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OSCILLATION CRITERIA FOR ODD-ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH ADVANCED AND DELAYED ARGUMENTS

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Abstract. This article presents oscillation criteria for n-th order nonlinear neutral mixed type differential equations of the form

$$\big((x(t)+ax(t-\tau_1)-bx(t+\tau_2))^{\alpha}\big)^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2),$$

$$\big((x(t)-ax(t-\tau_1)+bx(t+\tau_2))^{\alpha}\big)^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2),$$

$$\big((x(t)+ax(t-\tau_1)+bx(t+\tau_2))^{\alpha}\big)^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2)$$

where n is an odd positive integer, a and b are nonnegative constants, τ_1,τ_2,σ_1 and σ_2 are positive real constants, $q(t),p(t)\in C([t_0,\infty),(0,\infty))$ and α,β and γ are ratios of odd positive integers with $\beta,\gamma\geq 1$. Some examples are provided to illustrate the main results.

1. Introduction

In this article, we study the oscillatory behavior of all solutions of n-th order nonlinear neutral differential equations of the forms

$$((x(t)+ax(t-\tau_1)-bx(t+\tau_2))^{\alpha})^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2), \tag{1.1}$$

$$((x(t)-ax(t-\tau_1)+bx(t+\tau_2))^{\alpha})^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2), \tag{1.2}$$

$$((x(t)+ax(t-\tau_1)+bx(t+\tau_2))^{\alpha})^{(n)}=q(t)x^{\beta}(t-\sigma_1)+p(t)x^{\gamma}(t+\sigma_2) \tag{1.3}$$

where n is an odd positive integer, a and b are nonnegative constants, τ_1,τ_2,σ_1 and σ_2 are positive real constants, $q(t),p(t)\in C([t_0,\infty),(0,\infty))$ and α,β and γ are ratios of odd positive integers with $\beta,\gamma\geq 1$.

As is customary, a solution is called oscillatory if it has arbitrarily large zeros and non-oscillatory if it is eventually positive or eventually negative. Equations (1.1), (1.2) and (1.3) are called oscillatory if all its solutions are oscillatory.

Differential equations with advanced and delayed arguments (also called mixed differential equations or equations with mixed arguments) occur in many problems of economy, biology and physics (see for example [3, 7, 11, 12, 19]), because differential equations with mixed arguments are much more suitable than delay differential equations for an adequate treatment of dynamic phenomena. The concept of delay

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