be carried out rigorously, guaranteeing all the required precautions. This can also be a current problem, since many of the parks are in an advanced state of disrepair.

Based on the calculations made, and taking into account the type of asset, the most suitable depreciation curve would be that obtained by the sum of the digits method, since the asset does not degrade regularly and the remaining value considered is zero.

As for the economic models, any of them would give the same overall result, although the *MTAC* has the disadvantage of not considering the reduction to present value. This can lead to fallacious results when dealing with assets with a high economic cycle, as it does not take into account the variation in the value of money over the years.

Suggestions for future research based on this article are as follows:

- Seek to strengthen the quality of the national value average by increasing the database;
- Research and evaluation of assets on the field, or even in contact with the municipalities responsible;

- Research into the actual history of this type of asset;
- In addition to the data collection and calculations carried out, monitor the assets throughout their entire life cycle.

3.2 Assets of a heavy industry (*Celbi*'s Wood Debarking and Chipping Line)

In order to build on the research and mastery of life cycle models (both those explored in the previous chapter and the new ones - Life Cycle Investment and Life Cycle Recovery), the aim was to apply them to a real case in the heavy industry.

After a meeting with engineers from *Celbi*, part of the *Altri* group, it was concluded that it would be of mutual interest to study the life cycle of the Wood Debarking and Chipping Line acquired by the company in 2017.

This large asset consists of debarking and chipping wood so that it can later be transformed into paper pulp. It is, therefore, a critical and extremely important system for the company's operation. Upstream, the system is divided into two lines, the west line and the east line, which are responsible for debarking the wood. Downstream, these lines converge into a single line to cut the wood and turn it into chips. The company uses both market and domestically sourced wood.

3.2.1 Return on investment for the Wood Debarking and Chipping Line

In contrast to the case study presented in previous chapters, this type of asset brings a financial return to the company that acquires it, if the investment is meaningful. The return can be considered in two main ways:

- Considering the paper pulp sold and the operating costs of obtaining the final product. Therefore, the correct percentage of the cost would have to be considered, which corresponds to all the parameters involved, from wood acquisition and chip production to line maintenance costs. It's not a problem to obtain this data, but it is a problem to define the real percentage that considers these costs in relation to the paper pulp sold, bearing in mind that pulp is a final product that primarily goes through countless processes;
- The variation in the cost of chips when bought from external suppliers could be considered and compared with the costs of acquiring and producing them on the line. In this way, it's possible to make a direct comparison of the costs and define the return, taking into account the variation in the market price through the quantity of chips produced.

3.2.2 Life Cycle Models applied to the *Celbi*'s Wood Debarking and Chipping Line

As with the previous case study, the two types of life cycle will be analysed:

- Useful life cycle model;
- Economic life cycle model.

However, in addition to the *UAI*, *MTAC* and *MTAC-RPV*, this time two new models have been introduced to analyse the economic life cycle of this asset, which correspond to the state of the art. The Life Cycle Investment and Life Cycle of Physical Assets with Recovery models will be explained in detail in the following chapters.

Another major difference in this case study in comparison to the one of the children's playgrounds is that it takes into account real operating costs for the first 7 years of operation, in addition to maintenance costs. This time, it wasn't necessary to draw up any maintenance plan to be able to estimate these costs, since the data is provided directly by *Celbi*.

Perhaps, the main task will be to project future values since the data obtained shows some volatility for the period under consideration. Nevertheless, this is not a concern since the aim is to apply the models. In fact, predicting costs for the following periods doesn't need to be an exclusively "predictive" task. It can be much more than that, it can be an integral part of the company's strategic vision and planning, since if these future costs are seen as goals, it allows the company to control them and find reasons for possible deviations. If errors occur in management, operation, maintenance, or any area that influences the operability of equipment and production, and this causes a significant deviation from the pre-established targets, it makes it easier to understand why, readjust the parameters and most likely solve the problems.

3.2.2.1 Early considerations

In order to build any life cycle model, some initial considerations need to be taken into account, following the steps already explained in the previous chapters. What in the last case study required some research and estimates due to the scarcity of information, this time was made easier by the abundance and processing of data by *Celbi*.

Therefore, both the acquisition cost, the disposal value, the lifetime corresponding to the disposal value and the exploitation costs (operation and maintenance) were provided by the company, as shown in Tables 19 and 20.

Table 19 - *Celbi*'s Wood Debarking and Chipping Line acquisition data

Acquisition data		
Acquisition cost	*CA*	18 156 000.00 €
Cessation value	*VCn*	1.00 €
Lifetime corresponding to *VCn*	*N*	35
Average Availability	*A*	54%

Table 20 - *Celbi*'s Wood Debarking and Chipping Line annual explotation costs

Year	Annual operating cost	Annual maintenance cost	Annual Exploitation Cost
0	78 006.89 €	22 161.58 €	100 168.47 €
1	167 153.60 €	625 684.66 €	792 838.26 €
2	401 837.54 €	1 038 413.49 €	1 440 251.03 €
3	335 543.44 €	1 040 034.34 €	1 375 577.79 €
4	716 804.76 €	2 463 156.82 €	3 179 961.58 €
5	363 807.98 €	1 605 183.29 €	1 968 991.27 €
6	320 260.74 €	985 671.07 €	1 305 931.81 €
7	135 363.97 €	233 435.35 €	368 799.32 €
Total	2 518 778.92 €	8 013 740.61 €	10 532 519.52 €

Once again, as can be verified in Table 19, in order to show the application of all the above mentioned models, the cession value considered was 1.00€. Since there is no remaining value to consider, taking into account the cost of removing all the infrastructure.

A new indicator has also been provided: availability, in which case an average of 54 per cent. This information is useful when applying the latest models, as it is important to consider the availability of the equipment. Bearing in mind that while it is working it can be very profitable, but if it is susceptible to a lot of downtime, for example due to a high number of corrective maintenance interventions, or due to its operating characteristics it cannot work for too long without interruption, or because it can only work in certain weather conditions or times of day, among others, all of this can be associated with non-production costs. Once an investment has been made in the asset, it could, perhaps, be compared with other equipment of the same nature and with more availability, which would be a determining factor in its acquisition. Table 21 shows asset availability by year.

However, it should be kept in mind that these values represent the average availability of the two lines that make up this asset. The data was treated in this way, since it was not possible to distinguish the periods of time when the two lines were idle, and it was not possible to determine whether they worked simultaneously or not. This is nonetheless a relevant factor to consider in the calculations, knowing that the two lines can operate simultaneously and independently.

Table 21 - Asset availability by year

Year	Availability
1	60%
2	54%
3	57%
4	58%
5	52%
6	49%
7	48%
Mean	54%

With the introduction of new models, it was also necessary to collect more data than just the line availability. Table 22 shows the amount of wood that entered the company (with bark and without bark), as well as the amount of chips that had to be bought from external suppliers due to a lack of wood in Portugal.

Table 22 – Amount of wood entering the company per year

Wooden entries [m^3]						
Year	Chips (External supplier)	% Chips (External supplier)	Barked wood	Barkless wood	% Destroyed wood	Total
2018	529420	23.13%	973204	786064	76.87%	2288688
2019	569662	24.57%	1062966	686158	75.43%	2318786
2020	222047	10.24%	1199333	746775	89.76%	2168155
2021	495354	21.86%	1098956	672078	78.14%	2266388
2022	563043	24.10%	851659	922009	75.90%	2336711
2023	455543	20.67%	839031	908973	79.33%	2203547

At this time, the apparent rate was calculated both for the models already explained and for those that will be introduced in this case study. So, new variables were taken into account to calculate the taxes, which are an integral part of the *Fisher equation.* According to [66] "It is important to define the way the Apparent Rate (i_A) is calculated and influences the results of the models". The taxes considered are as follows:

- r = Interest rate;
- h = Inflation rate;
- p = Profit rate;
- R = Risk rate.

And are shown in equation 1.20, which corresponds to the *Fisher equation*:

$$(1 + i_A) = (1 + r) \times (1 + h) \times (1 + p) \times (1 + R) \tag{1.20}$$

The Interest rate and Inflation rate were collected from the *PORDATA* website[1]. As for the profit rate, a ratio of 20 per cent was agreed upon, since the production figures were not available.

A significant innovation with this approach is the consideration of the risk rate (R), since, according to [66] "it allows for many situations to be solved, including the impact of risk standards in organizations". The risk rate considered for this asset was obtained from the amount of chips that had to be acquired from external suppliers. According to the company's engineers, this only happens when there isn't enough wood to feed the machine.

Therefore, since this study focuses on the life cycle of the Wood Debarking and Chipping Line, it is considered that the risk for this investment is machine downtime due to a lack of raw materials. If the machine doesn't produce, it has to be bought out. If it is bought out, the investment in this asset will not be profitable. This consideration must be taken into account throughout the study, since although it is a crucial process in the production of paper pulp, its direct function is the production of chips. All these rates are shown in Table 23.

Table 23 - All rates

Inflation rate		Interest Rate		Risk		Profit Rate		Apparent Rate
Year	%	Year	%	Year	%	Year	%	%
2017	1.40	2017	2.73	2017	20.76	2017	20.00	50.95
2018	1.00	2018	2.41	2018	23.13	2018	20.00	52.83
2019	0.30	2019	2.28	2019	24.57	2019	20.00	53.35
2020	0.00	2020	2.00	2020	10.24	2020	20.00	34.93
2021	1.30	2021	2.03	2021	21.86	2021	20.00	51.14
2022	7.80	2022	2.71	2022	24.10	2022	20.00	64.89
2023	4.30	2023	5.47	2023	20.67	2023	20.00	59.29

[1] https://www.pordata.pt/, accessed 04/06/2024

For the projection of the following years, a constant value calculated from the average of the apparent rate numbers has been considered. This resulted in a 52.48% rate.

3.2.2.2 Prediction of future costs

In order to predict future costs (from period 7 onwards), the idea was to directly apply the constant mean of the values from the first 7 years. Since the variation in values from year to year is considerable, some algorithms were considered, such as simple exponential smoothing, exponential smoothing with adaptive alpha and *Excel*'s trend line tool. For these methods, the values for year 0 were excluded since it does not account for an entire period. In addition, the value for the seventh year was also ignored for the determination of the algorithm since it only contains values from the first quarter of 2024. This did not contribute to the good quality of the data when it came to calculating the error between forecasts and actual values.

An initial alpha of 0.3 was used for simple exponential smoothing. Equation 1.21 corresponds to the methodology used for this calculation, where S corresponds to the predicted value for period t and x the actual value for period t.

$$S_{t+1} = \alpha \times x_t + (1 - \alpha) \times S_t \tag{1.21}$$

The first forecast is made for the second year where $S_2 = x_1$. Then, the root mean square error (*RSME*) was calculated using the square root of the mean of the square of the error between each prediction and its actual value, using equation 1.22, where T is the number of periods considered.

$$RMSE = \sqrt{\sum_{t=1}^{T} \frac{(x_t - S_t)^2}{T}} \tag{1.22}$$

Finally, the *Excel* solve tool was used to minimise the *RSME* by changing the alpha. The conditions limiting the alpha to values between 0 and 1 were added, as can be seen in equation 1.23. Table 24 shows the obtained result.

$$0 \leq \alpha \leq 1 \tag{1.23}$$

Table 24 - Simple exponential smoothing of *CE*

Year	Annual Exploitation Cost	Prediction	Error^2	α
1	792 838.26 €	-	-	0.516816
2	1 440 251.03 €	792 838.26 €	4.19143E+11	
3	1 375 577.79 €	1 127 431.36 €	61576646306	
4	3 179 961.58 €	1 255 677.34 €	3.70287E+12	

5	1 968 991.27 €	2 250 177.71 €	79065814849	
6	1 305 931.81 €	2 104 856.13 €	6.3828E+11	
Mean			9.80187E+11	
RMSE			990 044.01 €	

Figure 16 shows the graph obtained with the real and estimated values, as well as the polynomial trend line of 4th degree obtained with the simple exponential smoothing of *CE*.

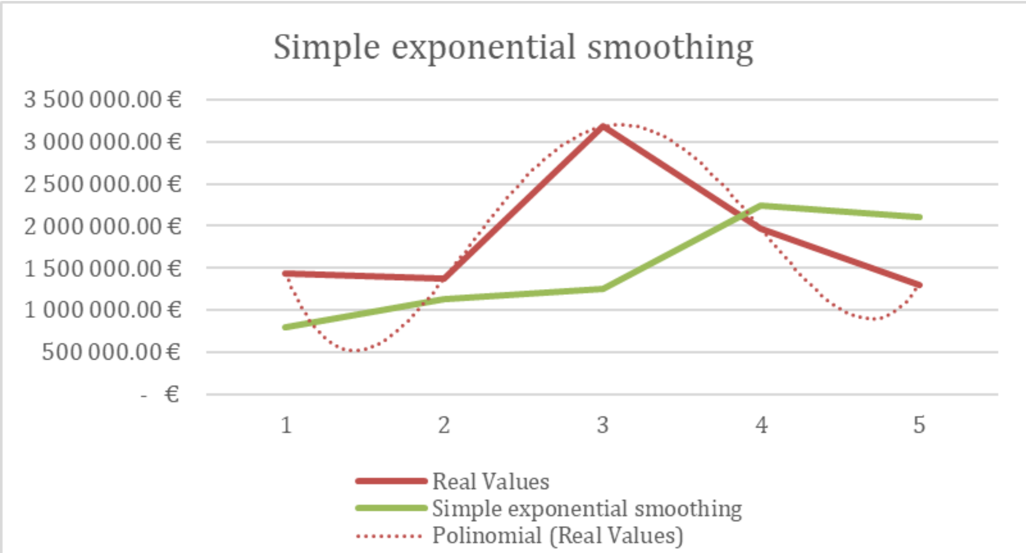

Figure 16 - Simple exponential smoothing of *CE*

In the end, an attempt was made to realise whether the error is greater by exponentially smoothing the *CE* or by summing the predictions obtained through simple exponential smoothing of the *CO* and *CM*. The conclusion was that, with this exponential smoothing, the first method provides a lower *RMSE*, as can be seen in Tables 25, 26 and 27.

Table 25 - Simple exponential smoothing of *CO*

Year	Annual Operation Cost	Prediction	Error^2	α
1	167 153.60 €	-	-	0.485415
2	401 837.54 €	167 153.60 €	55076552633	
3	335 543.44 €	281 072.74 €	2967058062	
4	716 804.76 €	307 513.64 €	1.67519E+11	
5	363 807.98 €	506 189.74 €	20272566888	
6	320 260.74 €	437 075.48 €	13645683461	
Mean			51896215297	
RMSE			227 807.41 €	

Table 26 - Simple exponential smoothing of *CM*

Year	Annual Maintenance Cost	Prediction	Error^2	α
1	625 684.66 €			0.51763
2	1 038 413.49 €	625 684.66 €	1.70345E+11	
3	1 040 034.34 €	839 325.35 €	40284099330	
4	2 463 156.82 €	943 218.28 €	2.31021E+12	
5	1 605 183.29 €	1 729 983.58 €	15575111098	
6	985 671.07 €	1 665 383.24 €	4.62009E+11	
Mean			5.99685E+11	
RMSE			774 393.45 €	

Table 27 - Sum of *CO* and *CM* predictions

Year	Annual Exploitation Cost	Prediction	Error^2
1	792 838.26 €		
2	1 440 251.03 €	792 838.26 €	4.19143E+11
3	1 375 577.79 €	1 120 398.09 €	65116679262
4	3 179 961.58 €	1 250 731.92 €	3.72193E+12
5	1 968 991.27 €	2 236 173.32 €	71386247543
6	1 305 931.81 €	2 102 458.72 €	6.34455E+11
Mean			9.82406E+11
RMSE			991 163.80 €

For the exponential smoothing methods with adaptive alpha, the same methodology was applied: to find, within each model, the calculation method that would lead to the lowest *RMSE*. Either by adaptive exponential smoothing of the sum of the operating costs, or by the sum of the predictions obtained by adaptive exponential smoothing. All the iterations were initiated with the variables to be modelled having a value of 0.3 and then solved using *Excel solver*.

In the first exponential smoothing with adaptive alpha, equations 1.22, 1.24-1.30 were used, where α_t is obtained as a function of the variables M_t, A_t, β and E_t.

$$E_t = x_t - S_t \tag{1.24}$$

$$A_t = \beta \times E_t + (1 - \beta) \times A_{t-1} \tag{1.25}$$

$$M_t = \beta \times |E_t| + (1 - \beta) \times M_{t-1} \tag{1.26}$$

$$\alpha_t = \left|\frac{A_t}{M_t}\right| \tag{1.27}$$

$$S_{t+1} = \alpha_t \times x_t + (1 - \alpha_t) \times S_t \tag{1.28}$$

$$0 \leq \alpha_t \leq 1 \tag{1.29}$$

$$0 \leq \beta \leq 1 \tag{1.30}$$

The obtained results for exponential smoothing with adaptive alpha of the sum of operating costs, and consequently where the smallest error lies, are shown in Table 28 and in the graph of Figure 17. The values achieved for the sum of the forecasts obtained by exponential smoothing with adaptive alpha are shown in Tables 29, 30 and 31.

Table 28 - Exponential smoothing with adaptive alpha 1 of CE

Year	Annual Exploitation Cost	Prediction	E_t	$E_t 2$	α	At	Mt	β
1	792 838.26 €	792 838.26 €	0	0	0.86	0.630165455	0.736346238	0.00
2	1 440 251.03 €	792 838.26 €	647 412.77 €	4.19143E+11	0.855800468	0.630165455	0.736346238	
3	1 375 577.79 €	1 346 894.41 €	28683.37457	822735977	0.855800468	0.630165455	0.736346238	
4	3 179 961.58 €	1 371 441.66 €	1808519.923	3.27074E+12	0.855800468	0.630165455	0.736346238	
5	1 968 991.27 €	2 919 173.85 €	-950182.5837	9.02847E+11	0.855800468	0.630165455	0.736346238	
6	1 305 931.81 €	2 106 007.15 €	-800075.3437	6.40121E+11	0.855800468	0.630165455	0.736346238	
Mean				1.04674E+12				
RMSE				1023100.957				

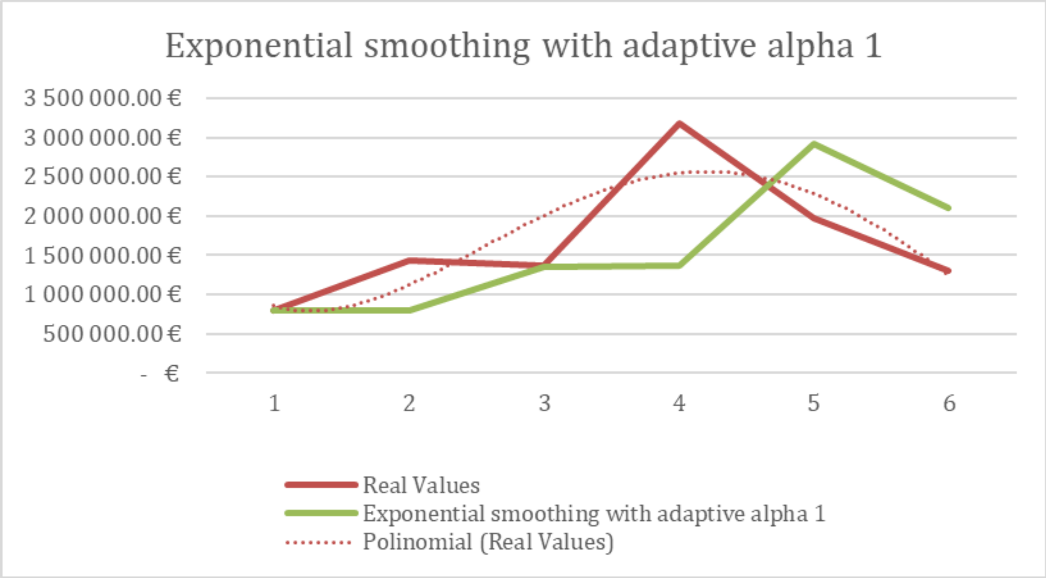

Figure 17 - Exponential smoothing with adaptive alpha 1 of *CE*

Table 29 - Exponential smoothing with adaptive alpha 1 of *CO*

Year	Annual Operation Cost	Prediction	E_t	E_t2	α	A_t	M_t	β
1	167 153.60 €	167 153.60 €	0	0	0.60	0.546574695	0.90703025	0.00
2	401 837.54 €	167 153.60 €	234 683.94 €	55076552633	0.602598088	0.546574695	0.90703025	
3	335 543.44 €	308 573.69 €	26969.75117	727367478.4	0.602598088	0.546574695	0.90703025	
4	716 804.76 €	324 825.61 €	391979.142	1.53648E+11	0.602598088	0.546574695	0.90703025	
5	363 807.98 €	561 031.49 €	-197223.516	38897115257	0.602598088	0.546574695	0.90703025	
6	320 260.74 €	442 184.98 €	-121924.2413	14865520608	0.602598088	0.546574695	0.90703025	
Mean				52642840747				
RMSE				229440.2771				

Table 30 - Exponential smoothing with adaptive alpha 1 of *CM*

Year	Annual Operation Cost	Prediction	E_t	E_t2	α	A_t	M_t	β
1	625 684.66 €	625 684.66 €	0	0	0.94	0.93896725	0.999458009	0.00
2	1 038 413.49 €	625 684.66 €	412 728.83 €	1.70345E+11	0.939476438	0.93896725	0.999458009	
3	1 040 034.34 €	1 013 433.67 €	26600.67092	707595693.1	0.939476438	0.93896725	0.999458009	
4	2 463 156.82 €	1 038 424.37 €	1424732.45	2.02986E+12	0.939476438	0.93896725	0.999458009	
5	1 605 183.29 €	2 376 926.94 €	-771743.6508	5.95588E+11	0.939476438	0.93896725	0.999458009	
6	985 671.07 €	1 651 891.97 €	-666220.8957	4.4385E+11	0.939476438	0.93896725	0.999458009	
Mean				6.48071E+11				
RMSE				805028.4195				

Table 31 - Sum of *CO* and *CM* predictions with exponential smoothing with adaptive alpha 1

Year	Annual Annual Exploitation Cost	Prediction	E_t	$E_t 2$
1	792 838.26 €	792 838.26 €	0	0
2	1 440 251.03 €	792 838.26 €	647 412.77 €	4.19143E+11
3	1 375 577.79 €	1 322 007.36 €	53570.42209	2869790123
4	3 179 961.58 €	1 363 249.99 €	1816711.592	3.30044E+12
5	1 968 991.27 €	2 937 958.44 €	-968967.1667	9.38897E+11
6	1 305 931.81 €	2 094 076.95 €	-788145.1369	6.21173E+11
Mean				1.0565E+12
RMSE				1027864.215

Equations 1.22, 1.28, 1.29, 1.31 and 1.32 were used to construct the exponential smoothing with adaptive alpha 2.

$$Er\%_{t+1} = \frac{x_t - S_t}{|x_t + S_t|} \tag{1.31}$$

$$\alpha_{t+1} = |\alpha_t \times (1 - Er\%_t)| \tag{1.32}$$

Tables 32-35 were thus calculated, showing that the sum of the *CO* and *CM* forecasts achieved through the respective exponential smoothing with adaptive alpha had the lowest *RMSE* value when compared to the exponential smoothing with adaptive alpha of *CE*. Therefore, the graph shown in Figure 18 is representative of the values provided in Table 35.

Equations 1.22, 1.28, 1.29, 1.31 and condition 1.33 were used for the exponential smoothing with adaptive alpha 3.

$$\alpha_t = MINIMUM(1, |\alpha_{t-1}(1 - E_{t-1})|) \tag{1.33}$$

Tables 36-39 and Figure 19 show the results and graphs achieved, respectively.

The last exponential smoothing with adaptive alpha used in this case study consisted of equations 1.24, 1.25, 1.28-1.31, 1.34 and 1.35.

$$M_t = \beta \times |E_{t-1}| + (1 - \beta) \times M_{t-1} \tag{1.34}$$

$$\alpha_t = MINIMUM\left(1, \left|\frac{A_t}{M_t}\right|\right), If\ 1 > \left|\frac{A_t}{M_t}\right|, \alpha_t = 0 \tag{1.35}$$

Tables 40-43 and Figure 20 respectively show the values and graph achieved.

Table 32 - Exponential smoothing with adaptive alpha 2 of *CE*

Year	Annual Exploitation Cost	Prediction	$Er\%_t$	α	E_t	E_t2
1	792 838.26 €	792 838.26 €	0%	0.66614243	0.00 €	0
2	1 440 251.03 €	792 838.26 €	0%	0.66614243	647 412.77 €	4.19143E+11
3	1 375 577.79 €	1 224 107.38 €	29%	0.66614243	151 470.41 €	22943285303
4	3 179 961.58 €	1 281 094.24 €	6%	0.376224434	1 898 867.33 €	3.6057E+12
5	1 968 991.27 €	1 884 857.21 €	43%	0.317959528	84 134.06 €	7078540555
6	1 305 931.81 €	1 893 918.00 €	2%	0.107694714	-587 986.19 €	3.45728E+11
Mean						8.80118E+11
RMSE						938146.0483

Table 33 - Exponential smoothing with adaptive alpha 2 of *CO*

Year	Annual Operation Cost	Prediction	$Er\%_t$	α	E_t	E_t2
1	167 153.60 €	167 153.60 €	0%	0.648738691	0.00 €	0
2	401 837.54 €	167 153.60 €	0%	0.648738691	234 683.94 €	55076552633
3	335 543.44 €	319 402.15 €	41%	0.648738691	16 141.29 €	260541326.7
4	716 804.76 €	323 216.06 €	2%	0.236282457	393 588.70 €	1.54912E+11
5	363 807.98 €	406 514.07 €	38%	0.211637215	-42 706.09 €	1823810386
6	320 260.74 €	399 390.44 €	-6%	0.166805884	-79 129.70 €	6261510126
Mean						43666895791
RMSE						208966.2551

Table 34 - Exponential smoothing with adaptive alpha 2 of *CM*

Year	Annual Maintenance Cost	Prediction	$Er\%_t$	α	E_t	E_t^2
1	625 684.66 €	625 684.66 €	0%	0.667744871	0.00 €	0
2	1 038 413.49 €	625 684.66 €	0%	0.667744871	412 728.83 €	1.70345E+11

3	1 040 034.34 €	901 282.22 €	25%	0.667744871	138 752.12 €	19252151595
4	2 463 156.82 €	959 520.00 €	7%	0.419725347	1 503 636.82 €	2.26092E+12
5	1 605 183.29 €	1 483 164.74 €	44%	0.348252139	122 018.55 €	14888526393
6	985 671.07 €	1 494 276.23 €	4%	0.091063899	-508 605.16 €	2.58679E+11
Mean						5.44818E+11
RMSE						738117.6958

Table 35 - Sum of *CO* and *CM* predictions with exponential smoothing with adaptive alpha 2

Year	Annual Exploitation Cost	Prediction	E_t	${E_t}^2$
1	792 838.26 €	792 838.26 €	0%	0
2	1 440 251.03 €	792 838.26 €	0%	4.19143E+11
3	1 375 577.79 €	1 220 684.37 €	29%	23991970149
4	3 179 961.58 €	1 282 736.06 €	6%	3.59946E+12
5	1 968 991.27 €	1 889 678.81 €	43%	6290465725
6	1 305 931.81 €	1 893 666.67 €	2%	3.45432E+11
Mean				8.78865E+11
RMSE				937477.7524

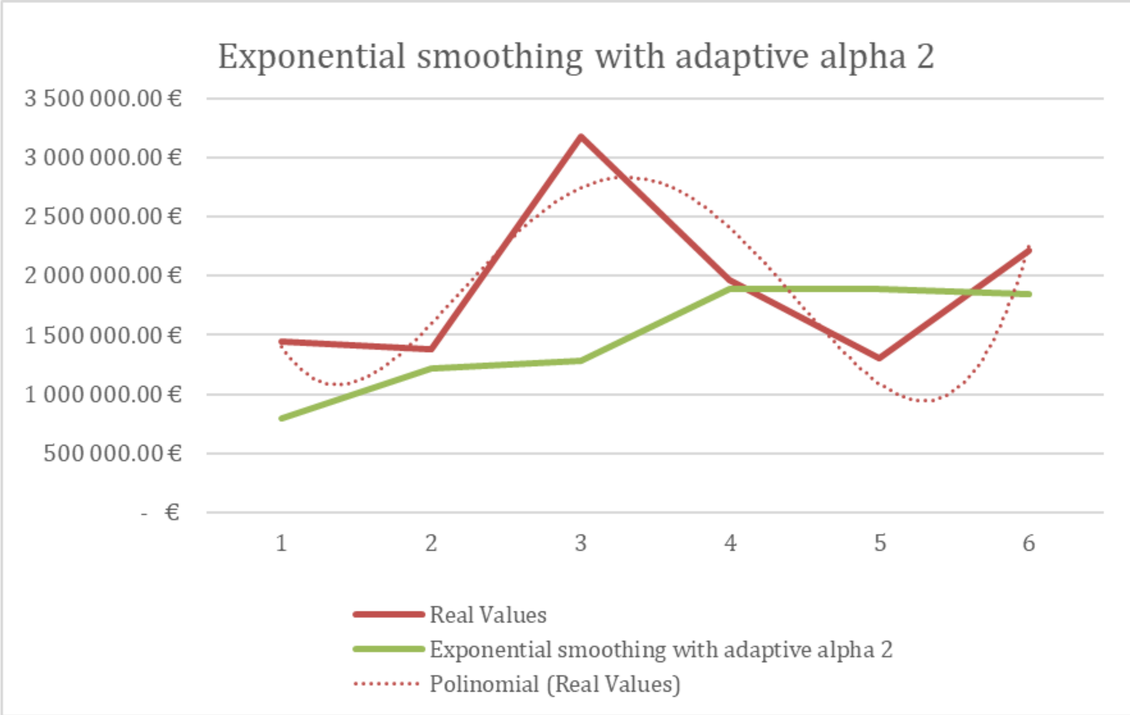

Figure 18 – Sum of exponential smoothing with adaptive alpha 2 of *CO* and *CM*

Table 36 - Exponential smoothing with adaptive alpha 3 of *CE*

Year	Annual Exploitation Cost	Prediction	$Er\%$	α	$\vert \alpha\ t\ (1-Er\%t) \vert$	E_t	$E_t 2$
1	792 838.26 €	792 838.26 €	0%	0.61011578	0.61011578	0	0
2	1 440 251.03 €	792 838.26 €	0%	0.61011578	0.61011578	647 412.77 €	4.19143E+11
3	1 375 577.79 €	1 187 835.01 €	29%	0.61011578	0.433232235	187742.7797	35247351330
4	3 179 961.58 €	1 269 171.23 €	7%	0.433232235	0.401502572	1910790.349	3.65112E+12
5	1 968 991.27 €	2 036 358.47 €	43%	0.401502572	0.229067342	-67367.20104	4538339776

6	1 305 931.81 €	2 020 926.85 €	-2%	0.229067342	0.232920096	-714995.0354	5.11218E+11
Mean							9.24253E+11
RMSE							961380.9493

Table 37 - Exponential smoothing with adaptive alpha 3 of *CO*

Year	Annual Operation Cost	Prediction	$Er\%$	α	$\|\alpha\ t\ (1-Er\%t)\|$	E_t	E_t2
1	167 153.60 €	167 153.60 €	0%	0.6299072	0.6299072	0	0
2	401 837.54 €	167 153.60 €	0%	0.6299072	0.6299072	234 683.94 €	55076552633
3	335 543.44 €	314 982.70 €	41%	0.6299072	0.370098048	20560.74125	422744080.7
4	716 804.76 €	322 592.19 €	3%	0.370098048	0.358400609	394212.5624	1.55404E+11
5	363 807.98 €	463 878.22 €	38%	0.358400609	0.222469844	-100070.2364	10014052205
6	320 260.74 €	441 615.61 €	-12%	0.222469844	0.249367248	-121354.8655	14727003381
Mean							47128779324
RMSE							217091.6381

Table 38 - Exponential smoothing with adaptive alpha 3 of *CM*

Year	Annual Maintenance Cost	Prediction	$Er\%$	α	$\|\alpha\ t\ (1-Er\%t)\|$	E_t	E_t2
1	625 684.66 €	625 684.66 €	0%	0.598117244	0.598117244	0	0
2	1 038 413.49 €	625 684.66 €	0%	0.598117244	0.598117244	412 728.83 €	1.70345E+11
3	1 040 034.34 €	872 544.89 €	25%	0.598117244	0.449772491	167489.4515	28052716374
4	2 463 156.82 €	947 877.04 €	9%	0.449772491	0.410384764	1515279.786	2.29607E+12
5	1 605 183.29 €	1 569 724.78 €	44%	0.410384764	0.228079996	35458.51533	1257306309
6	985 671.07 €	1 577 812.15 €	1%	0.228079996	0.225532716	-592141.0837	3.50631E+11
Mean							6.8062E+11

RMSE	824996.9583

Table 39 - Sum of *CO* and *CM* predictions with exponential smoothing with adaptive alpha 3

Year	Annual Exploitation Cost	Prediction	E_t	$E_t 2$
1	792 838.26 €	792 838.26 €	0	0
2	1 440 251.03 €	792 838.26 €	647412.772	4.19143E+11
3	1 375 577.79 €	1 187 527.59 €	188050.1928	35362875004
4	3 179 961.58 €	1 270 469.23 €	1909492.349	3.64616E+12
5	1 968 991.27 €	2 033 602.99 €	-64611.72103	4174674494
6	1 305 931.81 €	2 019 427.76 €	-713495.9492	5.09076E+11
Mean				9.22784E+11
RMSE				960616.2965

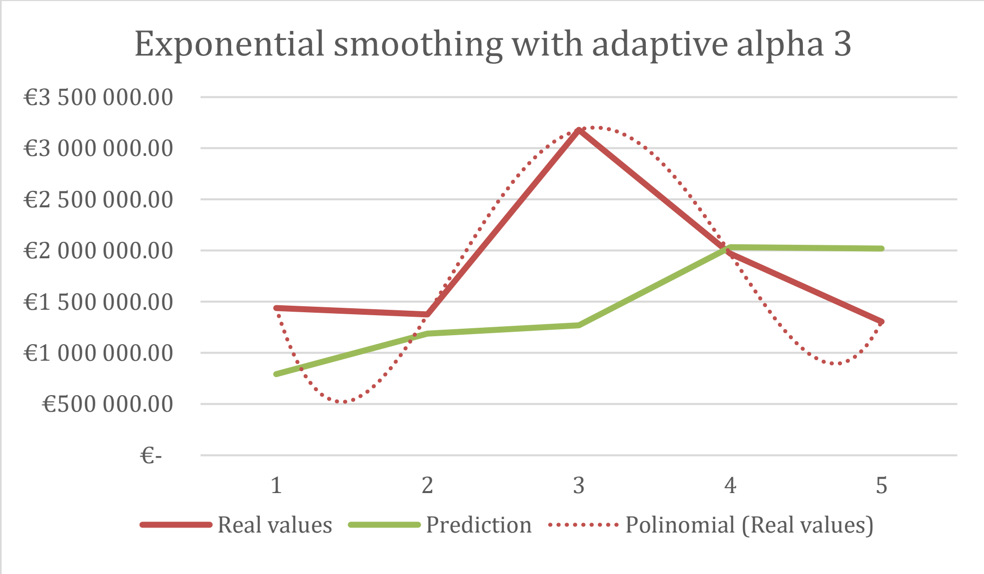

Figure 19 - Sum of exponential smoothing with adaptive alpha 3 of *CO* and *CM*

Table 40 - Exponential smoothing with adaptive alpha 4 of CE

Year	Annual Exploitation Cost	Prediction	$Er\%$	α	A_t	M_t	E_t	E_t2	β
1	792 838.26 €	792 838.26 €	0%	0	0.30000004	0.3	0.00 €	0	0.0300006
2	1 440 251.03 €	792 838.26 €	0%	1	19 423.05 €	0.290999828	647 412.77 €	4.19143E+11	
3	1 375 577.79 €	1 440 251.03 €	29%	1	16900.10907	19423.03736	-64 673.24 €	4182628489	
4	3 179 961.58 €	1 375 577.79 €	-2%	1	70525.6465	20780.56955	1 804 383.79 €	3.2558E+12	
5	1 968 991.27 €	3 179 961.58 €	40%	1	32080.03161	74289.69093	-1 210 970.31 €	1.46645E+12	
6	1 305 931.81 €	1 968 991.27 €	-24%	1	11225.44751	108390.7625	-663 059.46 €	4.39648E+11	
Mean								1.11704E+12	
RMSE								1056903.377	

Table 41 - Exponential smoothing with adaptive alpha 4 of *CO*

Year	Annual Operation Cost	Prediction	$Er\%$	α	A_t	M_t	E_t	E_t2	β
1	167 153.60 €	167 153.60 €	0%	0	0.60257217	0.3	0.00 €	0	0
2	401 837.54 €	167 153.60 €	0%	1	0.60257217	0.3	234 683.94 €	55076552633	
3	335 543.44 €	308 567.61 €	41%	0.60257217	0.60257217	0.3	26 975.83 €	727695611.1	
4	716 804.76 €	324 822.50 €	4%	0.60257217	0.60257217	0.3	391 982.26 €	1.5365E+11	
5	363 807.98 €	561 020.10 €	38%	0.60257217	0.60257217	0.3	-197 212.12 €	38892619449	
6	320 260.74 €	442 185.56 €	-21%	0.60257217	0.60257217	0.3	-121 924.82 €	14865662487	
Mean								52642524220	
RMSE								229439.5873	

Table 42 - Exponential smoothing with adaptive alpha 4 of *CM*

Year	Annual Maintenance Cost	Prediction	$Er\%$	α	At	Mt	E_t	E_t2	β
1	625 684.66 €	625 684.66 €	0%	0	0.300000049	0.3	0.00 €	0	0.0300008
2	1 038 413.49 €	625 684.66 €	0%	1	12382.50333	0.290999747	412 728.83 €	1.70345E+11	
3	1 040 034.34 €	1 038 413.49 €	25%	1	12059.64473	12382.4946	1 620.85 €	2627161.206	
4	2 463 156.82 €	1 040 034.34 €	0%	1	54392.71767	12059.63626	1 423 122.48 €	2.02528E+12	
5	1 605 183.29 €	2 463 156.82 €	41%	1	27020.96212	54392.70945	-857 973.53 €	7.36119E+11	
6	985 671.07 €	1 605 183.29 €	-21%	1	7624.422387	78500.81062	-619 512.22 €	3.83795E+11	
Mean								6.63108E+11	
RMSE								814314.3485	

Table 43 - Sum of *CO* and *CM* predictions with exponential smoothing with adaptive alpha 4

Year	Annual Exploitation Cost	Prediction	E_t	E_t2
1	792 838.26 €	792 838.26 €	0.00 €	0
2	1 440 251.03 €	792 838.26 €	647 412.77 €	4.19143E+11
3	1 375 577.79 €	1 346 981.10 €	28 596.69 €	817770440.7
4	3 179 961.58 €	1 364 856.84 €	1 815 104.74 €	3.29461E+12
5	1 968 991.27 €	3 024 176.92 €	-1 055 185.65 €	1.11342E+12
6	1 305 931.81 €	2 047 368.85 €	-741 437.04 €	5.49729E+11
Mean				1.07554E+12
RMSE				1037083.597

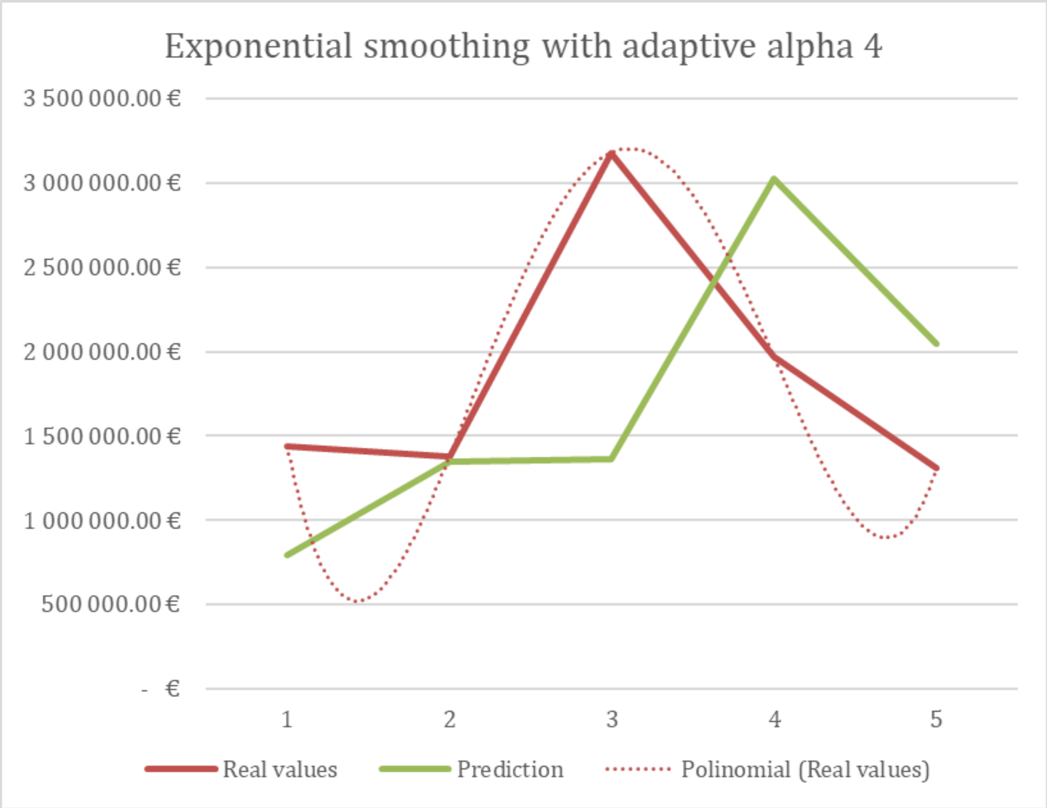

Figure 20 - Sum of exponential smoothing with adaptive alpha 4 of *CO* and *CM*

Gathering the minimum *RMSE* values for each method, it can be said that the lowest value is obtained using the exponential smoothing with adaptive alpha 2 approach, as can be seen in Table 44 and the graph in Figure 21.

Table 44 - Minimum values for all methods

Method	Minimum RMSE value
Simple exponential smoothing	990044.0062383
Exponential smoothing with adaptive alpha 1	1023100.9571935
Exponential smoothing with adaptive alpha 2	937477.7524179
Exponential smoothing with adaptive alpha 3	960616.2965279
Exponential smoothing with adaptive alpha 4	1037083.5971540

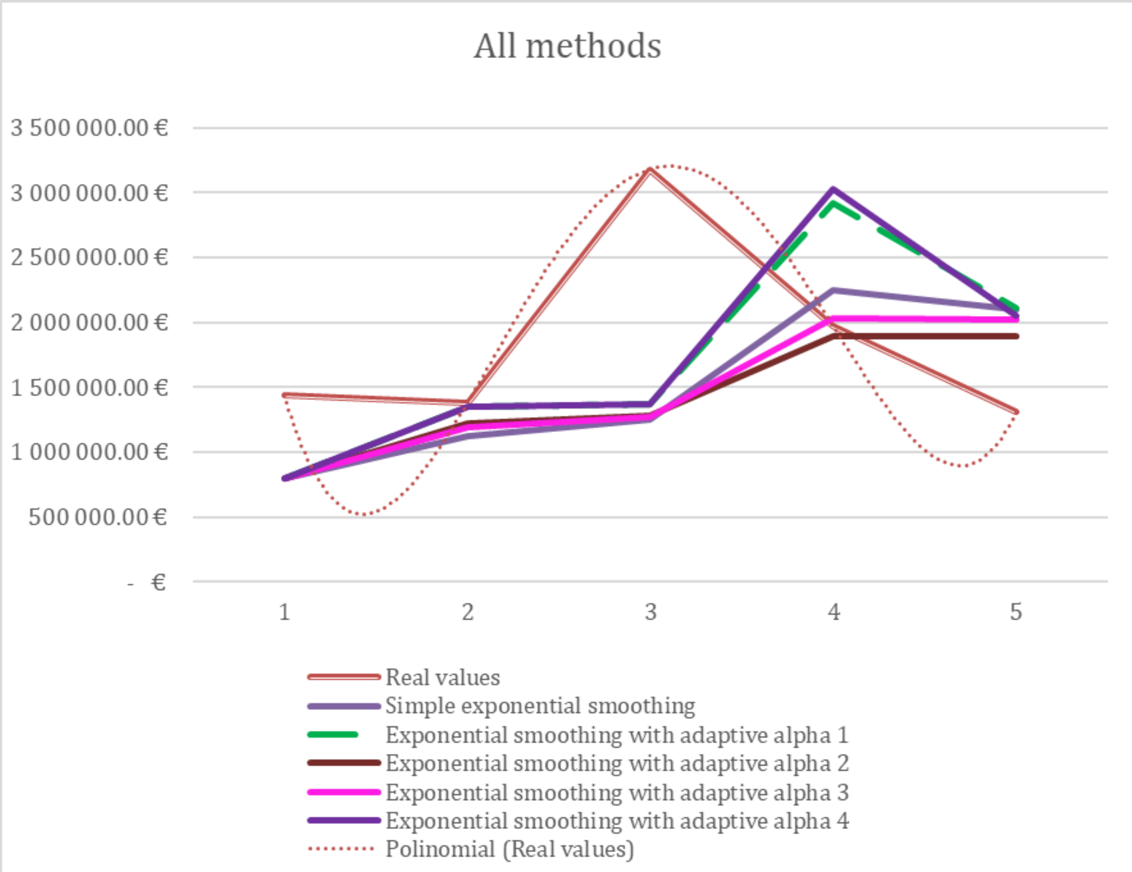

Figure 21 - All methods

Although it seems that using *Excel*'s degree 4 polynomial trend line is possible to produce relatively decent results, it is not at all the best option for this asset, given the large difference from year to year.

However, for any of the exponential smoothing methods, predictions can only be made for the period *t+1*, so it is not possible to make predictions up to the period considered for the cession value.

The company did not communicate its targets for maximum costs per period until the end of the asset's life, which would actually be the best option for the company's management and planning area. Furthermore, there was not enough time during the writing of this thesis to develop more advanced and complex forecasting algorithms in more depth, which provides a wide range of possibilities for future research on the subject.

As a result, it was chosen to use the values predicted by the exponential smoothing with adaptive alpha 2 method for period 8. But first it became necessary to determine the costs for the whole of year 7. The values for the first 2 months were then multiplied by 6 to obtain the value for the 12 months. Although the costs do not behave linearly over the course of a calendar year, this was considered the most appropriate approach in this case.

As a result, the following figures were obtained for the 7th period:

- CO_7 = 812 183.82€;
- CM_7 = 1 400 612.10€;
- CE_7 = 2 212 795.92€.

For the 8th year of operation, the following values were found using the exponential smoothing method with adaptive alpha 2:

- CO_8 = 524 776.68€;
- CM_8 = 1 450 746.57€;
- CE_8 = 1 975 523.25€.

To establish the remaining exploration cost amounts up to the 35th period, two different methods were employed. For the operating cost, a constant value was taken from calculating the average of the first 7 years of operation - 278 131.96€. For maintenance costs, a 20 per cent increase was considered each year until the end of equipment's life.

3.2.2.3 Life Cycle Models – Lifespan

In the same way as in the case of the playground assets, the lifespan model was calculated using equation 1.4. To calculate the apparent rate for this model, only the capitalization rate and the inflation rate were considered. The acquisition cost taken into account is the one provided by the company.

The equipment should be replaced around period 15, when the maintenance costs exceed 18 565 398.87€, which corresponds to the maintenance costs plus the capital amortization of equivalent new equipment. These data can be verified in Table 45 and Figure 22.

Table 45 - Lifetime method with apparent rate: annual values for *Celbi*'s equipment

Year	Maintenance Costs	Apparent Rate	Σ CM_j (Apparent Rate)
1	625 684.66 €	3.41%	605 052.37 €
2	1 038 413.49 €	2.58%	1 591 888.26 €
3	1 040 034.34 €	2.00%	2 571 935.84 €
4	2 463 156.82 €	3.33%	4 732 595.42 €
5	1 605 183.29 €	10.51%	5 706 500.73 €
6	985 671.07 €	9.77%	6 269 917.83 €
7	1 400 612.10 €	5.10%	7 258 413.79 €
8	1 450 746.57 €	5.10%	8 232 568.99 €

9	1 740 895.88 €	5.10%	9 344 784.56 €
10	2 089 075.06 €	5.10%	10 614 626.87 €
11	2 506 890.07 €	5.10%	12 064 435.28 €
12	3 008 268.09 €	5.10%	13 719 715.15 €
13	3 609 921.70 €	5.10%	15 609 586.57 €
14	4 331 906.04 €	5.10%	17 767 296.58 €
15	5 198 287.25 €	5.10%	20 230 804.13 €
16	6 237 944.70 €	5.10%	23 043 447.83 €
17	7 485 533.64 €	5.10%	26 254 708.34 €
18	8 982 640.37 €	5.10%	29 921 078.94 €
19	10 779 168.45 €	5.10%	34 107 059.25 €
20	12 935 002.13 €	5.10%	38 886 290.03 €
21	15 522 002.56 €	5.10%	44 342 848.63 €
22	18 626 403.07 €	5.10%	50 572 728.10 €
23	22 351 683.69 €	5.10%	57 685 525.94 €
24	26 822 020.43 €	5.10%	65 806 372.16 €
25	32 186 424.51 €	5.10%	75 078 130.58 €
26	38 623 709.41 €	5.10%	85 663 912.15 €
27	46 348 451.30 €	5.10%	97 749 944.41 €
28	55 618 141.56 €	5.10%	111 548 847.67 €
29	66 741 769.87 €	5.10%	127 303 375.48 €
30	80 090 123.84 €	5.10%	145 290 685.17 €
31	96 108 148.61 €	5.10%	165 827 213.71 €
32	115 329 778.33 €	5.10%	189 274 244.50 €
33	138 395 734.00 €	5.10%	216 044 263.22 €
34	166 074 880.80 €	5.10%	246 608 214.29 €
35	199 289 856.96 €	5.10%	281 503 785.90 €

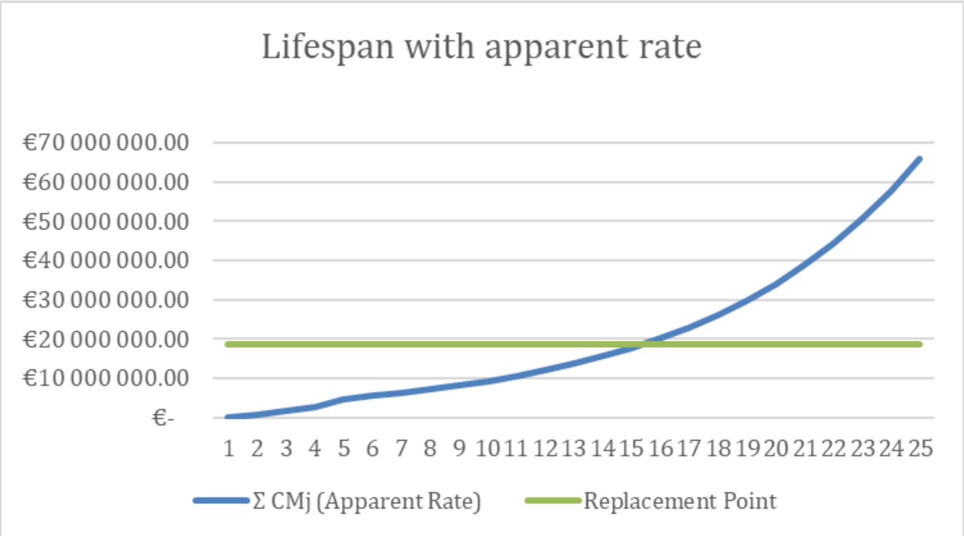

Figure 22 - Lifespan with apparent rate for *Celbi*'s equipment

3.2.2.4 Life Cycle Models - Economic

Similarly, to the previous case study, before applying the life cycle models mentioned above, it is necessary to calculate the depreciation of the assets. For this reason, the three depreciation methods explained in the previous chapters of this thesis were applied once again:

- Linear method – Equations 1.5 and 1.6;
- Exponential method – Equations 1.7 and 1.8;
- Sum of digits method – Equations 1.9, 1.10, 1.11 and 1.12.

To calculate the linear method, it was considered *CA* = 18 156 000.00€; *VCn* = 1.00€ for an *N* value of 35 years. This resulted in a *dl* = 518 742.83€, as shown in Table 46 and Figure 23.

Table 46 - Linear depreciation method: annual depreciation

Linear depreciation method		
dl	Annual Depreciation Rate	3 782.93 €
0	18 156 000.00 €	
1	17 637 257.17 €	
2	17 118 514.34 €	
3	16 599 771.51 €	
4	16 081 028.69 €	
5	15 562 285.86 €	
6	15 043 543.03 €	
7	14 524 800.20 €	
8	14 006 057.37 €	
9	13 487 314.54 €	
10	12 968 571.71 €	
11	12 449 828.89 €	
12	11 931 086.06 €	
13	11 412 343.23 €	
14	10 893 600.40 €	
15	10 374 857.57 €	
16	9 856 114.74 €	
17	9 337 371.91 €	
18	8 818 629.09 €	
19	8 299 886.26 €	
20	7 781 143.43 €	
21	7 262 400.60 €	
22	6 743 657.77 €	

23	6 224 914.94 €
24	5 706 172.11 €
25	5 187 429.29 €
26	4 668 686.46 €
27	4 149 943.63 €
28	3 631 200.80 €
29	3 112 457.97 €
30	2 593 715.14 €
31	2 074 972.31 €
32	1 556 229.49 €
33	1 037 486.66 €
34	518 743.83 €
35	1.00 €

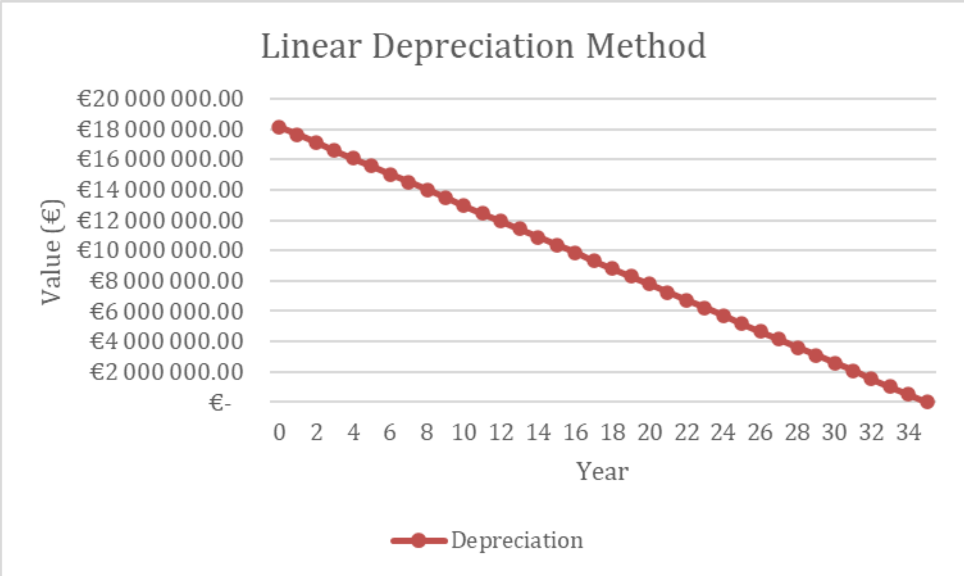

Figure 23 - Linear depreciation method applied to *Celbi*'s equipment

To calculate *T* for the exponential depreciation method, the same values were used for the *CA*, *VCn* and *N* variables. The results obtained are shown in Table 47 and Figure 24.

Table 47 - Exponential depreciation method: annual depreciation

Exponential method of depreciation	
T	0.379703367
Exponential depreciation	
0	18 156 000.00 €
1	11 262 105.66 €
2	6 985 846.22 €

3	4 333 296.89 €
4	2 687 929.47 €
5	1 667 313.60 €
6	1 034 229.01 €
7	641 528.77 €
8	397 938.14 €
9	246 839.69 €
10	153 113.83 €
11	94 975.99 €
12	58 913.29 €
13	36 543.71 €
14	22 667.94 €
15	14 060.85 €
16	8 721.90 €
17	5 410.16 €
18	3 355.91 €
19	2 081.66 €
20	1 291.24 €
21	800.95 €
22	496.83 €
23	308.18 €
24	191.16 €
25	118.58 €
26	73.55 €
27	45.63 €
28	28.30 €
29	17.56 €
30	10.89 €
31	6.75 €
32	4.19 €
33	2.60 €
34	1.61 €
35	1.00 €

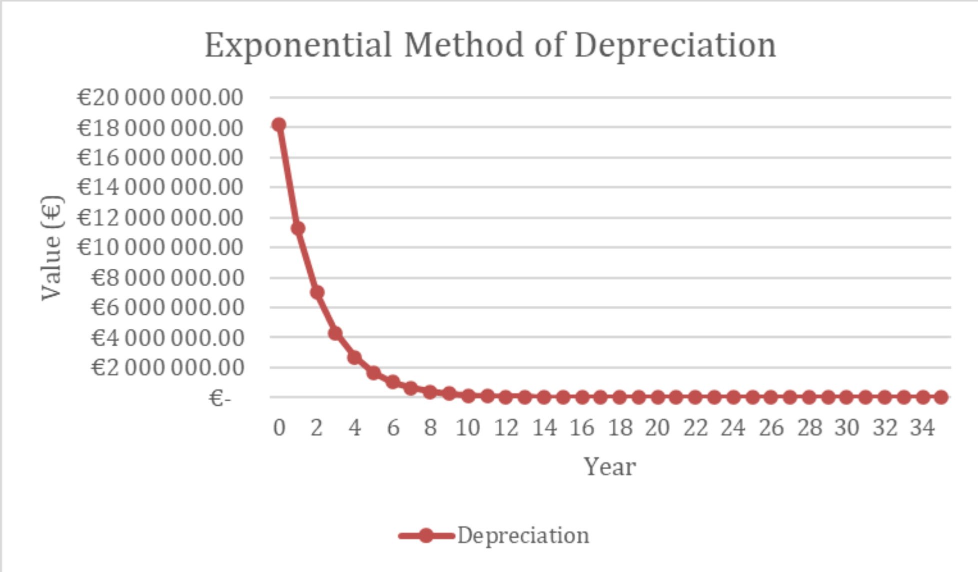

Figure 24 - Exponential method of depreciation applied to *Celbi*'s equipment

Similarly, for the sum of the digits depreciation method, the *SD* value was calculated using the equations mentioned above. The results can be seen in the Table 48 and Figure 25.

Table 48 - Sum of the digits depreciation method: annual depreciation

Sum of Digits Depreciation Method		
dl	Annual Depreciation Rate for year n	979 847.57 €
Vn	Value of equipment in a given period n	16 196 304.87 €
SD	630	
Year	*dl*	Sum of the Digits Depreciation
0		18 156 000.00 €
1	1 008 666.61 €	17 147 333.39 €
2	979 847.57 €	16 167 485.82 €
3	951 028.52 €	15 216 457.30 €
4	922 209.47 €	14 294 247.83 €
5	893 390.43 €	13 400 857.40 €
6	864 571.38 €	12 536 286.02 €

7	835 752.33 €	11 700 533.69 €
8	806 933.29 €	10 893 600.40 €
9	778 114.24 €	10 115 486.16 €
10	749 295.20 €	9 366 190.96 €
11	720 476.15 €	8 645 714.81 €
12	691 657.10 €	7 954 057.70 €
13	662 838.06 €	7 291 219.65 €
14	634 019.01 €	6 657 200.63 €
15	605 199.97 €	6 052 000.67 €
16	576 380.92 €	5 475 619.75 €
17	547 561.87 €	4 928 057.87 €
18	518 742.83 €	4 409 315.04 €
19	489 923.78 €	3 919 391.26 €
20	461 104.74 €	3 458 286.52 €
21	432 285.69 €	3 026 000.83 €
22	403 466.64 €	2 622 534.19 €
23	374 647.60 €	2 247 886.59 €
24	345 828.55 €	1 902 058.04 €
25	317 009.51 €	1 585 048.53 €
26	288 190.46 €	1 296 858.07 €
27	259 371.41 €	1 037 486.66 €
28	230 552.37 €	806 934.29 €
29	201 733.32 €	605 200.97 €
30	172 914.28 €	432 286.69 €
31	144 095.23 €	288 191.46 €
32	115 276.18 €	172 915.28 €
33	86 457.14 €	86 458.14 €
34	57 638.09 €	28 820.05 €
35	28 819.05 €	1.00 €

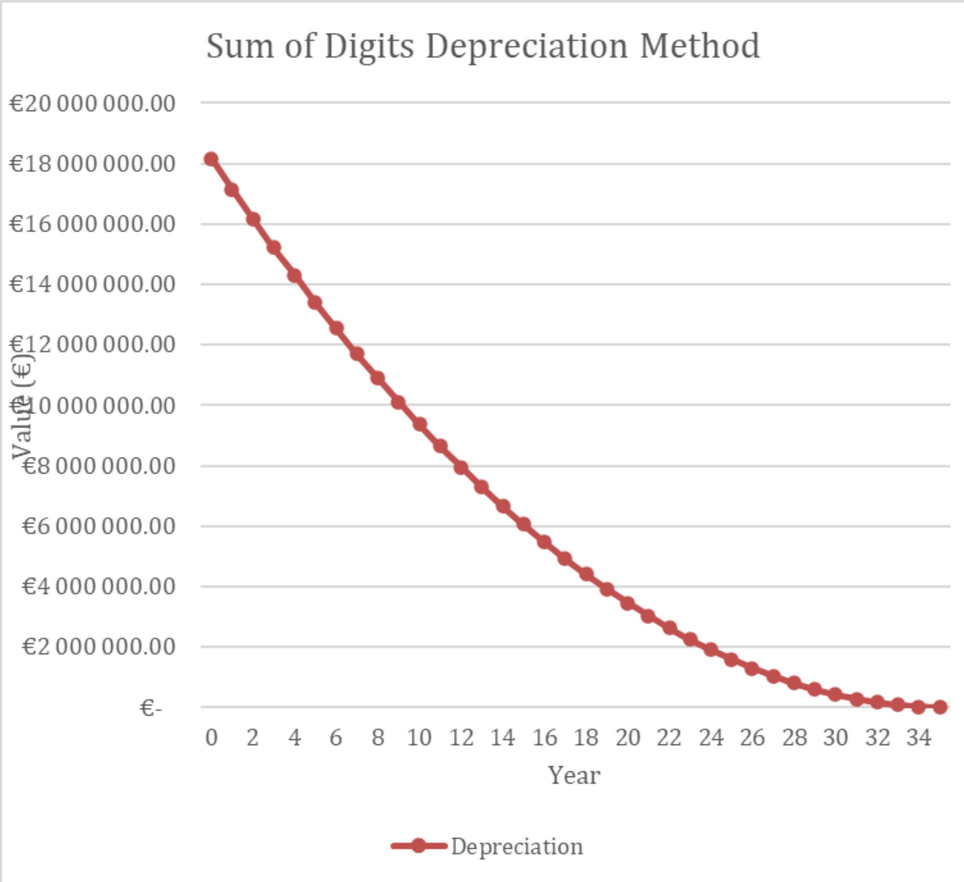

Figure 25 - Sum of the digits depreciation method applied to *Celbi*'s equipment

After the three different depreciation method curves had been obtained, the three life cycle models explained above were calculated:

- Income Annual Uniform Method (*UAI*) – Equations 1.13 and 1.20;
- Minimizing Total Average Cost Method (*MTAC*) – Equations 1.14, 1.15, 1.16 and 1.20;
- Minimizing Total Average Cost Reduced to Present Value Method (*MTAC-RPV*) – Equations 1.17, 1.18, 1.19 and 1.20.

The results obtained in the *UAI* model for the rates calculated above are shown in Table 49.

Similarly, the application of the *MTAC* method resulted in the values shown in Table 50.

Finally, in the *MTAC-RPV* method, the values shown in Table 51 were obtained.

Table 49 - Uniform annual income

i_A	50.95%	52.83%	53.35%	34.93%	51.14%	64.89%	59.29%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%
Linear Depreciation Method																		
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
CA	18 156 000.00 €																	
CE	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
Value P.	18 256 168.47 €	18 774 939.52 €	19 387 382.23 €	19 947 298.76 €	20 556 699.34 €	20 718 254.64 €	20 798 193.44 €	20 913 637.88 €	20 981 229.02 €	21 026 531.91 €	21 061 365.30 €	21 088 241.27 €	21 109 039.77 €	21 125 176.65 €	21 137 724.46 €	21 147 499.89 €	21 155 127.73 €	21 161 087.89 €
Assignment		17 637 257.17 €	17 118 514.34 €	16 599 771.51 €	16 081 028.69 €	15 562 285.86 €	15 043 543.03 €	14 524 800.20 €	14 006 057.37 €	13 487 314.54 €	12 968 571.71 €	12 449 828.89 €	11 931 086.06 €	11 412 343.23 €	10 893 600.40 €	10 374 857.57 €	9 856 114.74 €	9 337 371.91 €
Value P'.		11 540 435.07 €	7 279 362.47 €	6 756 787.23 €	3 081 731.65 €	1 276 882.10 €	920 846.52 €	757 777.74 €	479 207.42 €	302 627.98 €	190 832.21 €	120 143.11 €	75 507.73 €	47 365.51 €	29 650.69 €	18 519.17 €	11 537.75 €	7 168.30 €
P-P'		7 234 504.45 €	12 108 019.76 €	13 190 511.53 €	17 474 967.69 €	19 441 372.53 €	19 877 346.91 €	20 155 860.14 €	20 502 021.60 €	20 723 903.94 €	20 870 533.09 €	20 968 098.16 €	21 033 532.03 €	21 077 811.14 €	21 108 073.77 €	21 128 980.72 €	21 143 589.98 €	21 153 919.60 €
UAI		11 056 499.57 €	11 238 888.82 €	7 771 096.26 €	11 055 338.67 €	13 742 339.12 €	12 554 129.83 €	11 160 862.07 €	11 141 461.75 €	11 126 370.18 €	11 117 264.00 €	11 112 113.62 €	11 109 530.35 €	11 108 566.10 €	11 108 580.22 €	11 109 147.06 €	11 109 990.25 €	11 110 935.93 €
Exponential Depreciation Method																		
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
CA	18 156 000.00 €																	
CE	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
Value P.	18 256 168.47 €	18 774 939.52 €	19 387 382.23 €	19 947 298.76 €	20 556 699.34 €	20 718 254.64 €	20 798 193.44 €	20 913 637.88 €	20 981 229.02 €	21 026 531.91 €	21 061 365.30 €	21 088 241.27 €	21 109 039.77 €	21 125 176.65 €	21 137 724.46 €	21 147 499.89 €	21 155 127.73 €	21 161 087.89 €
Assignment		11 262 105.66 €	6 985 846.22 €	4 333 296.89 €	2 687 929.47 €	1 667 313.60 €	1 034 229.01 €	641 528.77 €	397 938.14 €	246 839.69 €	153 113.83 €	94 975.99 €	58 913.29 €	36 543.71 €	22 667.94 €	14 060.85 €	8 721.90 €	5 410.16 €
Value P'.		7 369 036.92 €	2 970 614.49 €	1 763 829.40 €	515 108.67 €	136 802.71 €	63 307.31 €	33 469.39 €	13 615.17 €	5 538.58 €	2 253.07 €	916.54 €	372.84 €	151.67 €	61.70 €	25.10 €	10.21 €	4.15 €
P-P'		11 405 902.60 €	16 416 767.74 €	18 183 469.36 €	20 041 590.67 €	20 581 451.93 €	20 734 886.13 €	20 880 168.49 €	20 967 613.85 €	21 020 993.33 €	21 059 112.24 €	21 087 324.74 €	21 108 666.93 €	21 125 024.98 €	21 137 662.76 €	21 147 474.79 €	21 155 117.52 €	21 161 083.74 €
UAI		17 431 651.08 €	15 238 348.72 €	10 712 661.93 €	12 679 083.38 €	14 548 216.26 €	13 095 734.23 €	11 561 931.81 €	11 394 479.64 €	11 285 873.26 €	11 217 715.87 €	11 175 298.14 €	11 149 215.24 €	11 133 449.04 €	11 124 152.07 €	11 118 870.82 €	11 116 047.44 €	11 114 698.85 €
Sum of Digits Depreciation Method																		

i_A	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%
Linear Depreciation Method																		
Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
CA																		
CE	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
Value P.	21 165 750.34 €	21 169 401.18 €	21 172 262.22 €	21 174 505.88 €	21 176 266.38 €	21 177 648.45 €	21 178 733.86 €	21 179 586.58 €	21 180 256.69 €	21 180 783.41 €	21 181 197.52 €	21 181 523.14 €	21 181 779.21 €	21 181 980.61 €	21 182 139.04 €	21 182 263.66 €	21 182 361.70 €	21 182 438.84 €
Assignment	8 818 629.09 €	8 299 886.26 €	7 781 143.43 €	7 262 400.60 €	6 743 657.77 €	6 224 914.94 €	5 706 172.11 €	5 187 429.29 €	4 668 686.46 €	4 149 943.63 €	3 631 200.80 €	3 112 457.97 €	2 593 715.14 €	2 074 972.31 €	1 556 229.49 €	1 037 486.66 €	518 743.83 €	1.00 €
Value P'.	4 439.85 €	2 740.41 €	1 684.85 €	1 031.28 €	628.01 €	380.17 €	228.54 €	136.25 €	80.42 €	46.88 €	26.90 €	15.12 €	8.26 €	4.34 €	2.13 €	0.93 €	0.31 €	0.00 €
P-P'	21 161 310.49 €	21 166 660.77 €	21 170 577.37 €	21 173 474.60 €	21 175 638.37 €	21 177 268.27 €	21 178 505.31 €	21 179 450.33 €	21 180 176.27 €	21 180 736.53 €	21 181 170.62 €	21 181 508.01 €	21 181 770.94 €	21 181 976.28 €	21 182 136.90 €	21 182 262.73 €	21 182 361.40 €	21 182 438.84 €
UAI	11 111 879.52 €	11 112 762.29 €	11 113 555.15 €	11 114 247.56 €	11 114 840.02 €	11 115 339.22 €	11 115 754.83 €	11 116 097.58 €	11 116 378.09 €	11 116 606.24 €	11 116 790.84 €	11 116 939.57 €	11 117 058.98 €	11 117 154.55 €	11 117 230.86 €	11 117 291.65 €	11 117 340.00 €	11 117 378.39 €
Exponential Depreciation Method																		
Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
CA																		
CE	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
Value P.	21 165 750.34 €	21 169 401.18 €	21 172 262.22 €	21 174 505.88 €	21 176 266.38 €	21 177 648.45 €	21 178 733.86 €	21 179 586.58 €	21 180 256.69 €	21 180 783.41 €	21 181 197.52 €	21 181 523.14 €	21 181 779.21 €	21 181 980.61 €	21 182 139.04 €	21 182 263.66 €	21 182 361.70 €	21 182 438.84 €
Assignment	3 355.91 €	2 081.66 €	1 291.24 €	800.95 €	496.83 €	308.18 €	191.16 €	118.58 €	73.55 €	45.63 €	28.30 €	17.56 €	10.89 €	6.75 €	4.19 €	2.60 €	1.61 €	1.00 €
Value P'.	1.69 €	0.69 €	0.28 €	0.11 €	0.05 €	0.02 €	0.01 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €
P-P'	21 165 748.65 €	21 169 400.49 €	21 172 261.94 €	21 174 505.76 €	21 176 266.34 €	21 177 648.43 €	21 178 733.85 €	21 179 586.58 €	21 180 256.69 €	21 180 783.41 €	21 181 197.52 €	21 181 523.14 €	21 181 779.21 €	21 181 980.61 €	21 182 139.04 €	21 182 263.66 €	21 182 361.70 €	21 182 438.84 €
UAI	11 114 210.02 €	11 114 200.68 €	11 114 439.47 €	11 114 788.83 €	11 115 169.63 €	11 115 538.75 €	11 115 874.78 €	11 116 169.09 €	11 116 420.30 €	11 116 630.84 €	11 116 804.96 €	11 116 947.51 €	11 117 063.31 €	11 117 156.83 €	11 117 231.98 €	11 117 292.14 €	11 117 340.16 €	11 117 378.39 €
Sum of Digits Depreciation Method																		

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
CA	18 156 000.00 €																	
CE	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
Value P.	18 256 168.47 €	18 774 939.52 €	19 387 382.23 €	19 947 298.76 €	20 556 699.34 €	20 718 254.64 €	20 798 193.44 €	20 913 637.88 €	20 981 229.02 €	21 026 531.91 €	21 061 365.30 €	21 088 241.27 €	21 109 039.77 €	21 125 176.65 €	21 137 724.46 €	21 147 499.89 €	21 155 127.73 €	21 161 087.89 €
Assignment		17 147 333.39 €	16 167 485.82 €	15 216 457.30 €	14 294 247.83 €	13 400 857.40 €	12 536 286.02 €	11 700 533.69 €	10 893 600.40 €	10 115 486.16 €	9 366 190.96 €	8 645 714.81 €	7 954 057.70 €	7 291 219.65 €	6 657 200.63 €	6 052 000.67 €	5 475 619.75 €	4 928 057.87 €
Value P'.		11 219 867.45 €	6 874 953.47 €	6 193 721.66 €	2 739 317.05 €	1 099 537.38 €	767 372.11 €	610 432.08 €	372 716.89 €	226 970.99 €	137 823.27 €	83 432.72 €	50 338.49 €	30 261.30 €	18 119.87 €	10 802.85 €	6 409.86 €	3 783.27 €
P-P'		7 555 072.07 €	12 512 428.76 €	13 753 577.10 €	17 817 382.29 €	19 618 717.26 €	20 030 821.32 €	20 303 205.80 €	20 608 512.13 €	20 799 560.92 €	20 923 542.03 €	21 004 808.55 €	21 058 701.28 €	21 094 915.35 €	21 119 604.59 €	21 136 697.04 €	21 148 717.87 €	21 157 304.62 €
UAI		11 546 423.35 €	11 614 268.77 €	8 102 822.34 €	11 271 963.36 €	13 867 697.11 €	12 651 061.16 €	11 242 451.47 €	11 199 332.15 €	11 166 989.34 €	11 145 500.67 €	11 131 568.42 €	11 122 824.29 €	11 117 580.47 €	11 114 648.56 €	11 113 204.12 €	11 112 684.72 €	11 112 713.89 €

Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
CA																		
CE	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
Value P.	21 165 750.34 €	21 169 401.18 €	21 172 262.22 €	21 174 505.88 €	21 176 266.38 €	21 177 648.45 €	21 178 733.86 €	21 179 586.58 €	21 180 256.69 €	21 180 783.41 €	21 181 197.52 €	21 181 523.14 €	21 181 779.21 €	21 181 980.61 €	21 182 139.04 €	21 182 263.66 €	21 182 361.70 €	21 182 438.84 €
Assignment	4 409 315.04 €	3 919 391.26 €	3 458 286.52 €	3 026 000.83 €	2 622 534.19 €	2 247 886.59 €	1 902 058.04 €	1 585 048.53 €	1 296 858.07 €	1 037 486.66 €	806 934.29 €	605 200.97 €	432 286.69 €	288 191.46 €	172 915.28 €	86 458.14 €	28 820.05 €	1.00 €
Value P'.	2 219.93 €	1 294.08 €	748.82 €	429.70 €	244.23 €	137.28 €	76.18 €	41.63 €	22.34 €	11.72 €	5.98 €	2.94 €	1.38 €	0.60 €	0.24 €	0.08 €	0.02 €	0.00 €
P-P'	21 163 530.42 €	21 168 107.10 €	21 171 513.40 €	21 174 076.18 €	21 176 022.16 €	21 177 511.16 €	21 178 657.68 €	21 179 544.95 €	21 180 234.35 €	21 180 771.69 €	21 181 191.54 €	21 181 520.19 €	21 181 777.83 €	21 181 980.01 €	21 182 138.80 €	21 182 263.58 €	21 182 361.68 €	21 182 438.84 €
UAI	11 113 045.21 €	11 113 521.63 €	11 114 046.52 €	11 114 563.33 €	11 115 041.46 €	11 115 466.70 €	11 115 834.80 €	11 116 147.24 €	11 116 408.58 €	11 116 624.69 €	11 116 801.82 €	11 116 945.96 €	11 117 062.59 €	11 117 156.51 €	11 117 231.85 €	11 117 292.10 €	11 117 340.15 €	11 117 378.39 €

Table 50 - Minimizing Total Average Cost Method

Linear Depreciation Method																		
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Acquisition (CA)	18 156 000.00 €																	
Exploration (CE)	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
C'n		893 006.73 €	1 166 628.88 €	1 236 278.51 €	1 722 199.28 €	1 771 557.68 €	1 693 953.37 €	1 768 073.73 €	1 794 004.92 €	1 819 007.47 €	1 873 827.42 €	1 956 663.30 €	2 067 474.69 €	2 207 519.23 €	2 379 127.71 €	2 585 613.81 €	2 685 439.13 €	2 984 158.34 €
Assignment (VN)		17 637 257.17 €	17 118 514.34 €	16 599 771.51 €	16 081 028.69 €	15 562 285.86 €	15 043 543.03 €	14 524 800.20 €	14 006 057.37 €	13 487 314.54 €	12 968 571.71 €	12 449 828.89 €	11 931 086.06 €	11 412 343.23 €	10 893 600.40 €	10 374 857.57 €	9 856 114.74 €	9 337 371.91 €
C''n		518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €
C'n+ C''n		1 411 749.56 €	1 685 371.71 €	1 755 021.34 €	2 240 942.11 €	2 290 300.51 €	2 212 696.20 €	2 286 816.56 €	2 312 747.75 €	2 337 750.30 €	2 392 570.25 €	2 475 406.13 €	2 586 217.52 €	2 726 262.06 €	2 897 870.54 €	3 104 356.64 €	3 204 181.96 €	3 502 901.16 €
Exponential Depreciation Method																		
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Acquisition (CA)	18 156 000.00 €																	

Linear Depreciation Method																		
Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acquisition (CA)																		
Exploration (CE)	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
C'n	3 332 859.11 €	3 396 012.31 €	3 886 868.39 €	4 454 166.78 €	4 861 263.02 €	5 633 808.79 €	6 528 239.77 €	7 278 842.16 €	8 495 111.36 €	9 907 388.10 €	11 128 973.59 €	13 056 246.98 €	15 299 980.61 €	17 528 813.00 €	20 593 784.79 €	23 656 077.01 €	27 853 045.70 €	32 065 745.95 €
Assignment (VN)	8 818 629.09 €	8 299 886.26 €	7 781 143.43 €	7 262 400.60 €	6 743 657.77 €	6 224 914.94 €	5 706 172.11 €	5 187 429.29 €	4 668 686.46 €	4 149 943.63 €	3 631 200.80 €	3 112 457.97 €	2 593 715.14 €	2 074 972.31 €	1 556 229.49 €	1 037 486.66 €	518 743.83 €	1.00 €
C''n	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €	518 742.83 €
C'n+ C''n	3 851 601.94 €	3 914 755.13 €	4 405 611.22 €	4 972 909.61 €	5 380 005.85 €	6 152 551.62 €	7 046 982.60 €	7 797 584.99 €	9 013 854.19 €	10 426 130.93 €	11 647 716.42 €	13 574 989.81 €	15 818 723.43 €	18 047 555.82 €	21 112 527.61 €	24 174 819.83 €	28 371 788.53 €	32 584 488.78 €
Exponential Depreciation Method																		
Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acquisition (CA)																		

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Exploration (CE)	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
C'n		893 006.73 €	1 166 628.88 €	1 236 278.51 €	1 722 199.28 €	1 771 557.68 €	1 693 953.37 €	1 768 073.73 €	1 794 004.92 €	1 819 007.47 €	1 873 827.42 €	1 956 663.30 €	2 067 474.69 €	2 207 519.23 €	2 379 127.71 €	2 585 613.81 €	2 685 439.13 €	2 984 158.34 €
Assignment (VN)		11 262 105.66 €	6 985 846.22 €	4 333 296.89 €	2 687 929.47 €	1 667 313.60 €	1 034 229.01 €	641 528.77 €	397 938.14 €	246 839.69 €	153 113.83 €	94 975.99 €	58 913.29 €	36 543.71 €	22 667.94 €	14 060.85 €	8 721.90 €	5 410.16 €
C''n		6 893 894.34 €	5 585 076.89 €	4 607 567.70 €	3 867 017.63 €	3 297 737.28 €	2 853 628.50 €	2 502 067.32 €	2 219 757.73 €	1 989 906.70 €	1 800 288.62 €	1 641 911.27 €	1 508 090.56 €	1 393 804.33 €	1 295 238.00 €	1 209 462.61 €	1 134 204.88 €	1 067 681.76 €
C'n+ C''n		7 786 901.06 €	6 751 705.77 €	5 843 846.22 €	5 589 216.91 €	5 069 294.96 €	4 547 581.87 €	4 270 141.05 €	4 013 762.66 €	3 808 914.17 €	3 674 116.04 €	3 598 574.57 €	3 575 565.25 €	3 601 323.56 €	3 674 365.72 €	3 795 076.42 €	3 819 644.01 €	4 051 840.09 €

	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Exploration (CE)	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
C'n	3 332 859.11 €	3 396 012.31 €	3 886 868.39 €	4 454 166.78 €	4 861 263.02 €	5 633 808.79 €	6 528 239.77 €	7 278 842.16 €	8 495 111.36 €	9 907 388.10 €	11 128 973.59 €	13 056 246.98 €	15 299 980.61 €	17 528 813.00 €	20 593 784.79 €	23 656 077.01 €	27 853 045.70 €	32 065 745.95 €
Assignment (VN)	3 355.91 €	2 081.66 €	1 291.24 €	800.95 €	496.83 €	308.18 €	191.16 €	118.58 €	73.55 €	45.63 €	28.30 €	17.56 €	10.89 €	6.75 €	4.19 €	2.60 €	1.61 €	1.00 €
C''n	1 008 480.23 €	955 469.39 €	907 735.44 €	864 533.29 €	825 250.14 €	789 377.91 €	756 492.03 €	726 235.26 €	698 304.86 €	672 442.75 €	648 427.56 €	626 068.36 €	605 199.64 €	585 677.20 €	567 374.87 €	550 181.74 €	533 999.95 €	518 742.83 €
C'n+ C''n	4 341 339.34 €	4 351 481.69 €	4 794 603.83 €	5 318 700.07 €	5 686 513.17 €	6 423 186.69 €	7 284 731.81 €	8 005 077.42 €	9 193 416.23 €	10 579 830.86 €	11 777 401.15 €	13 682 315.34 €	15 905 180.24 €	18 114 490.20 €	21 161 159.65 €	24 206 258.74 €	28 387 045.66 €	32 584 488.78 €

Sum of Digits Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Acquisition (CA)	18 156 000.00 €																	
Exploration (CE)	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €
C'n		893 006.73 €	1 166 628.88 €	1 236 278.51 €	1 722 199.28 €	1 771 557.68 €	1 693 953.37 €	1 768 073.73 €	1 794 004.92 €	1 819 007.47 €	1 873 827.42 €	1 956 663.30 €	2 067 474.69 €	2 207 519.23 €	2 379 127.71 €	2 585 613.81 €	2 685 439.13 €	2 984 158.34 €
Assignment (VN)		17 147 333.39 €	16 167 485.82 €	15 216 457.30 €	14 294 247.83 €	13 400 857.40 €	12 536 286.02 €	11 700 533.69 €	10 893 600.40 €	10 115 486.16 €	9 366 190.96 €	8 645 714.81 €	7 954 057.70 €	7 291 219.65 €	6 657 200.63 €	6 052 000.67 €	5 475 619.75 €	4 928 057.87 €
C''n		1 008 666.61 €	994 257.09 €	979 847.57 €	965 438.04 €	951 028.52 €	936 619.00 €	922 209.47 €	907 799.95 €	893 390.43 €	878 980.90 €	864 571.38 €	850 161.86 €	835 752.33 €	821 342.81 €	806 933.29 €	792 523.77 €	778 114.24 €
C'n+ C''n		1 901 673.34 €	2 160 885.97 €	2 216 126.08 €	2 687 637.32 €	2 722 586.20 €	2 630 572.36 €	2 690 283.20 €	2 701 804.87 €	2 712 397.90 €	2 752 808.33 €	2 821 234.68 €	2 917 636.55 €	3 043 271.56 €	3 200 470.53 €	3 392 547.10 €	3 477 962.90 €	3 762 272.58 €

Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acquisition (CA)																		
Exploration (CE)	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €
C'n	3 332 859.11 €	3 396 012.31 €	3 886 868.39 €	4 454 166.78 €	4 861 263.02 €	5 633 808.79 €	6 528 239.77 €	7 278 842.16 €	8 495 111.36 €	9 907 388.10 €	11 128 973.59 €	13 056 246.98 €	15 299 980.61 €	17 528 813.00 €	20 593 784.79 €	23 656 077.01 €	27 853 045.70 €	32 065 745.95 €
Assignment (VN)	4 409 315.04 €	3 919 391.26 €	3 458 286.52 €	3 026 000.83 €	2 622 534.19 €	2 247 886.59 €	1 902 058.04 €	1 585 048.53 €	1 296 858.07 €	1 037 486.66 €	806 934.29 €	605 200.97 €	432 286.69 €	288 191.46 €	172 915.28 €	86 458.14 €	28 820.05 €	1.00 €
C''n	763 704.72 €	749 295.20 €	734 885.67 €	720 476.15 €	706 066.63 €	691 657.10 €	677 247.58 €	662 838.06 €	648 428.54 €	634 019.01 €	619 609.49 €	605 199.97 €	590 790.44 €	576 380.92 €	561 971.40 €	547 561.87 €	533 152.35 €	518 742.83 €
C'n+ C''n	4 096 563.83 €	4 145 307.50 €	4 621 754.07 €	5 174 642.93 €	5 567 329.65 €	6 325 465.89 €	7 205 487.35 €	7 941 680.22 €	9 143 539.90 €	10 541 407.11 €	11 748 583.08 €	13 661 446.94 €	15 890 771.05 €	18 105 193.92 €	21 155 756.18 €	24 203 638.88 €	28 386 198.06 €	32 584 488.78 €

Table 51 - *MTAC* with reduction to present value

i_A	50.95 %	52.83 %	53.35 %	34.93 %	51.14 %	64.89 %	59.29 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %

i_A	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %

Linear Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Acquisition (CA)	18 156 000.00 €																	
Exploration (CE)	100 168.47 €	792 838.26 €	1 440 251.03 €	1 375 577.79 €	3 179 961.58 €	1 968 991.27 €	1 305 931.81 €	2 212 795.92 €	1 975 523.25 €	2 019 027.84 €	2 367 207.02 €	2 785 022.03 €	3 286 400.05 €	3 888 053.66 €	4 610 038.00 €	5 476 419.21 €	6 516 076.66 €	7 763 665.60 €

Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acquisition (CA)																		
Exploration (CE)	9 260 772.33 €	11 057 300.41 €	13 213 134.09 €	15 800 134.52 €	18 904 535.03 €	22 629 815.65 €	27 100 152.39 €	32 464 556.47 €	38 901 841.37 €	46 626 583.26 €	55 896 273.52 €	67 019 901.83 €	80 368 255.80 €	96 386 280.57 €	115 607 910.29 €	138 673 865.96 €	166 353 012.76 €	199 567 988.92 €

Present Value (P)	100 168.4 7 €	518 771.0 5 €	612 442.7 1 €	559 916.5 3 €	609 400.5 8 €	161 555.3 0 €	79 938.8 0 €	115 444.4 5 €	67 591.1 4 €	45 302.8 9 €	34 833.3 9 €	26 875.9 7 €	20 798.4 9 €	16 136.8 8 €	12 547.8 1 €	9 775.4 3 €	7 627.8 4 €	5 960.1 6 €	4 662.4 5 €	3 650.8 4 €	2 861.0 5 €	2 243.6 5 €	1 760.5 0 €	1 382.0 6 €	1 085.4 1 €	852.7 2 €	670.1 1 €	526.7 3 €	414.1 0 €	325.6 2 €	256.0 7 €	201.4 0 €	158.4 2 €	124.6 2 €	98.04 €	77.13 €
C'n		618 939.5 2 €	615 691.1 1 €	597 099.5 9 €	600 174.8 4 €	512 450.9 3 €	440 365.5 7 €	393 948.2 7 €	353 153.6 3 €	318 947.9 9 €	290 536.5 3 €	266 567.3 9 €	246 086.6 5 €	228 398.2 0 €	212 980.3 2 €	199 433.3 3 €	181 184.9 5 €	140 361.6 7 €	98 798.2 3 €	64 321.1 8 €	30 778.1 4 €	21 726.2 5 €	17 185.1 3 €	11 478.7 2 €	8 229.3 7 €	6 122.1 9 €	4 572.7 5 €	3 427.4 9 €	2 577.0 6 €	1 942.9 8 €	1 468.4 9 €	1 112.2 8 €	844.1 0 €	641.6 9 €	488.5 7 €	372.5 0 €
Assign ment (VN)		17 637 257.1 7 €	17 118 514.3 4 €	16 599 771.5 1 €	16 081 028.6 9 €	15 562 285.8 6 €	15 043 543.0 3 €	14 524 800.2 0 €	14 006 057.3 7 €	13 487 314.5 4 €	12 968 571.7 1 €	12 449 828.8 9 €	11 931 086.0 6 €	11 412 343.2 3 €	10 893 600.4 0 €	10 374 857.5 7 €	9 856 114.7 4 €	9 337 371.9 1 €	8 818 629.0 9 €	8 299 886.2 6 €	7 781 143.4 3 €	7 262 400.6 0 €	6 743 657.7 7 €	6 224 914.9 4 €	5 706 172.1 1 €	5 187 429.2 9 €	4 668 686.4 6 €	4 149 943.6 3 €	3 631 200.8 0 €	3 112 457.9 7 €	2 593 715.1 4 €	2 074 972.3 1 €	1 556 229.4 9 €	1 037 486.6 6 €	518 743.8 3 €	1.00 €
Present Value (P')		11 540 435.0 7 €	7 279 362.4 7 €	6 756 787.2 3 €	3 081 731.6 5 €	1 276 882.1 0 €	920 846.5 2 €	757 777.7 4 €	479 207.4 2 €	302 627.9 8 €	190 832.2 1 €	120 143.1 1 €	75 507.7 3 €	47 365.5 1 €	29 650.6 9 €	18 519.1 7 €	11 537.7 5 €	7 168.3 0 €	4 439.8 5 €	2 740.4 1 €	1 684.8 5 €	1 031.2 8 €	628.0 1 €	380.1 7 €	228.5 4 €	136.2 5 €	80.42 €	46.88 €	26.90 €	15.12 €	8.26 €	4.34 €	2.13 €	0.93 €	0.31 €	0.00 €
C''n		6 615 564.9 3 €	5 438 318.7 6 €	3 799 737.5 9 €	3 768 567.0 9 €	3 375 823.5 8 €	2 872 525.5 8 €	2 485 460.3 2 €	2 209 599.0 7 €	1 983 708.0 0 €	1 796 516.7 8 €	1 639 623.3 5 €	1 506 707.6 9 €	1 392 971.8 8 €	1 294 739.2 4 €	1 209 165.3 9 €	1 134 028.8 9 €	1 067 578.3 4 €	1 008 420.0 1 €	955 434.7 2 €	907 715.7 6 €	864 522.3 2 €	825 244.1 8 €	789 374.7 8 €	756 490.4 8 €	726 234.5 5 €	698 304.6 0 €	672 442.7 1 €	648 427.6 1 €	626 068.4 4 €	605 199.7 2 €	585 677.2 8 €	567 374.9 3 €	550 181.7 9 €	533 999.9 9 €	518 742.8 6 €
C'n + C''n		7 234 504.4 5 €	6 054 009.8 8 €	4 396 837.1 8 €	4 368 741.9 2 €	3 888 274.5 1 €	3 312 891.1 5 €	2 879 408.5 9 €	2 562 752.7 0 €	2 302 655.9 9 €	2 087 053.3 1 €	1 906 190.7 4 €	1 752 794.3 4 €	1 621 370.0 9 €	1 507 719.5 5 €	1 408 598.7 1 €	1 315 213.8 4 €	1 207 940.0 0 €	1 107 218.2 4 €	1 019 755.9 0 €	938 493.9 0 €	886 248.5 7 €	842 429.3 2 €	800 853.5 0 €	764 719.8 5 €	732 356.7 4 €	702 877.3 4 €	675 870.1 9 €	651 004.6 7 €	628 011.4 3 €	606 668.2 2 €	586 789.5 6 €	568 219.0 4 €	550 823.4 8 €	534 488.5 6 €	519 115.3 6 €

Exponential Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acqui sition (CA)	18 156 000.0 0 €																																			
Explo ration (CE)	100 168.4 7 €	792 838.2 6 €	1 440 251.0 3 €	1 375 577.7 9 €	3 179 961.5 8 €	1 968 991.2 7 €	1 305 931.8 1 €	2 212 795.9 2 €	1 975 523.2 5 €	2 019 027.8 4 €	2 367 207.0 2 €	2 785 022.0 3 €	3 286 400.0 5 €	3 888 053.6 6 €	4 610 038.0 0 €	5 476 419.2 1 €	6 516 076.6 6 €	7 763 665.6 0 €	9 260 772.3 3 €	11 057 300.4 1 €	13 213 134.0 9 €	15 800 134.5 2 €	18 904 535.0 3 €	22 629 815.6 5 €	27 100 152.3 9 €	32 464 556.4 7 €	38 901 841.3 7 €	46 626 583.2 6 €	55 896 273.5 2 €	67 019 901.8 3 €	80 368 255.8 0 €	96 386 280.5 7 €	115 607 910.2 9 €	138 673 865.9 6 €	166 353 012.7 6 €	199 567 988.9 2 €
Present Value (P)	100 168.4 7 €	518 771.0 5 €	612 442.7 1 €	559 916.5 3 €	609 400.5 8 €	161 555.3 0 €	79 938.8 0 €	115 444.4 5 €	67 591.1 4 €	45 302.8 9 €	34 833.3 9 €	26 875.9 7 €	20 798.4 9 €	16 136.8 8 €	12 547.8 1 €	9 775.4 3 €	7 627.8 4 €	5 960.1 6 €	4 662.4 5 €	3 650.8 4 €	2 861.0 5 €	2 243.6 5 €	1 760.5 0 €	1 382.0 6 €	1 085.4 1 €	852.7 2 €	670.1 1 €	526.7 3 €	414.1 0 €	325.6 2 €	256.0 7 €	201.4 0 €	158.4 2 €	124.6 2 €	98.04 €	77.13 €
C'n		618 939.5 2 €	615 691.1 1 €	597 099.5 9 €	600 174.8 4 €	512 450.9 3 €	440 365.5 7 €	393 948.2 7 €	353 153.6 3 €	318 947.9 9 €	290 536.5 3 €	266 567.3 9 €	246 086.6 5 €	228 398.2 0 €	212 980.3 2 €	199 433.3 3 €	181 184.9 5 €	140 361.6 7 €	98 798.2 3 €	64 321.1 8 €	30 778.1 4 €	21 726.2 5 €	17 185.1 3 €	11 478.7 2 €	8 229.3 7 €	6 122.1 9 €	4 572.7 5 €	3 427.4 9 €	2 577.0 6 €	1 942.9 8 €	1 468.4 9 €	1 112.2 8 €	844.1 0 €	641.6 9 €	488.5 7 €	372.5 0 €
Assign ment (VN)		11 262 105.6 6 €	6 985 846.2 2 €	4 333 296.8 9 €	2 687 929.4 7 €	1 667 313.6 0 €	1 034 229.0 1 €	641 528.7 7 €	397 938.1 4 €	246 839.6 9 €	153 113.8 3 €	94 975.9 9 €	58 913.2 9 €	36 543.7 1 €	22 667.9 4 €	14 060.8 5 €	8 721.9 0 €	5 410.1 6 €	3 355.9 1 €	2 081.6 6 €	1 291.2 4 €	800.9 5 €	496.8 3 €	308.1 8 €	191.1 6 €	118.5 8 €	73.55 €	45.63 €	28.30 €	17.56 €	10.89 €	6.75 €	4.19 €	2.60 €	1.61 €	1.00 €
Present Value (P')		7 369 036.9 2 €	2 970 614.4 9 €	1 763 829.4 0 €	515 108.6 7 €	136 802.7 1 €	63 307.3 1 €	33 469.3 9 €	13 615.1 7 €	5 538.5 8 €	2 253.0 7 €	916.5 4 €	372.8 4 €	151.6 7 €	61.70 €	25.10 €	10.21 €	4.15 €	1.69 €	0.69 €	0.28 €	0.11 €	0.05 €	0.02 €	0.01 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €
C''n		10 786 963.0 8 €	7 592 692.7 5 €	5 464 056.8 7 €	4 410 222.8 3 €	3 603 839.4 6 €	3 015 448.7 8 €	2 588 932.9 4 €	2 267 798.1 0 €	2 016 717.9 4 €	1 815 374.6 9 €	1 650 462.1 3 €	1 512 968.9 3 €	1 396 603.7 2 €	1 296 852.7 4 €	1 210 398.3 3 €	1 134 749.3 6 €	1 067 999.7 6 €	1 008 666.5 7 €	955 578.9 1 €	907 799.9 9 €	864 571.4 2 €	825 272.7 3 €	789 391.3 0 €	756 500.0 0 €	726 240.0 0 €	698 307.6 9 €	672 444.4 4 €	648 428.5 7 €	626 068.9 7 €	605 200.0 0 €	585 677.4 2 €	567 375.0 0 €	550 181.8 2 €	534 000.0 0 €	518 742.8 6 €
C'n + C''n		11 405 902.6 0 €	8 208 383.8 7 €	6 061 156.4 5 €	5 010 397.6 7 €	4 116 290.3 9 €	3 455 814.3 6 €	2 982 881.2 1 €	2 620 951.7 3 €	2 335 665.9 3 €	2 105 911.2 2 €	1 917 029.5 2 €	1 759 055.5 8 €	1 625 001.9 2 €	1 509 833.0 5 €	1 409 831.6 5 €	1 315 934.3 2 €	1 208 361.4 2 €	1 107 464.8 0 €	1 019 900.0 9 €	938 578.1 3 €	886 297.6 7 €	842 457.8 6 €	800 870.0 2 €	764 729.3 7 €	732 362.1 9 €	702 880.4 4 €	675 871.9 3 €	651 005.6 3 €	628 011.9 5 €	606 668.4 9 €	586 789.7 0 €	568 219.1 0 €	550 823.5 1 €	534 488.5 7 €	519 115.3 6 €

Sum of Digits Depreciation Method

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Acqui sition (CA)	18 156 000.0 0 €																																			
Explo ration (CE)	100 168.4 7 €	792 838.2 6 €	1 440 251.0 3 €	1 375 577.7 9 €	3 179 961.5 8 €	1 968 991.2 7 €	1 305 931.8 1 €	2 212 795.9 2 €	1 975 523.2 5 €	2 019 027.8 4 €	2 367 207.0 2 €	2 785 022.0 3 €	3 286 400.0 5 €	3 888 053.6 6 €	4 610 038.0 0 €	5 476 419.2 1 €	6 516 076.6 6 €	7 763 665.6 0 €	9 260 772.3 3 €	11 057 300.4 1 €	13 213 134.0 9 €	15 800 134.5 2 €	18 904 535.0 3 €	22 629 815.6 5 €	27 100 152.3 9 €	32 464 556.4 7 €	38 901 841.3 7 €	46 626 583.2 6 €	55 896 273.5 2 €	67 019 901.8 3 €	80 368 255.8 0 €	96 386 280.5 7 €	115 607 910.2 9 €	138 673 865.9 6 €	166 353 012.7 6 €	199 567 988.9 2 €

Present Value (P)	100 168.47 €	518 771.05 €	612 442.71 €	559 916.53 €	609 400.58 €	161 555.30 €	79 938.80 €	115 444.45 €	67 591.14 €	45 302.89 €	34 833.39 €	26 875.97 €
C'n		618 939.52 €	615 691.11 €	597 099.59 €	600 174.84 €	512 450.93 €	440 365.57 €	393 948.27 €	353 153.63 €	318 947.99 €	290 536.53 €	266 567.39 €
Assignment (VN)		17 147 333.39 €	16 167 485.82 €	15 216 457.30 €	14 294 247.83 €	13 400 857.40 €	12 536 286.02 €	11 700 533.69 €	10 893 600.40 €	10 115 486.16 €	9 366 190.96 €	8 645 714.81 €
Present Value (P')		11 219 867.45 €	6 874 953.47 €	6 193 721.66 €	2 739 317.05 €	1 099 537.38 €	767 372.11 €	610 432.08 €	372 716.89 €	226 970.99 €	137 823.27 €	83 432.72 €
C''n		6 936 132.55 €	5 640 523.27 €	3 987 426.11 €	3 854 170.74 €	3 411 292.52 €	2 898 104.65 €	2 506 509.70 €	2 222 910.39 €	1 992 114.33 €	1 801 817.67 €	1 642 960.66 €
C'n + C''n		7 555 072.07 €	6 256 214.38 €	4 584 525.70 €	4 454 345.57 €	3 923 743.45 €	3 338 470.22 €	2 900 457.97 €	2 576 064.02 €	2 311 062.32 €	2 092 354.20 €	1 909 528.05 €

Present Value (P)	20 798.49 €	16 136.88 €	12 547.81 €	9 775.43 €	7 627.84 €	5 960.16 €	4 662.45 €	3 650.84 €	2 861.05 €	2 243.65 €	1 760.50 €	1 382.06 €
C'n	246 086.65 €	228 398.20 €	212 980.32 €	199 433.33 €	181 184.95 €	140 361.67 €	98 798.23 €	64 321.18 €	30 778.14 €	21 726.25 €	17 185.13 €	11 478.72 €
Assignment (VN)	7 954 057.70 €	7 291 219.65 €	6 657 200.63 €	6 052 000.67 €	5 475 619.75 €	4 928 057.87 €	4 409 315.04 €	3 919 391.26 €	3 458 286.52 €	3 026 000.83 €	2 622 534.19 €	2 247 886.59 €
Present Value (P')	50 338.49 €	30 261.30 €	18 119.87 €	10 802.85 €	6 409.86 €	3 783.27 €	2 219.93 €	1 294.08 €	748.82 €	429.70 €	244.23 €	137.28 €
C''n	1 508 805.13 €	1 394 287.59 €	1 295 562.87 €	1 209 679.81 €	1 134 349.38 €	1 067 777.45 €	1 008 543.34 €	955 510.84 €	907 762.56 €	864 550.97 €	825 261.63 €	789 385.34 €
C'n + C''n	1 754 891.77 €	1 622 685.80 €	1 508 543.19 €	1 409 113.14 €	1 315 534.34 €	1 208 139.12 €	1 107 341.57 €	1 019 832.02 €	938 540.70 €	886 277.22 €	842 446.76 €	800 864.06 €

Present Value (P)	1 085.41 €	852.72 €	670.11 €	526.73 €	414.10 €	325.62 €	256.07 €	201.40 €	158.42 €	124.62 €	98.04 €	77.13 €
C'n	8 229.37 €	6 122.19 €	4 572.75 €	3 427.49 €	2 577.06 €	1 942.98 €	1 468.49 €	1 112.28 €	844.10 €	641.69 €	488.57 €	372.50 €
Assignment (VN)	1 902 058.04 €	1 585 048.53 €	1 296 858.07 €	1 037 486.66 €	806 934.29 €	605 200.97 €	432 286.69 €	288 191.46 €	172 915.28 €	86 458.14 €	28 820.05 €	1.00 €
Present Value (P')	76.18 €	41.63 €	22.34 €	11.72 €	5.98 €	2.94 €	1.38 €	0.60 €	0.24 €	0.08 €	0.02 €	0.00 €
C''n	756 496.83 €	726 238.33 €	698 306.83 €	672 444.01 €	648 428.36 €	626 068.86 €	605 199.95 €	585 677.40 €	567 374.99 €	550 181.82 €	534 000.00 €	518 742.86 €
C'n + C''n	764 726.19 €	732 360.52 €	702 879.58 €	675 871.50 €	651 005.42 €	628 011.85 €	606 668.45 €	586 789.68 €	568 219.10 €	550 823.51 €	534 488.57 €	519 115.36 €

After calculating these models, the *LCI* method was applied. From this point on, the profits made on the asset are taken into account, in addition to fees and expenses. In this way, in an asset where the financial return is tangible and decisive, it becomes a highly important factor (unlike in the previous case study with children's playgrounds). The *LCI* method can therefore be calculated using equation 1.36, where *GR* represents the global result from expenses and benefits in year n, B_j the value of benefits in year j, F_j the *CO* in year j and M_j the *CM* in year j.

$$GR_n = \sum_{j=0}^{n} \frac{B_j}{(1 + i_A)^j} - \sum_{j=0}^{n} \frac{F_j}{(1 + i_A)^j} - \sum_{j=0}^{n} \frac{M_j}{(1 + i_A)^j} \qquad (1.36)$$

Since actual production figures were not available, an alternative was sought to determine B_j values. Using the article[2] entitled "Altri aumenta preço da pasta de papel em 50 dólares por tonelada a 1 de abril" and the *€/ton* and *€/m³* references[3], it was possible to estimate production figures. These values include the one-off variation in pulp prices shown in Table 52, obtained from the trading economics website[4], and *Celbi*'s production values in *m³* shown in Table 53.

Table 52 - Paper pulp prices

Year	January Points	€/ton	€/m3
2017	87.9	841.31 €	921.43 €
2018	94.5	904.48 €	990.62 €
2019	100.4	960.95 €	1 052.47 €
2020	90.5	866.19 €	948.69 €
2021	87.3	835.57 €	915.14 €
2022	117.4	1 123.66 €	1 230.68 €
2023	138.4	1 324.66 €	1 450.81 €
2024	112.6	1 077.72 €	1 180.36 €

2 https://eco.sapo.pt/2022/03/22/altri-aumenta-preco-da-pasta-de-papel-em-50-dolares-por-tonelada-a-1-de-abril-e-o-terceiro-aumento-este-ano/, accessed 04/06/2024

3 https://www.bolsaflorestal.com/fornecimento-de-madeiras/, accessed 04/06/2024

4 https://tradingeconomics.com/euro-area/producer-prices-in-industry-manufacture-of-pulp-paper-paperboard-eurostat-data.html, accessed 04/06/2024

Table 53 - Production Values

Year	2018	2019	2020	2021	2022	2023	2024	Mean
m^3	757469.46	767671.30	848226.98	899520.43	719295.74	736546.80	779274.63	786857.91

The production value for 2024 was obtained by multiplying the actual value for the first 4 months of the year of 259758.21 m^3 by 3. In this way, and with no other information available to contribute to this calculation, it was considered that to the total value of pulp production, the production of chips would contribute 10%. The corresponding production amount for those years is shown in Table 54. To project the following years, the slope between pulp values from year to year was obtained and the average of these slopes was used to project future values using the average production in m^3 of the actual amounts.

Table 54 - Benefit

Year	Benefit
1	75 036 456.34 €
2	80 794 982.75 €
3	80 470 377.90 €
4	82 319 107.37 €
5	88 521 947.79 €
6	106 859 174.13 €
7	91 982 308.29 €
8	95 787 926.74 €
9	98 698 446.85 €
10	101 608 966.95 €
11	104 519 487.05 €
12	107 430 007.15 €
13	110 340 527.25 €
14	113 251 047.35 €
15	116 161 567.46 €
16	119 072 087.56 €
17	121 982 607.66 €
18	124 893 127.76 €
19	127 803 647.86 €
20	130 714 167.96 €
21	133 624 688.07 €
22	136 535 208.17 €
23	139 445 728.27 €
24	142 356 248.37 €
25	145 266 768.47 €

26	148 177 288.57 €
27	151 087 808.67 €
28	153 998 328.78 €
29	156 908 848.88 €
30	159 819 368.98 €
31	162 729 889.08 €
32	165 640 409.18 €
33	168 550 929.28 €
34	171 461 449.39 €
35	174 371 969.49 €

At this time, the formula was modified and the non-production costs NP_j were added. Two different methodologies were used for the non-production values. The first is directly dependent on risk, in other words, the probability of not having enough wood to produce chips on the line and therefore having to be bought in.

Using the values of chips bought from an external supplier in Table 22 and considering a generic value of $46€/m^3$, the non-production costs shown in Table 55 were obtained.

The other methodology was based on availability data. In this way, it was possible to translate the unavailability of the equipment into costs, which must be taken into account when making the investment. This translation is done using equation 1.37.

$$NP_j = \frac{(1 - A_j) \times B_j}{A_j} \qquad (1.37)$$

Equation 1.38 was thus obtained.

$$GR_n = \sum_{j=0}^{n} \frac{B_j}{(1+i_A)^j} - \sum_{j=0}^{n} \frac{F_j}{(1+i_A)^j} - \sum_{j=0}^{n} \frac{M_j}{(1+i_A)^j} - \sum_{j=0}^{n} \frac{NP_j}{(1+i_A)^j} \qquad (1.38)$$

The results are shown in Tables 56 and 57, with Table 56 relating to risk-dependent non-production and Table 57 dependent on asset availability.

Table 55 - Risk-dependent *NP*

Year	1	2	3	4	5	6
m^3 purchased	529420	569662	222047	495354	563043	455543
Risk-dependent *NP*	- 24 353 320.00 €	- 26 204 452.00 €	- 10 214 162.00 €	- 22 786 284.00 €	- 25 899 978.00 €	- 20 954 978.00 €

Table 56 - *LCI* with risk-dependent *NP*

i_A	0.00 %	52.83%	53.35 %	34.93 %	51.14 %	64.89 %	59.29 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
An (Availabity per year)		60%	54%	57%	58%	52%	49%	48%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%
Inicial Investment (II)	- 18 156 000.00 €																	
II Present Value (IIPV) - Return	- 18 156 000.00 €	- 11 879 859.62 €	- 7 720 535.93 €	- 7 390 235.99 €	- 3 479 374.42 €	- 1 489 695.77 €	- 1 111 366.48 €	- 947 222.17 €	- 621 194.79 €	- 407 383.81 €	- 267 165.09 €	- 175 208.70 €	- 114 903.07 €	- 75 354.22 €	- 49 417.81 €	- 32 408.54 €	- 21 253.74 €	- 13 938.35 €
Functioning (F)	- 78 006.89 €	- 167 153.60 €	- 401 837.54 €	- 335 543.44 €	- 716 804.76 €	- 363 807.98 €	- 320 260.74 €	- 812 183.82 €	- 524 776.68 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €
Present Value (FPV)		- 109 372.18 €	- 170 874.71 €	- 136 579.93 €	- 137 366.83 €	- 29 850.36 €	- 19 603.83 €	- 42 372.69 €	- 17 954.87 €	- 6 240.72 €	- 4 092.70 €	- 2 684.02 €	- 1 760.20 €	- 1 154.35 €	- 757.03 €	- 496.47 €	- 325.59 €	- 213.52 €
Accumulated FPV		- 18 265 372.18 €	- 18 436 246.88 €	- 18 572 826.82 €	- 18 710 193.64 €	- 18 740 044.01 €	- 18 759 647.83 €	- 18 802 020.52 €	- 18 819 975.39 €	- 18 826 216.10 €	- 18 830 308.81 €	- 18 832 992.83 €	- 18 834 753.03 €	- 18 835 907.39 €	- 18 836 664.42 €	- 18 837 160.89 €	- 18 837 486.47 €	- 18 837 699.99 €
Maintenance (M)	- 22 161.58 €	- 625 684.66 €	- 1 038 413.49 €	- 1 040 034.34 €	- 2 463 156.82 €	- 1 605 183.29 €	- 985 671.07 €	- 1 400 612.10 €	- 1 450 746.57 €	- 1 740 895.88 €	- 2 089 075.06 €	- 2 506 890.07 €	- 3 008 268.09 €	- 3 609 921.70 €	- 4 331 906.04 €	- 5 198 287.25 €	- 6 237 944.70 €	- 7 485 533.64 €
Present Value (MPV)		- 409 398.87 €	- 441 568.00 €	- 423 336.60 €	- 472 033.76 €	- 131 704.93 €	- 60 334.97 €	- 73 071.76 €	- 49 636.28 €	- 39 062.17 €	- 30 740.69 €	- 24 191.94 €	- 19 038.29 €	- 14 982.53 €	- 11 790.78 €	- 9 278.97 €	- 7 302.25 €	- 5 746.64 €

i_A	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %
Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
An (Availabity per year)	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%
Inicial Investment (II)																		
II Present Value (IIPV) - Return	- 9 140.87 €	- 5 994.64 €	- 3 931.33 €	- 2 578.19 €	- 1 690.80 €	- 1 108.84 €	- 727.18 €	- 476.89 €	- 312.75 €	- 205.10 €	- 134.51 €	- 88.21 €	- 57.85 €	- 37.94 €	- 24.88 €	- 16.32 €	- 10.70 €	- 7.02 €
Functioning (F)	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €	- 278 131.96 €
Present Value (FPV)	- 140.03 €	- 91.83 €	- 60.22 €	- 39.50 €	- 25.90 €	- 16.99 €	- 11.14 €	- 7.31 €	- 4.79 €	- 3.14 €	- 2.06 €	- 1.35 €	- 0.89 €	- 0.58 €	- 0.38 €	- 0.25 €	- 0.16 €	- 0.11 €
Accumulated FPV	- 18 837 840.02 €	- 18 837 931.85 €	- 18 837 992.08 €	- 18 838 031.57 €	- 18 838 057.48 €	- 18 838 074.46 €	- 18 838 085.60 €	- 18 838 092.91 €	- 18 838 097.70 €	- 18 838 100.84 €	- 18 838 102.90 €	- 18 838 104.25 €	- 18 838 105.14 €	- 18 838 105.72 €	- 18 838 106.10 €	- 18 838 106.35 €	- 18 838 106.51 €	- 18 838 106.62 €
Maintenance (M)	- 8 982 640.37 €	- 10 779 168.45 €	- 12 935 002.13 €	- 15 522 002.56 €	- 18 626 403.07 €	- 22 351 683.69 €	- 26 822 020.43 €	- 32 186 424.51 €	- 38 623 709.41 €	- 46 348 451.30 €	- 55 618 141.56 €	- 66 741 769.87 €	- 80 090 123.84 €	- 96 108 148.61 €	- 115 329 778.33 €	- 138 395 734.00 €	- 166 074 880.80 €	- 199 289 856.96 €
Present Value (MPV)	- 4 522.42 €	- 3 559.00 €	- 2 800.82 €	- 2 204.16 €	- 1 734.60 €	- 1 365.08 €	- 1 074.27 €	- 845.42 €	- 665.32 €	- 523.58 €	- 412.04 €	- 324.27 €	- 255.19 €	- 200.82 €	- 158.04 €	- 124.37 €	- 97.88 €	- 77.03 €

Accumulated MPV		- 409 398.87 €	- 850 966.87 €	- 1 274 303.47 €	- 1 746 337.23 €	- 1 878 042.16 €	- 1 938 377.13 €	- 2 011 448.89 €	- 2 061 085.17 €	- 2 100 147.34 €	- 2 130 888.03 €	- 2 155 079.97 €	- 2 174 118.26 €	- 2 189 100.79 €	- 2 200 891.57 €	- 2 210 170.54 €	- 2 217 472.79 €	- 2 223 219.43 €	- 2 227 741.85 €	- 2 231 300.85 €	- 2 234 101.68 €	- 2 236 305.83 €	- 2 238 040.44 €	- 2 239 405.51 €	- 2 240 479.79 €	- 2 241 325.20 €	- 2 241 990.52 €	- 2 242 514.10 €	- 2 242 926.15 €	- 2 243 250.41 €	- 2 243 505.60 €	- 2 243 706.42 €	- 2 243 864.47 €	- 2 243 988.84 €	- 2 244 086.72 €	- 2 244 163.74 €
Non Production (NP)		- 24 353 320.00 €	- 26 204 452.00 €	- 10 214 162.00 €	- 22 786 284.00 €	- 25 899 978.00 €	- 20 954 978.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €	- 21 735 529.00 €
Present Value (NPPV)		- 15 934 898.81 €	- 11 143 005.79 €	- 4 157 582.49 €	- 4 366 711.49 €	- 2 125 087.45 €	- 1 282 697.74 €	- 1 133 970.86 €	- 743 665.86 €	- 487 701.17 €	- 319 837.77 €	- 209 751.81 €	- 137 556.67 €	- 90 210.61 €	- 59 160.74 €	- 38 798.02 €	- 25 444.01 €	- 16 686.35 €	- 10 943.02 €	- 7 176.51 €	- 4 706.40 €	- 3 086.49 €	- 2 024.14 €	- 1 327.45 €	- 870.55 €	- 570.91 €	- 374.41 €	- 245.54 €	- 161.03 €	- 105.60 €	- 69.25 €	- 45.42 €	- 29.79 €	- 19.53 €	- 12.81 €	- 8.40 €
Accumulated NPPV		- 15 934 898.81 €	- 27 077 904.60 €	- 31 235 487.09 €	- 35 602 198.57 €	- 37 727 286.03 €	- 39 009 983.77 €	- 40 143 954.62 €	- 40 887 620.49 €	- 41 375 321.66 €	- 41 695 159.44 €	- 41 904 911.24 €	- 42 042 467.92 €	- 42 132 678.53 €	- 42 191 839.26 €	- 42 230 637.28 €	- 42 256 081.29 €	- 42 272 767.64 €	- 42 283 710.66 €	- 42 290 887.17 €	- 42 295 593.58 €	- 42 298 680.07 €	- 42 300 704.21 €	- 42 302 031.66 €	- 42 302 902.21 €	- 42 303 473.12 €	- 42 303 847.53 €	- 42 304 093.07 €	- 42 304 254.10 €	- 42 304 359.70 €	- 42 304 428.95 €	- 42 304 474.37 €	- 42 304 504.16 €	- 42 304 523.69 €	- 42 304 536.50 €	- 42 304 544.90 €
Accumulated Total Costs		- 34 609 669.86 €	- 46 365 118.36 €	- 51 082 617.38 €	- 56 058 729.44 €	- 58 345 372.19 €	- 59 708 008.74 €	- 60 957 424.04 €	- 61 768 681.04 €	- 62 301 685.10 €	- 62 656 356.27 €	- 62 892 984.05 €	- 63 051 339.21 €	- 63 157 686.71 €	- 63 229 395.25 €	- 63 277 968.70 €	- 63 311 040.54 €	- 63 333 687.06 €	- 63 349 292.54 €	- 63 360 119.88 €	- 63 367 687.33 €	- 63 373 017.48 €	- 63 376 802.13 €	- 63 379 511.64 €	- 63 381 467.60 €	- 63 382 891.23 €	- 63 383 935.75 €	- 63 384 708.01 €	- 63 385 283.14 €	- 63 385 714.36 €	- 63 386 039.69 €	- 63 386 286.51 €	- 63 386 474.72 €	- 63 386 618.88 €	- 63 386 729.73 €	- 63 386 815.27 €
Benefit (B)		75 036 456.34 €	80 794 982.75 €	80 470 377.90 €	82 319 107.37 €	88 521 947.79 €	106 859 174.13 €	91 982 308.29 €	95 787 926.74 €	98 698 446.85 €	101 608 966.95 €	104 519 487.05 €	107 430 007.15 €	110 340 527.25 €	113 251 047.35 €	116 161 567.46 €	119 072 087.56 €	121 982 607.66 €	124 893 127.76 €	127 803 647.86 €	130 714 167.96 €	133 624 688.07 €	136 535 208.17 €	139 445 728.27 €	142 356 248.37 €	145 266 768.47 €	148 177 288.57 €	151 087 808.67 €	153 998 328.78 €	156 908 848.88 €	159 819 368.98 €	162 729 889.08 €	165 640 409.18 €	168 550 929.28 €	171 461 449.39 €	174 371 969.49 €
Present Value (BPV)		49 097 960.31 €	34 356 717.73 €	32 754 741.30 €	15 775 445.95 €	7 263 206.19 €	6 541 072.07 €	4 798 836.82 €	3 277 316.66 €	2 214 592.91 €	1 495 173.45 €	1 008 632.05 €	679 887.49 €	457 954.63 €	308 251.77 €	207 348.92 €	139 387.95 €	93 645.98 €	62 879.01 €	42 197.47 €	28 303.60 €	18 975.00 €	12 714.98 €	8 516.32 €	5 701.64 €	3 815.62 €	2 552.45 €	1 706.79 €	1 140.89 €	762.34 €	509.22 €	340.03 €	226.98 €	151.47 €	101.05 €	67.40 €
Accumulated Benefits BPV		49 097 960.31 €	83 454 678.05 €	116 209 419.35 €	131 984 865.30 €	139 248 071.49 €	145 789 143.56 €	150 587 980.38 €	153 865 297.04 €	156 079 889.95 €	157 575 063.40 €	158 583 695.45 €	159 263 582.95 €	159 721 537.58 €	160 029 789.36 €	160 237 138.27 €	160 376 526.22 €	160 470 172.20 €	160 533 051.22 €	160 575 248.68 €	160 603 552.29 €	160 622 527.28 €	160 635 242.26 €	160 643 758.59 €	160 649 460.22 €	160 653 275.84 €	160 655 828.29 €	160 657 535.08 €	160 658 675.97 €	160 659 438.31 €	160 659 947.53 €	160 660 287.56 €	160 660 514.55 €	160 660 666.02 €	160 660 767.08 €	160 660 834.47 €
BPV+FPV+MPV +NPPV +IIPV		- 3 667 709.55 €	18 933 559.69 €	46 970 801.97 €	57 770 135.86 €	62 746 699.30 €	67 925 134.83 €	71 474 556.35 €	73 940 616.00 €	75 622 204.85 €	76 762 707.13 €	77 534 711.41 €	78 056 243.73 €	78 407 850.88 €	78 644 394.10 €	78 803 169.57 €	78 909 485.68 €	78 980 485.14 €	79 027 758.68 €	79 059 128.80 €	79 079 864.95 €	79 093 509.81 €	79 102 440.14 €	79 108 246.95 €	79 111 992.62 €	79 114 384.61 €	79 115 892.54 €	79 116 827.06 €	79 117 392.82 €	79 117 723.94 €	79 117 907.84 €	79 118 001.05 €	79 118 039.83 €	79 118 047.14 €	79 118 037.35 €	79 118 019.21 €

Table 57 - *LCI* with availability dependent *NP*

i_A	0.00 %	52.83%	53.35%	34.93 %	51.14 %	64.89 %	59.29 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %	52.48 %
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
An (Availabity per year)		60%	54%	57%	58%	52%	49%	48%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%
Inicial Investment (II)	- 18 156 000.00 €																																			
II Present Value (IIPV) - Return	- 18 156 000.00 €	- 11 879 859.62 €	- 7 720 535.93 €	- 7 390 235.99 €	- 3 479 374.42 €	- 1 489 695.77 €	- 1 111 366.48 €	- 947 222.17 €	- 621 194.79 €	- 407 383.81 €	- 267 165.09 €	- 175 208.70 €	- 114 903.07 €	- 75 354.22 €	- 49 417.81 €	- 32 408.54 €	- 21 253.74 €	- 13 938.35 €	- 9 140.87 €	- 5 994.64 €	- 3 931.33 €	- 2 578.19 €	- 1 690.80 €	- 1 108.84 €	- 727.18 €	- 476.89 €	- 312.75 €	- 205.10 €	- 134.51 €	- 88.21 €	- 57.85 €	- 37.94 €	- 24.88 €	- 16.32 €	- 10.70 €	- 7.02 €
Functioning (F)	- 78	- 167	- 401	- 335	- 716	- 363	- 320	- 812	- 524	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278	- 278

	006.89 €	153.60 €	837.54 €	543.44 €	804.76 €	807.98 €	260.74 €	183.82 €	776.68 €	131.96 €	131.96 €	131.96 €
Present Value (FPV)		- 109 372.18 €	- 170 874.71 €	- 136 579.93 €	- 137 366.83 €	- 29 850.36 €	- 19 603.83 €	- 42 372.69 €	- 17 954.87 €	- 6 240.72 €	- 4 092.70 €	- 2 684.02 €
Accumulated FPV		- 18 265 372.18 €	- 18 436 246.88 €	- 18 572 826.82 €	- 18 710 193.64 €	- 18 740 044.01 €	- 18 759 647.83 €	- 18 802 020.52 €	- 18 819 975.39 €	- 18 826 216.10 €	- 18 830 308.81 €	- 18 832 992.83 €
Maintenance (M)	- 22 161.58 €	- 625 684.66 €	- 1 038 413.49 €	- 1 040 034.34 €	- 2 463 156.82 €	- 1 605 183.29 €	- 985 671.07 €	- 1 400 612.10 €	- 1 450 746.57 €	- 1 740 895.88 €	- 2 089 075.06 €	- 2 506 890.07 €
Present Value (MPV)		- 409 398.87 €	- 441 568.00 €	- 423 336.60 €	- 472 033.76 €	- 131 704.93 €	- 60 334.97 €	- 73 071.76 €	- 49 636.28 €	- 39 062.17 €	- 30 740.69 €	- 24 191.94 €
Accumulated MPV		- 409 398.87 €	- 850 966.87 €	- 1 274 303.47 €	- 1 746 337.23 €	- 1 878 042.16 €	- 1 938 377.13 €	- 2 011 448.89 €	- 2 061 085.17 €	- 2 100 147.34 €	- 2 130 888.03 €	- 2 155 079.97 €
Non Production (NP)		- 50 509 367.64 €	- 68 215 974.45 €	- 59 820 700.28 €	- 58 477 541.50 €	- 82 026 879.94 €	- 112 902 623.57 €	- 99 747 359.85 €	- 81 447 348.74 €	- 83 922 129.79 €	- 86 396 910.84 €	- 88 871 691.89 €
Present Value (NPPV)		- 33 049 360.92 €	- 29 007 704.43 €	- 24 349 476.33 €	- 11 206 502.66 €	- 6 730 287.31 €	- 6 911 004.16 €	- 5 203 949.67 €	- 2 786 663.85 €	- 1 883 042.33 €	- 1 271 328.42 €	- 857 627.98 €
Accumulated NPPV		- 33 049 360.92 €	- 62 057 065.35 €	- 86 406 541.68 €	- 97 613 044.34 €	- 104 343 331.65 €	- 111 254 335.82 €	- 116 458 285.49 €	- 119 244 949.34 €	- 121 127 991.68 €	- 122 399 320.09 €	- 123 256 948.08 €
Accumulated Total Costs		- 51 724 131.97 €	- 81 344 279.11 €	- 106 253 671.97 €	- 118 069 575.21 €	- 124 961 417.82 €	- 131 952 360.78 €	- 137 271 754.90 €	- 140 126 009.90 €	- 142 054 355.12 €	- 143 360 516.93 €	- 144 245 020.88 €
Benefit (B)		75 036 456.34 €	80 794 982.75 €	80 470 377.90 €	82 319 107.37 €	88 521 947.79 €	106 859 174.13 €	91 982 308.29 €	95 787 926.74 €	98 698 446.85 €	101 608 966.95 €	104 519 487.05 €
Present Value (BPV)		49 097 960.31 €	34 356 717.73 €	32 754 741.30 €	15 775 445.95 €	7 263 206.19 €	6 541 072.07 €	4 798 836.82 €	3 277 316.66 €	2 214 592.91 €	1 495 173.45 €	1 008 632.05 €
Accumulated Benefits BPV		49 097 960.31 €	83 454 678.05 €	116 209 419.35 €	131 984 865.30 €	139 248 071.49 €	145 789 143.56 €	150 587 980.38 €	153 865 297.04 €	156 079 889.95 €	157 575 063.40 €	158 583 695.45 €
BPV+FPV+MPV+NPPV+IIPV		- 20 782 171.66 €	- 16 045 601.06 €	- 8 200 252.62 €	- 4 240 709.91 €	- 3 869 346.33 €	- 4 319 217.22 €	- 4 839 774.52 €	- 4 416 712.85 €	- 4 130 465.17 €	- 3 941 453.53 €	- 3 817 325.43 €

	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €
Present Value (FPV)	- 1 760.20 €	- 1 154.35 €	- 757.03 €	- 496.47 €	- 325.59 €	- 213.52 €	- 140.03 €	- 91.83 €	- 60.22 €	- 39.50 €	- 25.90 €	- 16.99 €
Accumulated FPV	- 18 834 753.03 €	- 18 835 907.39 €	- 18 836 664.42 €	- 18 837 160.89 €	- 18 837 486.47 €	- 18 837 699.99 €	- 18 837 840.02 €	- 18 837 931.85 €	- 18 837 992.08 €	- 18 838 031.57 €	- 18 838 057.48 €	- 18 838 074.46 €
Maintenance (M)	- 3 008 268.09 €	- 3 609 921.70 €	- 4 331 906.04 €	- 5 198 287.25 €	- 6 237 944.70 €	- 7 485 533.64 €	- 8 982 640.37 €	- 10 779 168.45 €	- 12 935 002.13 €	- 15 522 002.56 €	- 18 626 403.07 €	- 22 351 683.69 €
Present Value (MPV)	- 19 038.29 €	- 14 982.53 €	- 11 790.78 €	- 9 278.97 €	- 7 302.25 €	- 5 746.64 €	- 4 522.42 €	- 3 559.00 €	- 2 800.82 €	- 2 204.16 €	- 1 734.60 €	- 1 365.08 €
Accumulated MPV	- 2 174 118.26 €	- 2 189 100.79 €	- 2 200 891.57 €	- 2 210 170.54 €	- 2 217 472.79 €	- 2 223 219.43 €	- 2 227 741.85 €	- 2 231 300.85 €	- 2 234 101.68 €	- 2 236 305.83 €	- 2 238 040.44 €	- 2 239 405.51 €
Non Production (NP)	- 91 346 472.94 €	- 93 821 253.99 €	- 96 296 035.03 €	- 98 770 816.08 €	- 101 245 597.13 €	- 103 720 378.18 €	- 106 195 159.23 €	- 108 669 940.27 €	- 111 144 721.32 €	- 113 619 502.37 €	- 116 094 283.42 €	- 118 569 064.47 €
Present Value (NPPV)	- 578 100.35 €	- 389 393.44 €	- 262 102.86 €	- 176 306.35 €	- 118 519.94 €	- 79 626.08 €	- 53 465.29 €	- 35 880.01 €	- 24 066.22 €	- 16 134.22 €	- 10 811.40 €	- 7 241.33 €
Accumulated NPPV	- 123 835 048.43 €	- 124 224 441.87 €	- 124 486 544.73 €	- 124 662 851.08 €	- 124 781 371.02 €	- 124 860 997.09 €	- 124 914 462.38 €	- 124 950 342.39 €	- 124 974 408.61 €	- 124 990 542.83 €	- 125 001 354.23 €	- 125 008 595.56 €
Accumulated Total Costs	- 144 843 919.72 €	- 145 249 450.05 €	- 145 524 100.72 €	- 145 710 182.50 €	- 145 836 330.28 €	- 145 921 916.51 €	- 145 980 044.25 €	- 146 019 575.10 €	- 146 046 502.37 €	- 146 064 880.24 €	- 146 077 452.14 €	- 146 086 075.53 €
Benefit (B)	107 430 007.15 €	110 340 527.25 €	113 251 047.35 €	116 161 567.46 €	119 072 087.56 €	121 982 607.66 €	124 893 127.76 €	127 803 647.86 €	130 714 167.96 €	133 624 688.07 €	136 535 208.17 €	139 445 728.27 €
Present Value (BPV)	679 887.49 €	457 954.63 €	308 251.77 €	207 348.92 €	139 387.95 €	93 645.98 €	62 879.01 €	42 197.47 €	28 303.60 €	18 975.00 €	12 714.98 €	8 516.32 €
Accumulated Benefits BPV	159 263 582.95 €	159 721 537.58 €	160 029 789.36 €	160 237 138.27 €	160 376 526.22 €	160 470 172.20 €	160 533 051.22 €	160 575 248.68 €	160 603 552.29 €	160 622 527.28 €	160 635 242.26 €	160 643 758.59 €
BPV+FPV+MPV+NPPV+IIPV	- 3 736 336.78 €	- 3 683 912.47 €	- 3 650 311.37 €	- 3 629 044.23 €	- 3 615 804.05 €	- 3 607 744.31 €	- 3 602 993.04 €	- 3 600 326.42 €	- 3 598 950.08 €	- 3 598 352.95 €	- 3 598 209.88 €	- 3 598 316.95 €

	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €	131.96 €
Present Value (FPV)	- 11.14 €	- 7.31 €	- 4.79 €	- 3.14 €	- 2.06 €	- 1.35 €	- 0.89 €	- 0.58 €	- 0.38 €	- 0.25 €	- 0.16 €	- 0.11 €
Accumulated FPV	- 18 838 085.60 €	- 18 838 092.91 €	- 18 838 097.70 €	- 18 838 100.84 €	- 18 838 102.90 €	- 18 838 104.25 €	- 18 838 105.14 €	- 18 838 105.72 €	- 18 838 106.10 €	- 18 838 106.35 €	- 18 838 106.51 €	- 18 838 106.62 €
Maintenance (M)	- 26 822 020.43 €	- 32 186 424.51 €	- 38 623 709.41 €	- 46 348 451.30 €	- 55 618 141.56 €	- 66 741 769.87 €	- 80 090 123.84 €	- 96 108 148.61 €	- 115 329 778.33 €	- 138 395 734.00 €	- 166 074 880.80 €	- 199 289 856.96 €
Present Value (MPV)	- 1 074.27 €	- 845.42 €	- 665.32 €	- 523.58 €	- 412.04 €	- 324.27 €	- 255.19 €	- 200.82 €	- 158.04 €	- 124.37 €	- 97.88 €	- 77.03 €
Accumulated MPV	- 2 240 479.79 €	- 2 241 325.20 €	- 2 241 990.52 €	- 2 242 514.10 €	- 2 242 926.15 €	- 2 243 250.41 €	- 2 243 505.60 €	- 2 243 706.42 €	- 2 243 864.47 €	- 2 243 988.84 €	- 2 244 086.72 €	- 2 244 163.74 €
Non Production (NP)	- 121 043 845.51 €	- 123 518 626.56 €	- 125 993 407.61 €	- 128 468 188.66 €	- 130 942 969.71 €	- 133 417 750.75 €	- 135 892 531.80 €	- 138 367 312.85 €	- 140 842 093.90 €	- 143 316 874.95 €	- 145 791 655.99 €	- 148 266 437.04 €
Present Value (NPPV)	- 4 848.03 €	- 3 244.38 €	- 2 170.31 €	- 1 451.26 €	- 970.08 €	- 648.21 €	- 432.99 €	- 289.13 €	- 193.00 €	- 128.80 €	- 85.92 €	- 57.31 €
Accumulated NPPV	- 125 013 443.59 €	- 125 016 687.97 €	- 125 018 858.28 €	- 125 020 309.55 €	- 125 021 279.63 €	- 125 021 927.84 €	- 125 022 360.83 €	- 125 022 649.95 €	- 125 022 842.96 €	- 125 022 971.75 €	- 125 023 057.68 €	- 125 023 114.98 €
Accumulated Total Costs	- 146 092 008.98 €	- 146 096 106.08 €	- 146 098 946.50 €	- 146 100 924.49 €	- 146 102 308.68 €	- 146 103 282.50 €	- 146 103 971.56 €	- 146 104 462.10 €	- 146 104 813.52 €	- 146 105 066.94 €	- 146 105 250.91 €	- 146 105 385.35 €
Benefit (B)	142 356 248.37 €	145 266 768.47 €	148 177 288.57 €	151 087 808.67 €	153 998 328.78 €	156 908 848.88 €	159 819 368.98 €	162 729 889.08 €	165 640 409.18 €	168 550 929.28 €	171 461 449.39 €	174 371 969.49 €
Present Value (BPV)	5 701.64 €	3 815.62 €	2 552.45 €	1 706.79 €	1 140.89 €	762.34 €	509.22 €	340.03 €	226.98 €	151.47 €	101.05 €	67.40 €
Accumulated Benefits BPV	160 649 460.22 €	160 653 275.84 €	160 655 828.29 €	160 657 535.08 €	160 658 675.97 €	160 659 438.31 €	160 659 947.53 €	160 660 287.56 €	160 660 514.55 €	160 660 666.02 €	160 660 767.08 €	160 660 834.47 €
BPV+FPV+MPV+NPPV+IIPV	- 3 598 548.76 €	- 3 598 830.24 €	- 3 599 118.21 €	- 3 599 389.41 €	- 3 599 632.71 €	- 3 599 844.20 €	- 3 600 024.03 €	- 3 600 174.53 €	- 3 600 298.97 €	- 3 600 400.92 €	- 3 600 483.83 €	- 3 600 550.88 €

The *LCR* method, on the other hand, not only considers the devaluation of the asset that has been acquired or that is to be acquired according to the rates, but also the value of a new equivalent at the same rates. In this way, as well as companies being able to consider the costs of renewing and maintaining assets, it is easier to determine the best time to replace them, based on the change in the market value of the new one.

Equation 1.39 relates the apparent rate to the devaluation of the physical asset, where C_{0D}^{t} represents the value of the asset in period *t* and C_{0D}^{t-1} represents the value of the asset in the previous period.

$$C_{0D}^{t} = C_{0D}^{t-1} \times i_A \tag{1.39}$$

Equation 1.40 represents the same relationship, but this time from the point of view of acquiring equivalent new equipment, where C_{0N}^{t} represents the value of the new physical asset at time *t* and C_{0N}^{t-1} the value of the new physical asset at time *t-1*.

$$C_{0N}^{t} = C_{0N}^{t-1} \times i_A \tag{1.40}$$

The total net present value, considering the devaluation of physical assets (NPV_{TD}), depends on the net present value of the movement in financial production (NPV_{FpD}) and the net present value of the financial expenses movement (NPV_{Fe}), as can be seen in equation 1.41.

$$NPV_{TD} = NPV_{FpD} + NPV_{Fe} - C_{0D}^{t} \tag{1.41}$$

Similarly, equation 1.42 represents the total net present value considering a new physical asset (NPV_{TN}), where NPV_{FpN} represents the net present value of the financial production movement considering the acquisition of the equivalent new equipment.

$$NPV_{TN} = NPV_{FpN} + NPV_{Fe} - C_{0N}^{t} \tag{1.42}$$

NPV_{Fe} is calculated using equation 1.43.

$$NPV_{Fe} = -\sum_{j=0}^{n} \frac{F_j}{(1+i_A)^j} - \sum_{j=0}^{n} \frac{M_j}{(1+i_A)^j} \tag{1.43}$$

The results of the application of this method can be seen in Table 58.

Table 58 - *LCR*

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
i_A	50.95%	52.83%	53.35%	34.93%	51.14%	64.89%	59.29%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%
Initial Investment (II) (C0)	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €
Initial Investment (II) (C0) with Devaluation (NPV)		-12 027 478 €	-7 720 536 €	-7 390 236 €	-3 479 374 €	-1 489 696 €	-1 111 366 €	-947 222 €	-621 195 €	-407 384 €	-267 165 €	-175 209 €	-114 903 €	-75 354 €	-49 418 €	-32 409 €	-21 254 €	-13 938 €
Initial Investment (II) (C0) considering NEW Asset (NPV)		-27 407 269 €	-42 696 561 €	-44 604 846 €	-94 741 265 €	-221 280 305 €	-296 608 132 €	-348 007 413 €	-530 655 343 €	-809 164 065 €	-1 233 845 079 €	-1 881 415 334 €	-2 868 855 839 €	-4 374 543 820 €	-6 670 475 866 €	-10 171 403 032 €	-15 509 753 984 €	-23 649 880 739 €
Fp Financial Movement (Production)		75 036 456 €	80 794 983 €	80 470 378 €	82 319 107 €	88 521 948 €	106 859 174 €	91 982 308 €	95 787 927 €	98 698 447 €	101 608 967 €	104 519 487 €	107 430 007 €	110 340 527 €	113 251 047 €	116 161 567 €	119 072 088 €	121 982 608 €
Fp Financial Movement (NPV)		49 708 050 €	34 356 718 €	32 754 741 €	15 775 446 €	7 263 206 €	6 541 072 €	4 798 837 €	3 277 317 €	2 214 593 €	1 495 173 €	1 008 632 €	679 887 €	457 955 €	308 252 €	207 349 €	139 388 €	93 646 €
Fp Financial Movement (NPV) Accumulated		49 708 050 €	84 064 768 €	116 819 509 €	132 594 955 €	139 858 161 €	146 399 233 €	151 198 070 €	154 475 387 €	156 689 980 €	158 185 153 €	159 193 785 €	159 873 673 €	160 331 628 €	160 639 879 €	160 847 228 €	160 986 616 €	161 080 262 €
Functioning Costs	-78 007 €	-167 154 €	-401 838 €	-335 543 €	-716 805 €	-363 808 €	-320 261 €	-812 184 €	-524 777 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €
Functioning Costs (NPV)		-110 731 €	-170 875 €	-136 580 €	-137 367 €	-29 850 €	-19 604 €	-42 373 €	-17 955 €	-6 241 €	-4 093 €	-2 684 €	-1 760 €	-1 154 €	-757 €	-496 €	-326 €	-214 €
Functioning Costs (NPV) Accumulated		-110 731 €	-281 606 €	-418 186 €	-555 553 €	-585 403 €	-605 007 €	-647 380 €	-665 334 €	-671 575 €	-675 668 €	-678 352 €	-680 112 €	-681 266 €	-682 023 €	-682 520 €	-682 846 €	-683 059 €
Maintenance Costs	-22 162 €	-625 685 €	-1 038 413 €	-1 040 034 €	-2 463 157 €	-1 605 183 €	-985 671 €	-1 400 612 €	-1 450 747 €	-1 740 896 €	-2 089 075 €	-2 506 890 €	-3 008 268 €	-3 609 922 €	-4 331 906 €	-5 198 287 €	-6 237 945 €	-7 485 534 €
Maintenance Costs (NPV)		-414 486 €	-441 568 €	-423 337 €	-472 034 €	-131 705 €	-60 335 €	-73 072 €	-49 636 €	-39 062 €	-30 741 €	-24 192 €	-19 038 €	-14 983 €	-11 791 €	-9 279 €	-7 302 €	-5 747 €

Year	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
i_A	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%	52.48%
Initial Investment (II) (C0)	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €	-18 156 000 €
Initial Investment (II) (C0) with Devaluation (NPV)	-9 141 €	-5 995 €	-3 931 €	-2 578 €	-1 691 €	-1 109 €	-727 €	-477 €	-313 €	-205 €	-135 €	-88 €	-58 €	-38 €	-25 €	-16 €	-11 €	-7 €
Initial Investment (II) (C0) considering NEW Asset (NPV)	-36 062 265 045 €	-54 989 155 106 €	-83 849 618 859 €	-127 857 185 099 €	-194 961 646 863 €	-297 285 160 140 €	-453 312 063 484 €	-691 228 000 764 €	-1 054 011 546 411 €	-1 607 198 115 151 €	-2 450 718 675 846 €	-3 736 951 886 342 €	-5 698 250 696 203 €	-8 688 915 989 378 €	-13 249 199 639 599 €	-20 202 898 877 665 €	-30 806 172 007 650 €	-46 974 458 443 390 €
Fp Financial Movement (Production)	124 893 128 €	127 803 648 €	130 714 168 €	133 624 688 €	136 535 208 €	139 445 728 €	142 356 248 €	145 266 768 €	148 177 289 €	151 087 809 €	153 998 329 €	156 908 849 €	159 819 369 €	162 729 889 €	165 640 409 €	168 550 929 €	171 461 449 €	174 371 969 €
Fp Financial Movement (NPV)	62 879 €	42 197 €	28 304 €	18 975 €	12 715 €	8 516 €	5 702 €	3 816 €	2 552 €	1 707 €	1 141 €	762 €	509 €	340 €	227 €	151 €	101 €	67 €
Fp Financial Movement (NPV) Accumulated	161 143 141 €	161 185 339 €	161 213 642 €	161 232 617 €	161 245 332 €	161 253 849 €	161 259 550 €	161 263 366 €	161 265 918 €	161 267 625 €	161 268 766 €	161 269 528 €	161 270 037 €	161 270 377 €	161 270 604 €	161 270 756 €	161 270 857 €	161 270 924 €
Functioning Costs	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €	-278 132 €
Functioning Costs (NPV)	-140 €	-92 €	-60 €	-39 €	-26 €	-17 €	-11 €	-7 €	-5 €	-3 €	-2 €	-1 €	-1 €	-1 €	-0 €	-0 €	-0 €	-0 €
Functioning Costs (NPV) Accumulated	-683 199 €	-683 291 €	-683 351 €	-683 391 €	-683 417 €	-683 434 €	-683 445 €	-683 452 €	-683 457 €	-683 460 €	-683 462 €	-683 463 €	-683 464 €	-683 465 €	-683 465 €	-683 465 €	-683 466 €	-683 466 €
Maintenance Costs	-8 982 640 €	-10 779 168 €	-12 935 002 €	-15 522 003 €	-18 626 403 €	-22 351 684 €	-26 822 020 €	-32 186 425 €	-38 623 709 €	-46 348 451 €	-55 618 142 €	-66 741 770 €	-80 090 124 €	-96 108 149 €	-115 329 778 €	-138 395 734 €	-166 074 881 €	-199 289 857 €
Maintenance Costs (NPV)	-4 522 €	-3 559 €	-2 801 €	-2 204 €	-1 735 €	-1 365 €	-1 074 €	-845 €	-665 €	-524 €	-412 €	-324 €	-255 €	-201 €	-158 €	-124 €	-98 €	-77 €

Maintenance Costs (NPV) Accumulated		- 414 486 €	- 856 054 €	- 1 279 391 €	- 1 751 424 €	- 1 883 129 €	- 1 943 464 €	- 2 016 536 €	- 2 066 172 €	- 2 105 235 €	- 2 135 975 €	- 2 160 167 €	- 2 179 205 €	- 2 194 188 €	- 2 205 979 €	- 2 215 258 €
Total Costs (NPV) Accumulated	- €	- 525 217 €	- 1 137 660 €	- 1 697 577 €	- 2 306 977 €	- 2 468 532 €	- 2 548 471 €	- 2 663 916 €	- 2 731 507 €	- 2 776 810 €	- 2 811 643 €	- 2 838 519 €	- 2 859 318 €	- 2 875 454 €	- 2 888 002 €	- 2 897 778 €
FT Net Present Value (NPV) (With II Devaluated)		37 155 355 €	75 206 572 €	107 731 697 €	126 808 604 €	135 899 933 €	142 739 396 €	147 586 932 €	151 122 685 €	153 505 786 €	155 106 345 €	156 180 058 €	156 899 452 €	157 380 819 €	157 702 459 €	157 917 042 €
FT Net Present Value (With II New Asset)		21 775 564 €	40 230 547 €	70 517 087 €	35 546 713 €	- 83 890 676 €	- 152 757 369 €	- 199 473 259 €	- 378 911 463 €	- 655 250 895 €	- 1 078 471 569 €	- 1 725 060 068 €	- 2 711 841 484 €	- 4 217 087 647 €	- 6 512 723 989 €	- 10 013 453 582 €

Maintenance Costs (NPV) Accumulated	- 2 222 560 €	- 2 228 307 €	- 2 232 829 €	- 2 236 388 €	- 2 239 189 €	- 2 241 393 €	- 2 243 128 €	- 2 244 493 €	- 2 245 567 €	- 2 246 412 €	- 2 247 078 €	- 2 247 601 €	- 2 248 013 €	- 2 248 338 €	- 2 248 593 €	- 2 248 794 €	- 2 248 952 €	- 2 249 076 €	- 2 249 174 €	- 2 249 251 €
Total Costs (NPV) Accumulated	- 2 905 405 €	- 2 911 366 €	- 2 916 028 €	- 2 919 679 €	- 2 922 540 €	- 2 924 784 €	- 2 926 544 €	- 2 927 926 €	- 2 929 012 €	- 2 929 864 €	- 2 930 534 €	- 2 931 061 €	- 2 931 475 €	- 2 931 801 €	- 2 932 057 €	- 2 932 258 €	- 2 932 417 €	- 2 932 541 €	- 2 932 639 €	- 2 932 717 €
FT Net Present Value (NPV) (With II Devaluated)	158 059 957 €	158 154 958 €	158 217 972 €	158 259 665 €	158 287 171 €	158 305 255 €	158 317 097 €	158 324 813 €	158 329 811 €	158 333 025 €	158 335 071 €	158 336 359 €	158 337 156 €	158 337 639 €	158 337 923 €	158 338 081 €	158 338 163 €	158 338 198 €	158 338 207 €	158 338 201 €
FT Net Present Value (With II New Asset)	- 15 351 672 774 €	- 23 491 711 843 €	- 35 904 037 932 €	- 54 830 889 446 €	- 83 691 327 757 €	- 127 698 877 266 €	- 194 803 328 074 €	- 297 126 834 218 €	- 453 153 732 946 €	- 691 069 667 262 €	- 1 053 853 211 027 €	- 1 607 039 778 588 €	- 2 450 560 338 556 €	- 3 736 793 548 615 €	- 5 698 092 358 223 €	- 8 688 757 651 259 €	- 13 249 041 301 411 €	- 20 202 740 539 451 €	- 30 806 013 669 432 €	- 46 974 300 105 182 €

3.2.2.5 Results comparison

In this subchapter, the graphs for all the economic models will be presented first. This will be followed by a brief reflection on the data obtained. The economic life cycle models will be compared in a divided format: those that do not consider production value and those that do consider production value.

As with the first case study, the three depreciation models were analysed for the three methods. Therefore, Figure 26 corresponds to the values obtained by applying the linear depreciation method to *MTAC-RPV*, *MTAC* and *UAI*. Following the same logic, Figures 27 and 28 correspond to the application of the exponential depreciation method and the sum of the digits depreciation method, respectively.

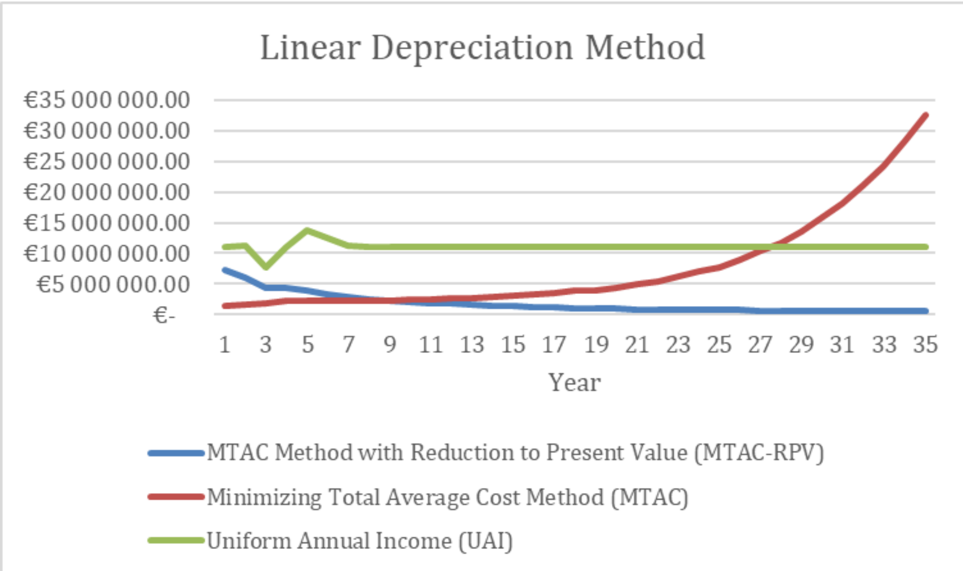

Figure 26 - Linear depreciation method applied to *Celbi*'s equipment: comparison between economic models

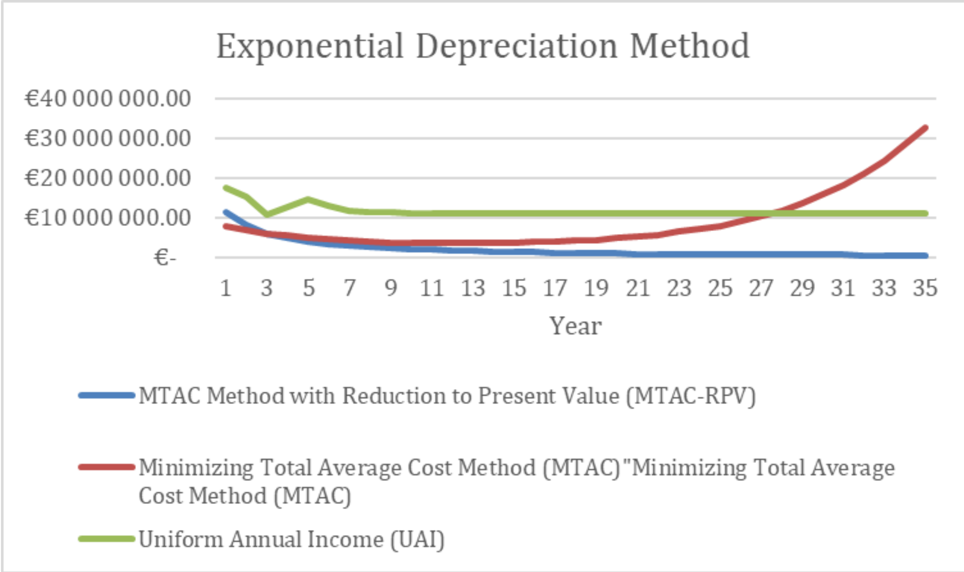

Figure 27 - Exponential method of depreciation applied to *Celbi*'s equipment: comparison between economic models

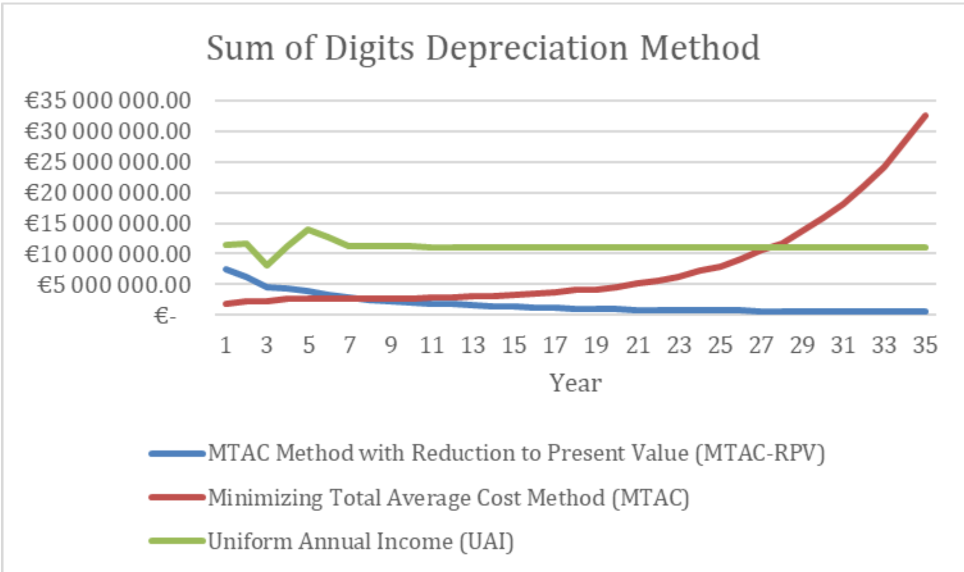

Figure 28 - Sum of the digits depreciation method applied to *Celbi*'s equipment: comparison of economic models

As for the models which consider benefits, the figures which represent the graphs obtained are shown below. Figure 29 shows the graphs generated by applying the *LCI* method (with risk-dependent *NP* and availability-dependent *NP*). Figure 30 represents the lines obtained with the *LCI* method and compares them with the values obtained with the *LCR*, considering the acquisition of new equivalent equipment and the devaluation of the equipment already acquired.

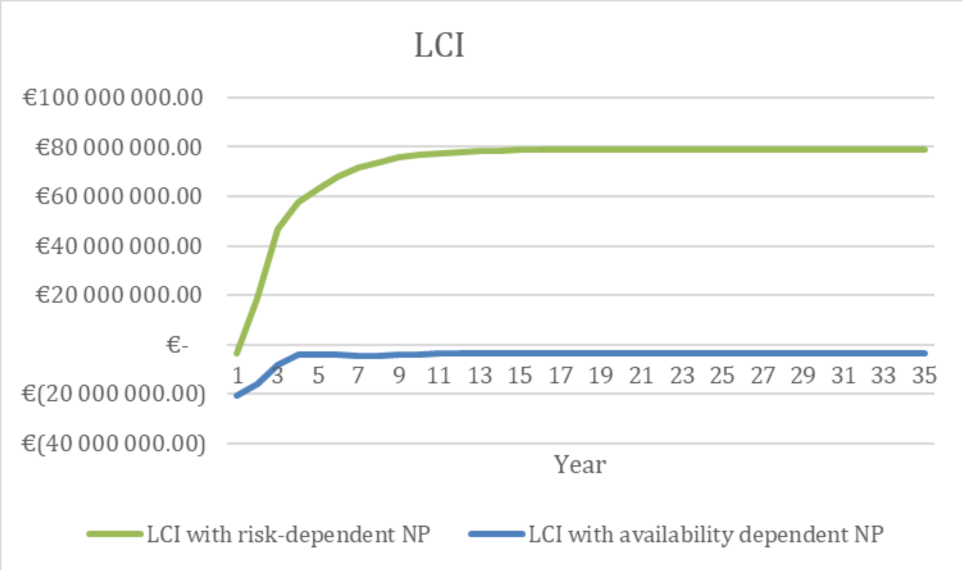

Figure 29 – *LCI* applied to *Celbi*'s equipment: comparison of economic models

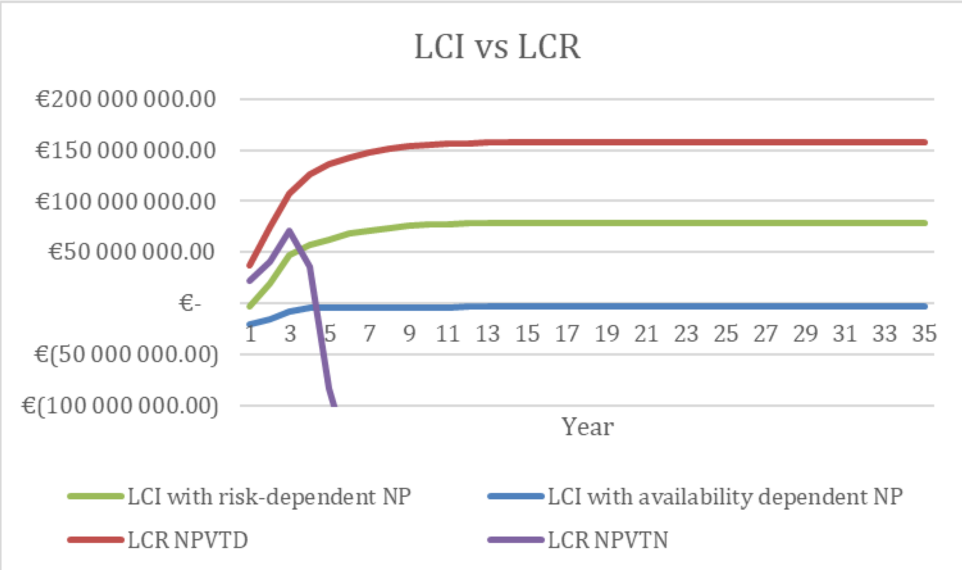

Figure 30 - *LCI* vs *LCR* applied to *Celbi*'s equipment: comparison of economic models

It is possible to see some differences between the results of the methods applied. Starting with the Lifespan method, which indicates that the equipment should be replaced around the 15th period, which is less than half the estimated lifetime of the minimum transfer value.

Interesting aspects can be seen in the first three models. With the exception of *MTAC* using the exponential depreciation method, which considers the 12th period to be the optimum time to replace the asset, the other models consider either the first year, the third year or the last year. The irregularity of the data for the first 7 years (real data) may influence these results since it does not take production values into account, in addition to the fact that the projections made for costs take into account multiplicative and linear factors.

In the case of *UAI*, all the methods applied result in the 3rd year being indicated as the optimal period for replacing the asset. In addition to the aspect mentioned in the previous paragraph, this time period coincides with the year of lockdown due to covid-19 (2020). This is a completely atypical year and should not be included in the assessment of the asset's life cycle. If the first 4 years are disregarded from the study, these models suggest substituting the linear depreciation method in the 13th period, the exponential method in the 19th period and the sum of the digits method in the 16th period, as can be seen in the graphs in Figures 31, 32 and 33, respectively.

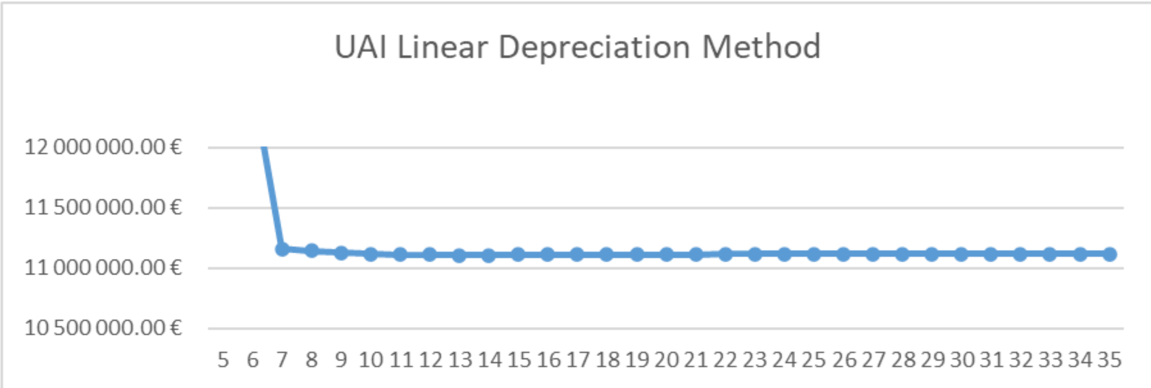

Figure 31 - *UAI* applied to *Celbi*'s equipment from 5th period with Linear Depreciation Method

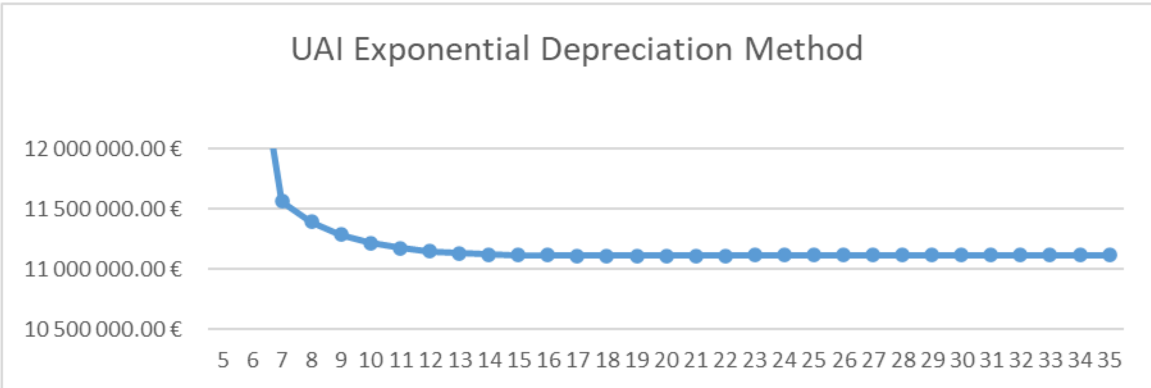

Figure 32 - *UAI* applied to *Celbi*'s equipment from 5th period with Exponential Depreciation Method

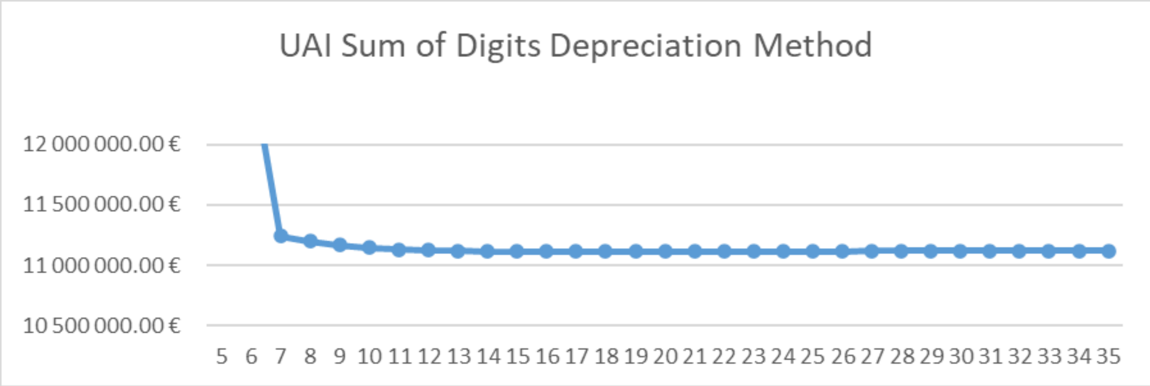

Figure 33 - *UAI* applied to *Celbi*'s equipment from 5th period with Sum of Digits Depreciation Method

The fact that these methods do not take production figures into consideration can also distort reality, since more operating costs can often mean more production. Figure 34 relates these two figures.

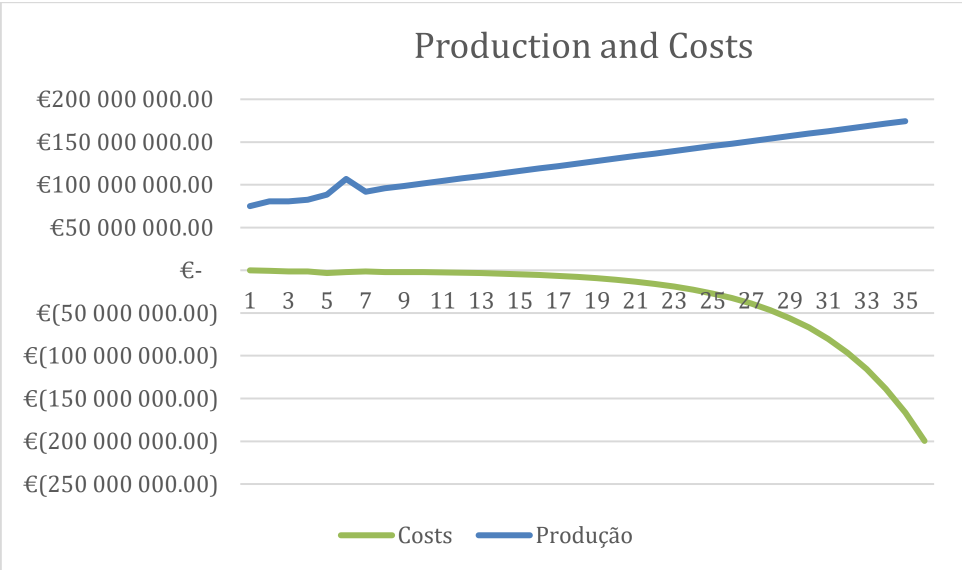

Figure 34 - Production and Costs

As the graph shows, in the first 7 years (real values), more operating costs meant a disproportionate increase in production values.

The *LCI* with risk-dependent *NP* method confirms the return on investment in the second year of operation. It also suggests replacing the equipment in the 33rd operating period. On the other hand, using non-production costs dependent on the availability of the equipment, the return on the equipment is never verified, suggesting that it be replaced in the 22nd period. Bearing in mind that no data was provided on the redundancy of operation between the west and east lines and that an average availability was therefore used for calculation purposes, this could lead to misleading interpretations.

This model was then calculated for 100 per cent availability. Although it's not possible to achieve availability of this order of magnitude at the moment, it's still a relevant assessment. This calculation shows how profitable the asset could be by minimizing non-production costs. It also motivates the monitoring of this type of data, since the more data obtained, the more easily it can be fine-tuned, increasing the company's control over its production and making it possible to maximize its investments. Figure 35 shows the results obtained, where it can be seen that the return could have been made in the first year of operation and replacement would be advised in the 34th period.

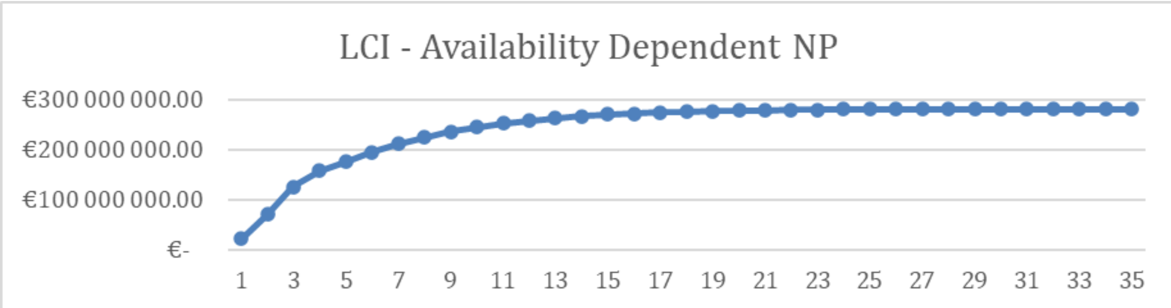

Figure 35 - *LCI* Availability Dependent without *NP* costs applied to *Celbi*'s equipment

As with the latter, the *LCR* method suggests replacing the asset in the 34th operating period. While the recommended time interval for acquiring equivalent new equipment would be around the 3rd year.

However, given that it wasn't possible to access all the asset's real data, the aim was to demonstrate one of the great potentialities of the *LCI* and *LCR* models, which is to establish goals and objectives for the company. In addition to creating hypothetical scenarios that can allow the organisation to work towards or away from them, depending on its objectives. For all the following scenarios, the profit parameter that Fisher considered in his equation was null.

Figures 36, 37 and 38 show that what happens, given that previously a constant average profit (indicated by the company as a reference value) had been considered, is a " transfer " of values. The periods recommended for replacement have remained the same, but the periods in which the return occurs have changed. For the *LCI* with risk-dependent *NP* method, the return occurs in the first year. For the *LCI* with availability dependent *NP* method, it occurs in the third year of operation. For the *LCR* method, the purchase of equivalent new equipment is more favourable in the 7th year of use.

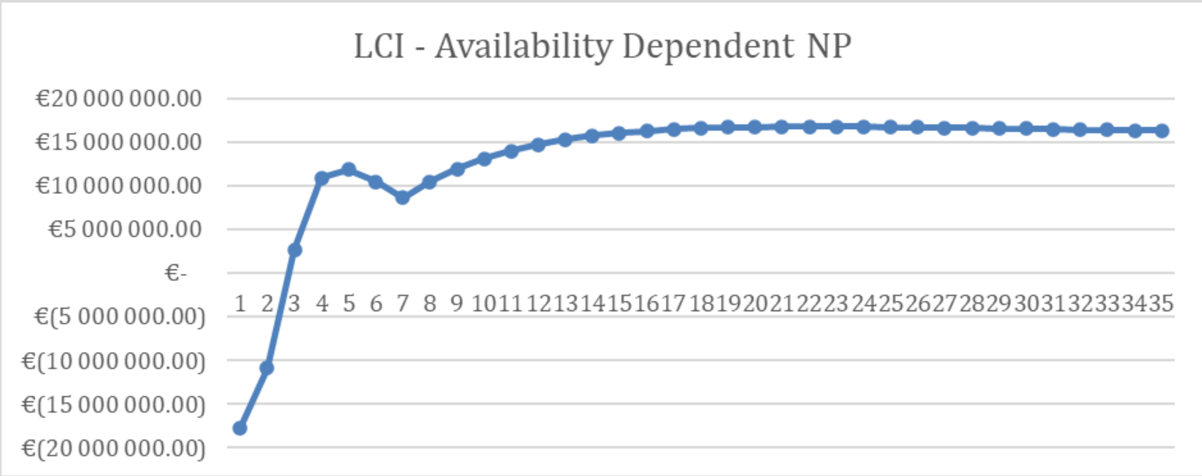

Figure 36 - *LCI* - Availability Dependent with null fisher equation profit applied to *Celbi*'s equipment

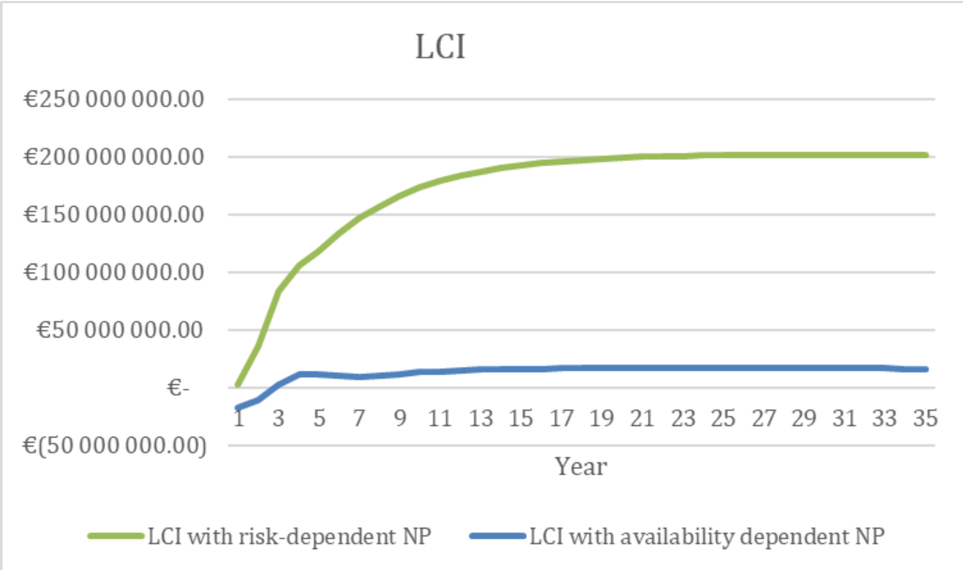

Figure 37 – *LCI* with null fisher equation profit applied to *Celbi*'s equipment

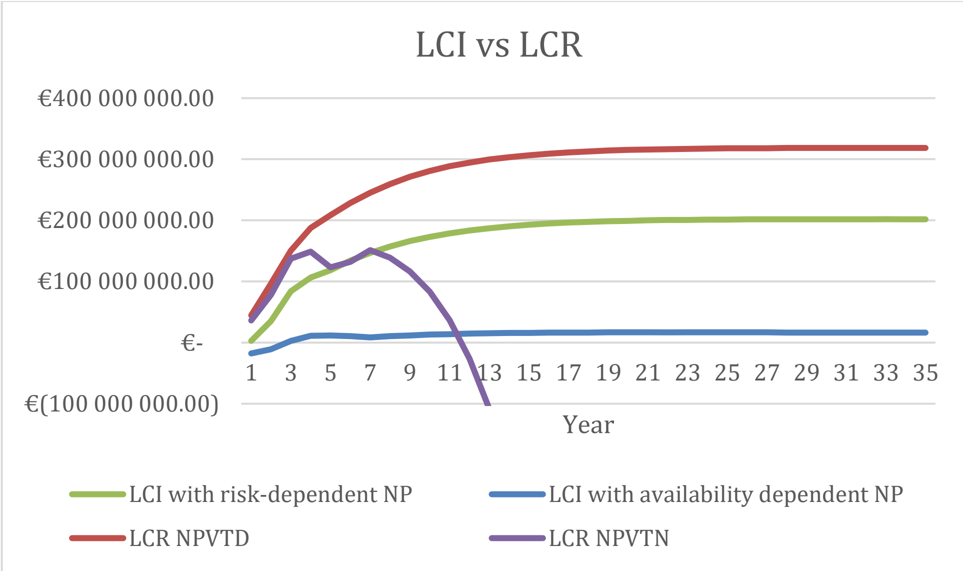

Figure 38 - *LCI* vs *LCR* with null fisher equation profit applied to *Celbi*'s equipment

The next step was to consider zero risk. In other words, a scenario in which there would never be a shortage of wood to feed into the line. In this way, the apparent rate was calculated using only the capitalisation rate and inflation.

The *LCI* with risk-dependent *NP* method thus realises its return in the first year, while replacement is now recommended in the 34th year of operation. As can be seen in Figure 39.

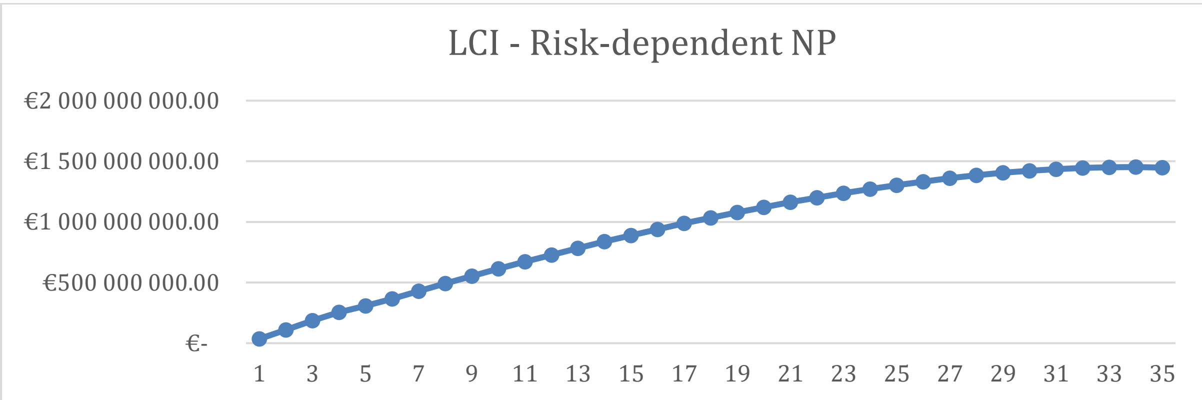

Figure 39 - *LCI* - Risk Dependent with 0% risk applied to *Celbi*'s equipment

The *LCR* method, on the other hand, verifies a match between the replacement of the asset and the acquisition of the equivalent new equipment, with both occurring between the 33rd and 34th periods of operation, as can be seen in the graph in Figure 40.

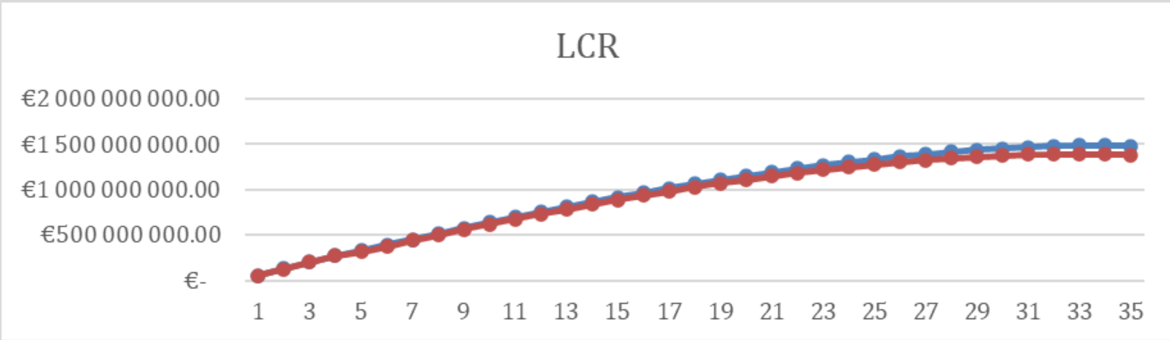

Figure 40 - *LCR* with 0% risk applied to *Celbi*'s equipment

Subsequently, by resetting the risk values, an attempt was made to identify the relevance of the price of chip from external suppliers for the life cycle models applied. A price lower than the original was considered, with the new value being 20 €/*m*³. The results are shown in Figure 41, which suggests replacing the *LCI* with risk-dependent *NP* method in the 33rd year.

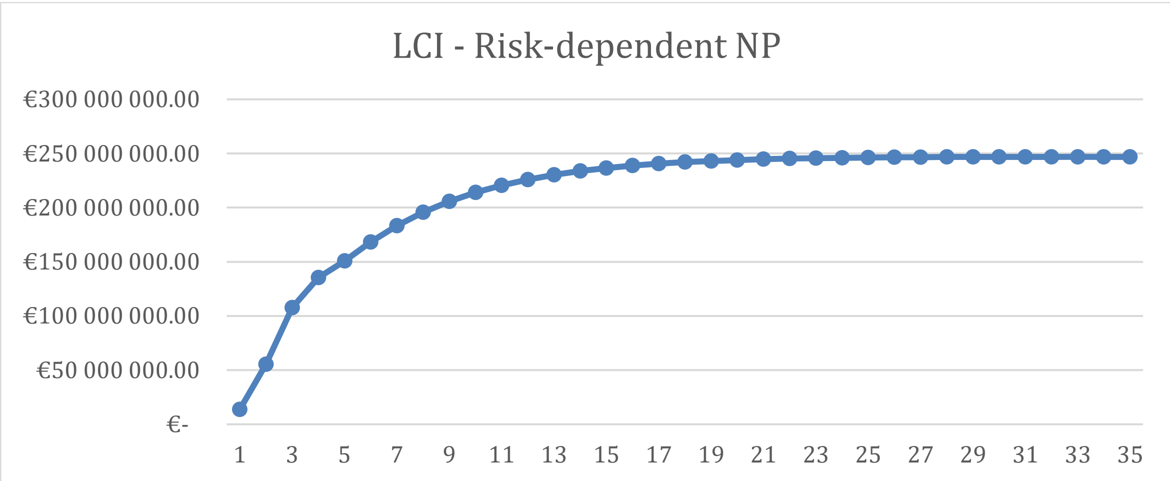

Figure 41 - *LCI* with risk-dependent *NP* with external chip price of 20 €/*m*³ applied to *Celbi*'s equipment

For the next scenario, the chip price of 46€/*m*³ was reset and a new coefficient for the increase in maintenance costs of 10% was considered. What happened was an increase in the life cycle of the equipment in the *LCI* and *LCR* methods. They all suggest replacement in the period when the transfer value is minimal, as can be seen in the graph in Figure 42.

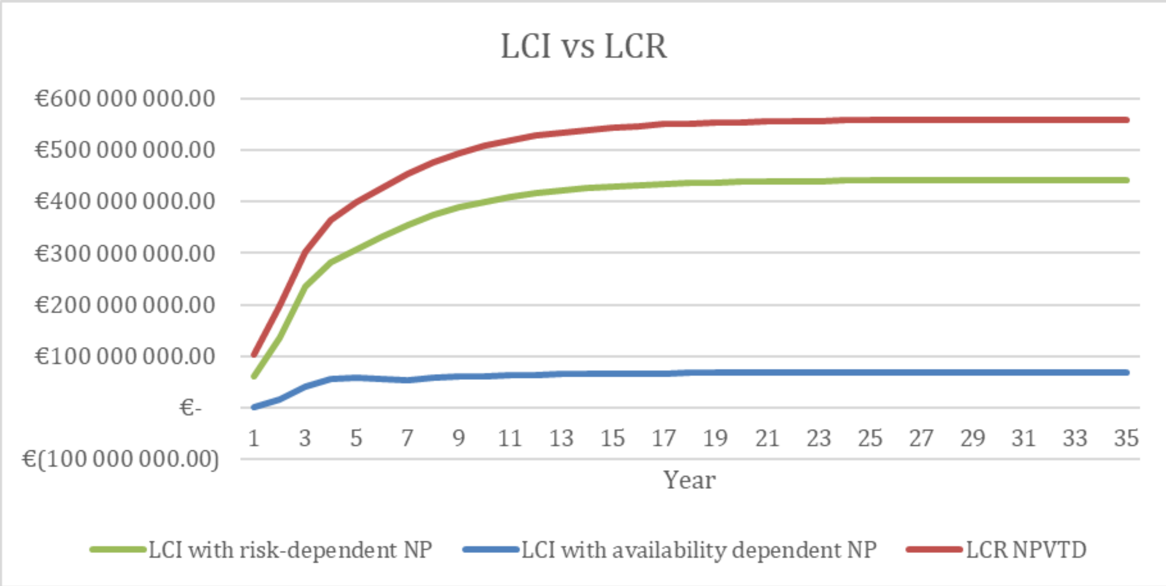

Figure 42 - *LCI* vs *LCR* with a 10% coefficient of increase in maintenance costs applied to *Celbi*'s equipment

In the last scenario created in this demonstration, the coefficient for increasing maintenance costs was maintained and the value of chip production in the pulp manufacturing process was increased to 40%. The *LCI* with risk-dependent *NP* method behaved in a similar way, but the *LCI* with availability dependent *NP* method showed significant changes. It now considers the optimal interval for asset replacement to be the 31st operating period, as can be seen in Figure 43.

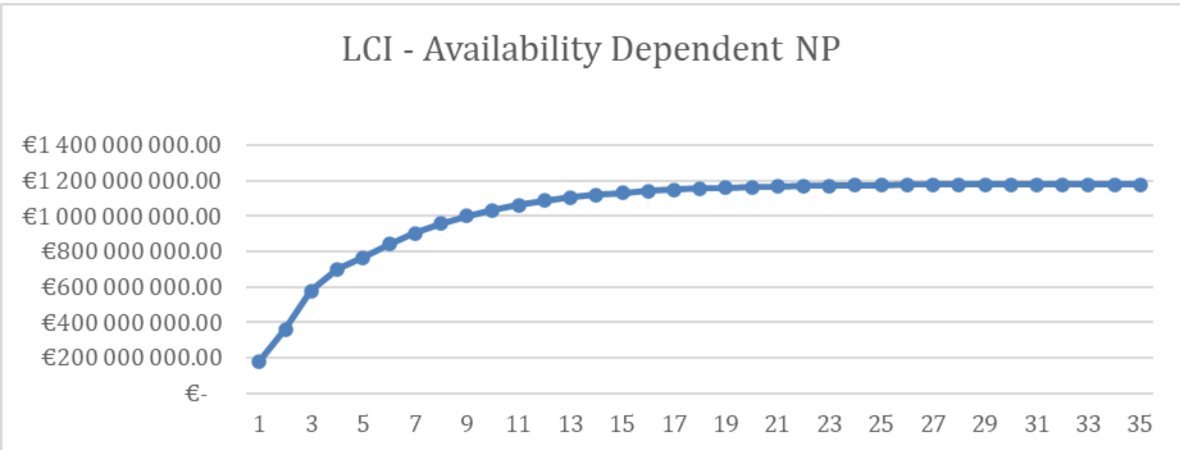

Figure 43 - *LCI* Availability dependent *NP* with a 10% coefficient of increase in maintenance costs and 40% of value of chip production in the pulp manufacturing process

Meanwhile, the *LCR* method indicates replacement in the last period of use, while the purchase of new equipment is recommended in the 9th period of operation, as can be seen in Figure 44.

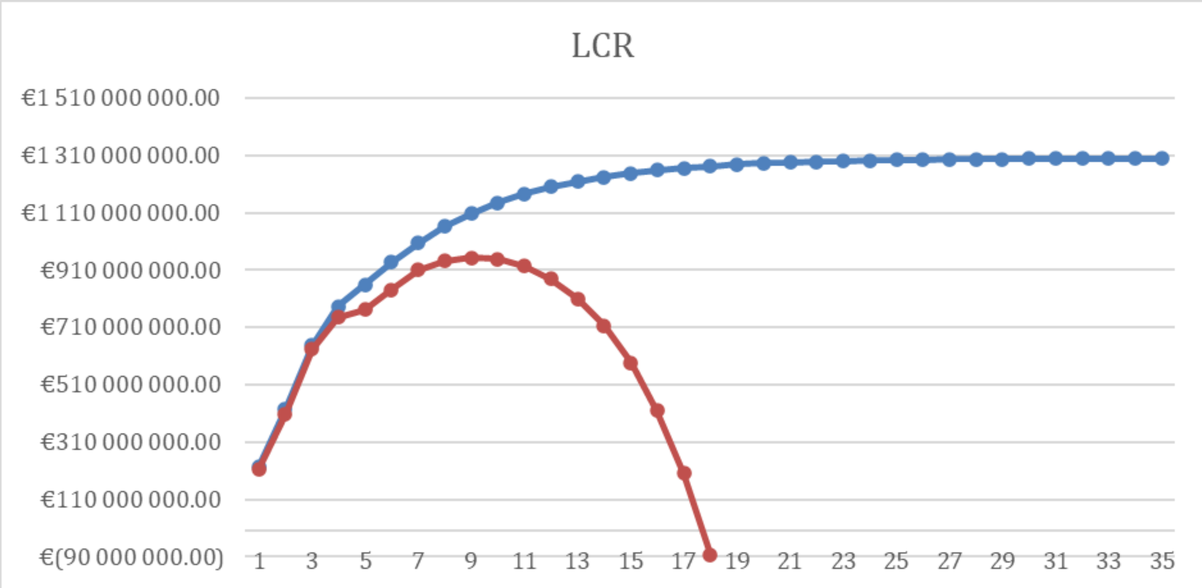

Figure 44 - *LCR* with a 10% coefficient of increase in maintenance costs and 40% of value of chip production in the pulp manufacturing process

3.2.2.6 Conclusions

The lifespan method is not very suitable for evaluating the life cycle of this type of asset, as it "ignores" very important points when it comes to production values. This type of asset is particularly important because of the direct financial return it brings.

The *MTAC-RPV* and *UAI* models, on the other hand, stand out in comparison to *MTAC*, since the latter does not take into account the updating of the value of money, which turns out to be a fundamental point in this type of evaluation. These methods focus on devaluing the equipment and minimising the associated costs. They are a good valuation tool for assets where the equipment's production is more or less regular over the periods considered. As it is essential to consider the depreciation of the asset, a great deal of attention is required when choosing the method. This choice turns out to be a challenge due to the number of factors that can influence depreciation and which the maths of the methods used may not cover. Both the exponential method and the sum of the digits method will be the closest to reality.

The *LCI* and *LCR* methods prove tremendously useful in evaluating this type of asset, taking into account all the factors involved. Accounting for the accumulated values of all costs and production gives a panoramic view of what is really happening with the asset. With the addition of taking into account the acquisition of equivalent new equipment, in the case of the *LCR* method. This is a highly important factor, since not only should the time at which the asset is replaced be taken into account due to its profitability characteristics, but also the effort the company will have to make to replace it. Sometimes the ideal time to replace equipment may not correspond to the ideal time to buy new equivalent equipment, and all of this must be analysed.

4 CONCLUSIONS AND FURTHER RESEARCH

The concern to identify all the factors that are related to and directly affect the operation of the asset, as well as its profitability, should be a priority. Both in the case of playgrounds and in the case study of the industry with a measurable financial return, the importance of collecting and processing data is emphasised.

The greater the knowledge of the asset and the way it operates, the more profit can be made from it and the greater the maximisation of the investment. This study should not only be carried out when making the decision to acquire the asset, but throughout its life. All projections made must be constantly updated and all initial plans must be constantly adjusted in line with changing forecasts. To do this, it is necessary to account for and identify all the parameters, from operating costs, maintenance costs, production values, non-production values, risk, availability and so on, depending on the type of asset.

The path of engineering and asset management has been paved with the philosophy of continuous improvement and optimisation of resources. It can be seen, especially in the second case study, that changing small parameters can result in drastic changes in the values obtained by the methods. Just changing the period indicated for replacing or acquiring equivalent new equipment translates into a difference of hundreds of thousands or even millions of euros in the result of accumulated costs. Often this difference in the company's financial performance is not even emphasised because these figures are underestimated. Hence the importance of these methodologies, which allow not only calculation but also visualisation of the parameters.

If an organisation wants to remain competitive and at the forefront of its business, it needs to look for new strategies and approaches to the market and, in this case, to business management and planning.

This study reveals the long road that has been travelled so far, but also how much more can be done to increase the certainty of the projections made. In this master's thesis, various cost prediction methods were superficially covered, as well as various exponential smoothing methods. However, with the introduction of artificial intelligence into everyday life in today's society, it is becoming increasingly possible to predict future values using it.

Suggestions for future research include the following:

- Deepen the models in order to correlate the data obtained in the *LCI* method with that of the *LCR* method, seeking to introduce new parameters for calculating the optimum period for acquiring equivalent new equipment;
- Aim to introduce artificial intelligence into the calculation of depreciation methods, making it depend not only on static mathematical models, but

on what is happening in the world at present, including new technologies and the weight these have on the market price of older ones;

- Aiming to introduce artificial intelligence into the projection of costs and production values of assets, taking into account the stock market price of raw materials, companies and the end product;
- Look for new methods and strategies for monitoring equipment, enabling all the factors considered in the models applied to be analysed in real time.

REFERENCES

[1] J. M. T. Farinha, "Asset Maintenance Engineering Methodologies," 1st ed. CRC Press, May 2018, ISBN-10: 1138035890, ISBN-13: 978-1138035898.

[2] J. E. Amadi-Echendu, R. J. Willet, K. A. Brown, T. Hope, J. Lee, J. Matthew, N. Vyas, and B. Yang, "What Is Engineering Asset Management?," Engineering asset management review, 2010, pp. 3–16, doi: 10.1007/978-1-84996-178-3_1.

[3] K. El-Akruti, R. Dwight, and T. Zhang, "The strategic role of Engineering Asset Management," Int J Prod Econ, vol. 146, no. 1, pp. 227–239, Nov. 2013, doi: 10.1016/j.ijpe.2013.07.002.

[4] S. Burnett and P. J. Vlok, "A simplified numerical decision-making methodology for physical asset management decisions," South African Journal of Industrial Engineering, vol. 25, no. 1, pp. 162–175, 2014, doi: 10.7166/25-1-689.

[5] M. S. Somia Alfatih, M. S. Leong, and L. M. Hee, "Definition of Engineering Asset Management: A Review," Applied Mechanics and Materials, vol. 773–774, pp. 794–798, Jul. 2015, doi: 10.4028/www.scientific.net/amm.773-774.794.

[6] L. Katičić, D. Lisjak, and Ž. Dulčić, "Asset governance as strategy for physical asset," in International Conference of the School of Economics and Business (ICES 2014.), University of Sarajevo, School of Economics and Business, 2014, pp. 208–221, ISBN: 9958250934.

[7] V. Frolov, L. Ma, Y. Sun, and W. Bandara, "Identifying core functions of asset management," Engineering Asset Management Review, vol. 1, pp. 19–30, 2010, doi: 10.1007/978-1-84996-178-3_2.

[8] J. S. Mitchell, "Physical Asset Management Handbook," 3rd ed. Clarion Technical Publishers, Jan. 2002, ISBN-10: 0971794510, ISBN-13: 978-0971794511.

[9] M. Ben-Daya, S. O. Duffuaa, A. Raouf, J. Knezevic, and D. Ait-Kadi, "Handbook of Maintenance Management and Engineering," 2009th ed. Springer London, Aug. 2009, ISBN-10: 1848824718, ISBN-13: 978-1848824713.

[10] J. T. Farinha, H. N. Raposo, and D. Galar, "Life cycle cost versus life cycle investment – a new approach," WSEAS Transactions on Systems and Control, vol. 15, pp. 743–753, 2020, doi: 10.37394/23203.2020.15.75.

[11] S. Toniolo, A. Mazzi, G. Mazzarotto, and A. Scipioni, "International standards with a life cycle perspective: which dimension of sustainability is addressed?," Int J Life Cycle Assess, vol. 24, pp. 1765–1777, 2019, doi: 10.1007/s11367-019-01606-w.

[12] B. H. Goh and Y. Sun, "The development of life-cycle costing for buildings," Building Research & Information, vol. 44, no. 3, pp. 319–333, Jan. 2015, doi: 10.1080/09613218.2014.993566.

[13] Y. Asiedu and P. Gu, "Product life cycle cost analysis: State of the art review," Int J Prod Res, vol. 36, no. 4, pp. 883–908, 1998, doi: 10.1080/002075498193444.

[14] B. Kianian, M. Kurdve, and C. Andersson, "Comparing life cycle costing and performance part costing in assessing acquisition and operational cost of new manufacturing technologies," Procedia CIRP, vol. 80, pp. 428–433, Jan. 2019, doi: 10.1016/j.procir.2019.01.025.

[15] B. Eicher, "Selection of asset investment models by hospitals: examination of influencing factors, using Switzerland as an example," I The International Journal of Health Planning and Management, vol. 31, no. 4, pp. 554–579, May 2016, doi: 10.1002/hpm.2341.

[16] J. M. T. Farinha, H. N. Raposo, E. A. Pais, and M. Mendes, "Physical Assets Life Cycle Evaluation Models – A Comparative Analysis Aiming the Sustainability," Sep. 2023, doi: 10.20944/preprints202309.1321.v1.

[17] W. Klöpffer, "Life cycle sustainability assessment of products," The International Journal of Life Cycle Assessment, vol. 13, no 2, pp. 89–95, Feb. 2008, doi: 10.1065/lca2008.02.376.

[18] E. Pais, J. T. Farinha, A. J. M. Cardoso, and H. Raposo, "Optimizing the life cycle of physical assets – A review," WSEAS Transactions on Systems and Control, vol. 15. World Scientific and Engineering Academy and Society, pp. 417–430, 2020. doi: 10.37394/23203.2020.15.42.

[19] S. Nakajima, "Introduction to TPM: Total Productive Maintenance," ed. Eleventh Printing edition, Productivity Pr, Jan. 1988, ISBN-10: 0915299232, ISBN-13: 978-0915299232.

[20] T. M. Husband, "Maintenance Management and Terotechnology," Saxon House, Jan. 1976, ISBN-10: 0566001462, ISBN-13: 978-0566001468.

[21] Y. Takahashi, "Total Productive Maintenance, a new task for plant managers in Japan," Terotechnica, vol. 2, pp. 79–88, 1981.

[22] J. M. T. Farinha, "Uma abordagem terológica da manutenção dos equipamentos hospitalares," Faculdade de Engenharia da Universidade do Porto, Porto, Portugal, 1994. doi: 10.1111/j.1600-0404.1976.tb04330.x.

[23] R. C. M. Yam, P. W. Tse, L. Li, and P. Tu, "Intelligent predictive decision support system for condition-based maintenance," The International Journal of Advanced Manufacturing Technology/International Journal, Advanced Manufacturing Technology, vol. 17, no. 5, pp. 383–391, Feb. 2001, doi: 10.1007/s001700170173.

[24] K. C. Sarma and H. Adeli, "Life-cycle cost optimization of steel structures," International Journal for Numerical Methods in Engineering, vol. 55, no. 12, pp. 1451–1462, Sep. 2002, doi: 10.1002/nme.549.

[25] D. M. Frangopol and M. Liu, "Maintenance and management of civil infrastructure based on condition, safety, optimization, and life-cycle cost," in Routledge eBooks, 2019, pp. 96–108. doi: 10.1201/9781351182805-6.

[26] A. Lindholm and P. Suomala, “Present and future of life cycle costing: reflections from Finnish companies,” LTA, vol. 2, no. 05, pp. 282–292, 2005.

[27] H. Estevan, B. Schaefer, and A. Adell, “Life Cycle Costing State of the art report,” Local Governments for Sustainability, European Secretariat, 2017.

[28] H. Raposo, J. T. Farinha, L. Ferreira, and D. Galar, “An integrated econometric model for bus replacement and determination of reserve fleet size based on predictive maintenance,” Eksploatacja i Niezawodnosc, vol. 19, no. 3, pp. 358–368, 2017, doi: 10.17531/ein.2017.3.6.

[29] H. M. Malano, B. A. George, and B. Davidson, “Asset management modelling framework for irrigation and drainage systems: Principles and case study application,” Irrigation and Drainage Systems, vol. 19, no. 2, pp. 107–127, May 2005, doi: 10.1007/s10795-005-2972-7.

[30] G. Schuh, P. Jussen, and F. Optehostert, “Iterative cost assessment of maintenance services,” Procedia CIRP, vol. 80, pp. 488–493, Jan. 2019, doi: 10.1016/j.procir.2019.01.067.

[31] M. Bengtsson and M. Kurdve, “Machining equipment life cycle costing model with dynamic maintenance cost,” Procedia CIRP, vol. 48, pp. 102–107, Jan. 2016, doi: 10.1016/j.procir.2016.03.110.

[32] N. Hastings, “Physical asset management,”, ed. 2010th, Springer, Nov. 2014, ISBN-10: 1447157478, ISBN-13: 978-1447157472.

[33] J. E. Amadi-Echendu, "Managing physical assets is a paradigm shift from maintenance," 2004 IEEE International Engineering Management Conference (IEEE Cat. No.04CH37574), Singapore, 2004, pp. 1156-1160 Vol.3, doi: 10.1109/IEMC.2004.1408874.

[34] J. Nilsson and L. Bertling, “Maintenance management of wind power systems using condition monitoring systems—life cycle cost analysis for two case studies,” IEEE Transactions on Energy Conversion, vol. 22, no. 1, pp. 223–229, Mar. 2007, doi: 10.1109/tec.2006.889623.

[35] C. A. Schuman and A. C. Brent, “Asset life cycle management: Towards improving physical asset performance in the process industry,” International Journal of Operations and Production Management, vol. 25, no. 6, pp. 566–579, 2005, doi: 10.1108/01443570510599728.

[36] J. Moubray, “Reliability-centered maintenance,” ed. 2nd, Industrial Press Inc., Jan. 2001, ISBN-10: 0831131462, ISBN-13: 978-0831131463.

[37] B. K. N. Rao, “Handbook of condition monitoring,” ed. 1st, Elsevier, Dec. 1996, ISBN-10: 1856172341, ISBN-13: 978-1856172349.

[38] A. Davies, “Handbook of condition monitoring: techniques and methodology,” ed. 1998th, Springer, Dec. 1997, ISBN-10: 0412613204, ISBN-13: 978-3764384098.

[39] I. Fonseca, T. Farinha, and M. Barbosa, “A computer system for predictive maintenance of wind generators,” Annual Conference on Computers, pp. 928–

933, Jul. 2008, [Online]. Available: http://www.wseas.us/e-library/conferences/2008/crete/Computers/147-computers.pdf.

[40] J. M. T. Farinha and B. C. Vasconcelos, "SMITH-Sistema modular integrado de terologia hospitalar," in Actas do 4º Congresso Nacional de Manutenção Industrial, Porto, 1994.

[41] T. Farinha, I. Fonseca, V. Marques, A. Brito, A. Marimba, N. Pincho, A. Simões, "A global view of maintenance management- from maintenance diagnosis to know-how retention and sharing.," WSEAS Transactions on Systems, vol. 3, no. 4, pp. 1703–1711, Jun. 2004, ISSN: 1109-2777.

[42] I. Fonseca, J. T. Farinha, and F. M. Barbosa, "Maintenance planning in wind farms with allocation of teams using genetic algorithms," Revista IEEE América Latina, vol. 12, no. 6, pp. 1062–1070, Sep. 2014, doi: 10.1109/tla.2014.6894001.

[43] R. R. da Motta and G. M. Calôba, "Análise de investimentos: tomada de decisão em projetos industriais," Atlas, Jan. 2002, ISBN-10: 8522430799, ISBN-13: 978-8522430796.

[44] J. L. Rogers and J. C. Hartman, "Equipment replacement under continuous and discontinuous technological change," IMA Journal of Management Mathematics, vol. 16, no. 1, pp. 23–36, Jan. 2005, doi: 10.1093/imaman/dph027.

[45] M. Spickova and R. Myskova, "Costs efficiency evaluation using life cycle costing as strategic method," Procedia Economics and Finance, vol. 34, pp. 337–343, Jan. 2015, doi: 10.1016/s2212-5671(15)01638-x.

[46] D. G. Woodward, "Life cycle costing—Theory, information acquisition and application," International Journal of Project Management, vol. 15, no. 6, pp. 335–344, Dec. 1997, doi: 10.1016/s0263-7863(96)00089-0.

[47] H. Raposo, A. Meireles, E. Pais, and J. T. Farinha, "Análise de Investimento versus Análise do Ciclo de Vida no Contexto das ISO 55000 com ênfase na Gestão de Manutenção. 14," in º Congresso Nacional de Manutenção. Maia, Portugal, 2017.

[48] J. S. Cabral, "Organização e gestão da manutenção: dos conceitos à prática," ed. 6th, Lidel, Jan. 2006, ISBN-10: 978-9727574407, ISBN-13: 978-9727574407.

[49] M. Aoudia, O. Belmokhtar, and G. Zwingelstein, "Economic impact of maintenance management ineffectiveness of an oil and gas company," J Qual Maint Eng, vol. 14, no. 3, pp. 237–261, 2008, doi: 10.1108/13552510810899454.

[50] R. Assis and J. Julião, "Maintenance Management or Asset Management? (Costs over the lifecycle)," in Communication 10th National Congress Maintenance, Figueira da Foz, Portugal: APMI, 2009.

[51] R. Assis, "Apoio à decisão em manutenção na gestão de activos físicos," Lisboa: Lidel, 2014.

[52] F. Bescherer, "Established Life Cysle Concepts in the Business Environment-Introduction and Terminology," Laboratory of Industrial Management Report Series, Helsinki University, Helsinki, Finland, Rep. no. 1/2005, 2005, Accessed:

Dec. 19, 2023. [Online]. Available: https://research.aalto.fi/en/publications/established-life-cysle-concepts-in-the-business-environment-intro.

[53] E. Korpi and T. Ala-Risku, "Life cycle costing: A review of published case studies," Managerial Auditing Journal, vol. 23, no. 3. pp. 240–261, 2008. doi: 10.1108/02686900810857703.

[54] A. Lindholm and P. Suomala, "The Possibilities of Life Cycle Costing in Outsourcing Decision Making,", 2004. [Online]. Available: https://www.researchgate.net/publication/241720050.

[55] J. M. T. Farinha, "Manutenção. A terologia e as novas ferramentas de gestão," ed 1st, Monitor, ISBN-10: 9729413827, ISBN-13: 978-9729413827.

[56] W. G. Sullivan, T. N. McDonald, and E. M. Van Aken, "Equipment replacement decisions and lean manufacturing," Robotics and Computer-integrated Manufacturing, vol. 18, no. 3–4, pp. 255–265, Jun. 2002, doi: 10.1016/s0736-5845(02)00016-9.

[57] N. Hritonenko and Y. Yatsenko, "Optimal equipment replacement without paradoxes: A continuous analysis," Operations Research Letters, vol. 35, no. 2, pp. 245–250, Mar. 2007, doi: 10.1016/j.orl.2006.03.001.

[58] A. Assaf Neto, "Corporate finance and value," Atlas, São Paulo, 2005, ISBN: 9788522460144.

[59] F. N. Casarotto, "Investment analysis-financial mathematics, economic engineering, decision making, business strategy," São Paulo, Atlas, 2000, ISBN: 85-224-2572-8.

[60] I. H. Vey and R. M. Rosa, "Fleet replacement in municipal passenger transportation company: a case study," Federal University of Santa Maria, Electronic Accounting Journal, vol. 1, no. 1, pp. 150–173, 2004.

[61] J. A. N. de Oliveira, Engenharia economica: uma abordagem as decisões de investimento. McGraw-Hill, 1982. [Online]. Available: https://books.google.pt/books?id=vL_9kQEACAAJ.

[62] M. R. Seyedabadi and U. Eicker, "A critical review of urban scale life cycle assessment of the built environment," Sustainability Analytics and Modeling, vol. 3, p. 100026, Jan. 2023, doi: 10.1016/j.samod.2023.100026.

[63] A. Hussien, A. Saleem, E. Mushtah, N. Jannat, A. Al-Shammaa, S. Ali, S. Assi, and D. Al-Jumeily, "A statistical analysis of life cycle assessment for buildings and buildings' refurbishment research," Ain Shams Engineering Journal/Ain Shams Engineering Journal, vol. 14, no. 10, p. 102143, Oct. 2023, doi: 10.1016/j.asej.2023.102143.

[64] R. Wittmanová, M. Šutúš, J. Hrudka, and I. Škultétyová, "Life cycle assessment of urban stormwater management approaches: A review," in AIP Conference Proceedings, AIP Publishing, 2023.

[65] S. Hellweg, E. Benetto, M. A. J. Huijbregts, F. Verones, and R. Wood, "Life-cycle assessment to guide solutions for the triple planetary crisisNature Reviews. Earth & Environment, vol. 4, no. 7, pp. 471–486, Jul. 2023, doi: 10.1038/s43017-023-00449-2.

[66] J. T. Farinha, H. D. N. Raposo, J. E. de-Almeida-e-Pais, and M. Mendes, "Physical Asset Life Cycle Evaluation Models—A Comparative Analysis towards Sustainability," Sustainability, vol. 15, no. 22, 2023, doi: 10.3390/su152215754.

APPENDIX

Two tables have been drawn up with indicators that analyse the references in this thesis. This evaluation is made using quantitative parameters that may reflect the credibility and quality of the references, if properly interpreted. To obtain this data the *Scimagojr*[5] platform was used to evaluate the journals in which the articles were published. As for the authors, the website *Scopus*[6] was always used.

The table "Average indicators of references" shows the average values of the data obtained and the table "Information on references" shows the data on which it is based.

Notes:

- To be sure that the authors were indeed those listed on the platform, the information was cross-referenced with other databases such as *Google Scholar* and *ResearchGate*. When it wasn't clear that it was the same author, it was left blank.
- Some of the evaluations may not correspond to the truth, bearing in mind that many authors have several of their articles associated with different signatures and, consequently, accounts on the *Scopus* platform. This often results in a drop in the number of publications and the h-index. In these cases, the account associated with the highest number of each of these parameters was always considered.
- Many of the fields referring to the evaluation of journals on the *Scimago* platform were left blank because the references are books or conference papers. Therefore, they are not included in the type of document submitted for assessment on this site.
- To calculate the averages, only parameters for which it was possible to obtain values were taken into account.

Table 59 - Average indicators of references

Average Journal *H-Index*	88.06
Average *Quartile* by Journal	2.00
Authors' *H-Index* Average	16.43056
Average number of documents per author	82.29

[5] https://www.scimagojr.com platform, accessed 13/05/2024
[6] https://www.scopus.com, accessed 13/05/2024

Table 60 - Information on references

Reference	Journal (Scimago)	Journal H-index	Quartiles	Authors	Authors Index (Scopus)	Authors Documents
[1]	-	-	-	Torres Farinha J.	14	57
[2]	Engineering Asset Management Review	7	Q4 (2016)	Joe E. Amadi-Echendu	12	122
				Roger J. Willett	16	1
				Kerry A. Brown	23	101
				Tony Hope	4	10
				Jay Lee	60	361
				Joseph Mathew	1	2
				Nalinaksh Vyas	16	72
				Bo-Suk Yang	-	-
[3]	International Journal of Production Economics	231	Q1 (2013)	El-Akruti K.	7	11
				Dwight R.	13	38
				Zhang T.	19	49
[4]	South African Journal of Industrial Engineering	23	Q3 (2013)	Burnett S.	-	-
				Vlok P.	8	38
[5]	Applied Mechanics and Materials	41	Q4 (2015)	Somia Alfatih M.	17	78
				Leong M.	20	97
				Hee L.	2	3

[6]	International Conference of the School of Economics and Business	-	-	Katičić L.	2	2
				Lisjak D.	8	40
				Dulčić Ž.	4	13
[7]	Engineering Asset Management Review	7	Q4 (2011)	Frolov V.	2	2
				Ma L.	30	132
				Sun Y.	17	69
				Bandara W.	20	106
[8]	-	-	-	Mitchell J.	-	-
				Bond T. H.	-	-
				Nodianos N.	-	-
				Brotherton T.	-	-
				Fitch J. C.	-	-
				Gaberson H.	-	-
				Motylenski R. J.	-	-
				Murnane T.	-	-
				Nicholas J. R.	-	-
[9]	-	-	-	Ben-Daya M.	38	96
				Duffuaa S.	30	124
				Raouf A.	16	96
				Knezevic J.	14	52
				Ait-Kadi D.	31	181
[10]	WSEAS Transactions on Systems and Control	20	Q4 (2020)	Torres Farinha J.	14	57
				Raposo H.	6	15
				Galar D.	24	156

[11]	International Journal of Life Cycle Assessment	131	Q1 (2019)	Toniolo S.	16	40
				Mazzi A.	26	47
				Mazzarotto G.	1	2
				Scipioni A.	30	67
[12]	Building Research & Information	105	Q1 (2016)	Goh B.	4	9
				Sun Y.	2	3
[13]	International Journal of Production Research	186	Q1 (1999)	Asiedu Y.	7	15
				Gu P.	42	237
[14]	Procedia Cirp	103	-	Kianian B.	7	16
				Kurdve M.	15	41
				Andersson C.	12	27
[15]	International journal of health planning and management	49	Q2 (2016)	Eicher B.	4	15
[16]	-	-	-	Torres Farinha J.	14	57
				Raposo H.	6	15
				Pais E.	4	5
				Mendes M.	12	56

[17]	International Journal of Life Cycle Assessment	131	Q1 (2008)	Klöpffer W.	31	134
[18]	WSEAS Transactions on Systems and Control	20	Q4 (2020)	Pais E.	4	5
				Torres Farinha J.	14	57
				Cardoso A.	51	429
				Raposo H.	6	15
[19]	Productivity Press	-	-	Nakajima S.	2	2
[20]	-	-	-	Husband T.	9	33
[21]	-	-	-	Takahashi Y.	-	-
[22]	-	-	-	Torres Farinha J.	14	57
[23]	International Journal of Advanced Manufacturing Technology	161	Q1 (2001)	Yam R.	38	86
				Tse P.	43	201
				Li L.	109	1100
				Tu P.	30	187
[24]	International Journal for Numerical Methods in Engineering	194	Q1 (2002)	Sarma K.	10	11
				Adeli H.	117	545
[25]	-	-	-	Frangopol D.	82	678
				Liu M.	-	-

[26]	-	-	-	Lindholm A.	4	4
				Suomala P.	15	37
[27]	-	-	-	Estevan H.	-	-
				Schaefer B.	-	-
				Adell A.	-	-
[28]	Eksploatacja i Niezawodnosc	34	Q2 (2017)	Raposo H.	6	15
				Torres Farinha J.	14	57
				Ferreira L.	11	51
				Galar D.	24	156
[29]	Irrigation and Drainage Systems	38	Q2 (2005)	Malano H.	26	123
				George B.	23	77
				Davidson B.	14	51
[30]	Procedia CIRP	103	-	Schuh G.	28	769
				Jussen P.	7	24
				Optehostert F.	3	5
[31]	Procedia CIRP	103	-	Bengtsson M.	9	30
				Kurdve M.	15	41
[32]	-	-	-	Hastings N.	9	21
[33]	IEEE International Engineering Management Conference	21	-	Joe E. Amadi-Echendu	12	122
[34]	IEEE Transactions	200	Q1 (2007)	Nilsson J.	4	5

	on energy conversion			Bertling L.	29	129
[35]	International Journal of Operations and Production Management	163	Q1 (2005)	Schuman C.	-	-
				Brent A.	35	230
[36]	-	-	-	Moubray J.	-	-
[37]	-	-	-	Rao B.	10	32
[38]	-	-	-	Davies A.	-	-
[39]	-	-	-	Fonseca I.	8	32
				Torres Farinha J.	14	57
				Barbosa M.	11	55
[40]	-	-	-	Torres Farinha J.	14	57
				Vasconcelos B.	-	-
[41]	WSEAS Transactions on Systems	28	Q3 (2006)	Torres Farinha J.	14	57
				Fonseca I.	8	32
				Marques V.	3	15
				Brito A.	5	34
				Marimba A.	-	-
				Pincho N.	1	1
				Simões A.	2	4
[42]	IEEE Latin America Transactions	36	Q2 (2014)	Fonseca I.	8	32
				Torres Farinha J.	14	57
				Barbosa F.	11	55
[43]	-	-	-	Motta R.	2	5

				Calôba G.	3	11
[44]	IMA Journal of Management Mathematics	40	Q4 (2005)	Rogers J.	2	4
				Hartman J.	20	91
[45]	-	-	-	Spickova M.	0	1
				Myskova R.	9	30
[46]	-	-	-	Woodward D.	11	21
[47]	-	-	-	Raposo H.	6	15
				Meireles A.	-	-
				Pais E.	4	5
				Torres Farinha J.	14	57
[48]	-	-	-	Cabral J.	-	-
[49]	Journal of Quality in Maintenance Engineering	62	Q1 (2008)	Aoudia M.	3	7
				Belmokhtar O.	3	9
				Zwingelstein G.	4	23
[50]	-	-	-	Assis R.	3	8
				Jorge J.	7	16
[51]	-	-	-	Assis R.	3	8
[52]	-	-	-	Bescherer F.	-	-
[53]	Managerial Auditing Journal	71	Q2 (2008)	Korpi E.	1	1
				Ala-Risku T.	16	21
[54]	-	-	-	Lindholm A.	4	4
				Suomala P.	15	37
[55]	-	-	-	Torres Farinha J.	14	57

[56]	Robotics and computer-integrated manufacturing	121	Q1 (2002)	Sullivan W.	10	63
				McDonald T.	4	13
				Van Aken E.	23	129
[57]	Operations Research Letters	82	Q1 (2007)	Hritonenko N.	20	103
				Yatsenko Y.	20	112
[58]	-	-	-	Assaf Neto A.	1	2
[59]	-	-	-	Casarotto F.	-	-
[60]	-	-	-	Vey I.	-	-
				Rosa R.	-	-
[61]	-	-	-	Oliveira J.	-	-
[62]	-	-	-	Seyedabadi M.	6	10
				Eicker U.	41	234
[63]	Ain Shams Engineering Journal	76	Q1 (2023)	Hussien A.	13	34
				Saleem A.	9	18
				Mushtaha E.	12	72
				Jannat N.	7	10
				Al-Shammaa A.	31	265
				Ali S.	3	4
				Assi S.	16	74
				Al-Jumeily D.	30	289
[64]	AIP Conference Proceedings	83	-	Wittmanová R.	2	16
				Šutúš M.	2	10
				Hrudka J.	4	41
				Škultétyová I.	5	51
[65]		67		Hellweg S.	64	210

	Nature Reviews Earth & Environment		Q1 (2023)	Benetto E.	41	120
				Huijbregts M.	72	347
				Verones F.	34	89
				Wood R.	55	130
[66]	Sustainability	169	Q2 (2023)	Torres Farinha J.	14	57
				Raposo H.	6	15
				Pais E.	4	5
				Mendes M.	12	56

Instituto Superior
de Engenharia
Politécnico de Coimbra