Blind Source Separation Using Time-Delayed Signals

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Abstract—**In this work a modified version of AMUSE, called dAMUSE, is proposed. The main modification consists in increasing the dimension of the data vectors by joining delayed versions of the observed mixed signals. With the new data a matrix pencil is computed and its generalized eigendecomposition is performed as in AMUSE. We will show that in this case the output (or independent) signals are filtered versions of the source signals. Some numerical simulations using artificially mixed signals as well as biological data (RR and QT intervals of Electrocardiogram) are presented.**

I. Introduction

Blind Source Separation (BSS) methods consider the separation of observed sensor signals into their underlying independent source signals knowing neither the source signals nor the mixing process. Independent component analysis (ICA) techniques using either higher order statistics of random variables or the (time-)correlation structure in the data can solve the BSS problem. Methods based on a generalized eigenvalue decomposition (GEVD) of a matrix pencil, estimated using second order statistics, are sensitive to noise [1], [2]. There are several proposals to improve efficiency and robustness of these algorithms when noise is present [1], [3]. They are mainly based on joint approximative diagonalization procedures of a set of correlation or cumulant matrices. Further there are denoising techniques whose first step consists in increasing the dimension of the data set by joining time-delayed versions of the signals [4], [5]. Then the sensor signals are projected into a high-dimensional feature space. The deterministic sensor signals, however, reside in a lower dimensional subspace only. Hence de-noising is achieved by back-projection of the data into this lower dimensional subspace corresponding to the dimension of the underlying source signals.

In this work we present a modified version of AMUSE [6], called dAMUSE, which joins both strategies and alleviates the sensitivity of GEVD methods against noise. Hence the new algorithm solves the BSS problem and simultaneously reduces the noise present in the signals. AMUSE [7] is an algorithm using second-order statistics which is based on the simultaneous diagonalization of a matrix pencil formed with a correlation matrix and a matrix of time-delayed correlations. The proposed algorithm dAMUSE also comprises this simultaneous diagonalization but the pencil is computed in a high-dimensional feature space, i.e. after increasing the data set dimension by joining delayed versions of each sensor signal. In the following section we will show, considering a model of linearly mixed sensor signals with additive noise, that the computed independent signals correspond to filtered versions of the underlying source signals. Hence the additional information represented in the time-correlations leads to an indeterminacy in the form of an arbitrary filtering. We will also present an algorithm to compute the eigenvector matrix of the matrix pencil which involves a two step procedure based on the standard eigenvalue decomposition (EVD) approach [8]. The advantage of this procedure is concerned with a dimension reduction between the two steps and a concomitant reduction in the number of independent signals which helps to reduce the computational load with high-dimensional problems. Finally, some simulations with artificially mixed signals are discussed to illustrate the proposed method. The dAMUSE algorithm is also applied to a vector sequence whose components are the RR and QT intervals of the ECG signal. These preliminary results show that the independent signals estimated are related to LF (Low Frequency) and HF (High Frequency) fluctuations of cardiovascular signals [9].

II. Generalized Eigendecomposition Using Time-Delayed Signals

Considering a group of N sensor signals $x^{(n)} = \left[\begin{array}{cccc} x_1, & x_2, & \cdots & x_N \end{array}\right]^T$ where each component, x_i, $i = 1 \ldots N$, includes M delayed versions of the sensor signal

$$x_i = \left[\begin{array}{cccc} x_i[n], & x_i[n-1], & \cdots & x_i[n-(M-1)] \end{array}\right]$$

The data vector $x^{(n)}$ is used to compute a pair of time-delayed correlation matrices

$$R_x(k_i) = E\left\{x^{(n)} x^{(n-k_i)^T}\right\} \tag{1}$$

Note that additive Gaussian white noise components cancel out for non-zero k_i. The generalized eigendecomposition of the pair $(R_x(k_1), R_x(k_2))$, with size $NM \times NM$, in particular its eigenvector matrix E_x, will be used to transform the NM dimensional sensor signals $x^{(n)}, i.e,\ y^{(n)} = E_x^T x^{(n)}$. The signals y turn out to be filtered versions of the source signals s as we will prove next.

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Assuming that each sensor signal is a linear combination of N underlying but unknown source signals, the vector of the source signals $s^{(n)} = \begin{bmatrix} s_1, & s_2, & \dots & s_N \end{bmatrix}^T$ has components s_i, $i = 1 \dots N$, with M delayed versions of each source signal,

$$s_i = \begin{bmatrix} s_i[n], & s_i[n-1], & \cdots & s_i[n-(M-1)] \end{bmatrix}.$$

Writing the vector of sensor signals $x^{(n)} = Hs^{(n)}$, the mixing matrix H, given by the Kronecker product $h \otimes I$, will be a block matrix with a diagonal matrix in each block

$$H = \begin{bmatrix} h_{11}I_{M\times M} & h_{12}I_{M\times M} & \cdots & h_{1N}I_{M\times M} \\ h_{21}I_{M\times M} & h_{22}I_{M\times M} & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1}I_{M\times M} & \cdots & \cdots & h_{NN}I_{M\times M} \end{bmatrix} \quad (2)$$

where I_{MxM} is the identity matrix and h_{ij} is the mixing coefficient that relates the sensor signal i with the source signal j. As we are dealing with an instantaneous mixing model all delayed versions of a signal are related by the same coefficient.

Using the linear mixture model $x^{(n)} = Hs^{(n)}$ where some of the source vector components represent noise contributions, we obtain

$$R_x(k_i) = HE\left\{s^{(n)}s^{(n-k_i)^T}\right\}H^T = HR_s(k_i)H^T \quad (3)$$

Equation (3) relates the time delayed correlation matrices $R_x(k_i)$ of the sensor signals with the identical matrices $R_s(k_i)$ computed using source signals. The two pairs of matrices $(R_x(k_1), R_x(k_2))$ and $(R_s(k_1), R_s(k_2))$ represent congruent pencils [10] hence:

- the eigenvalues are the same,i.e., the eigenvalue matrices of both pencils are identical: $D_x = D_s$
- And consequently having unique eigenvalues (distinct values in the diagonal of the matrix $D_x = D_s$), the eigenvectors are related by $E_s = H^T E_x$

Assuming that all sources are uncorrelated the matrices $R_s(k_i)$ are block diagonal having block matrices $R_{mm}(k_i) = E\left\{s_m^{(n)} s_m^{(n-k_i)}\right\}$ along the diagonal

$$R_s(k_i) = \begin{bmatrix} R_{11}(k_i) & 0 & \cdots & 0 \\ 0 & R_{22}(k_i) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{NN}(k_i) \end{bmatrix}$$

The eigenvector matrix of the generalized eigendecomposition of the pencil $(R_s(k_1), R_s(k_2))$ can be written as

$$E_s = \begin{bmatrix} E_{11} & 0 & \cdots & 0 \\ 0 & E_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{NN} \end{bmatrix} \quad (4)$$

where E_{mm} is the $M \times M$ eigenvector matrix of the generalized eigendecomposition of the pencil $(R_{mm}(k_1), R_{mm}(k_2))$.

The sensor signals are linearly transformed by E_x^T,i.e.

$$y^{(n)} = E_x^T Hs^{(n)} = E_s^T s^{(n)}$$

As the eigenvector matrix E_s (eqn. 4) is a block diagonal matrix, there are M signals in the vector y which are a linear combination of one of the source signals and its delayed versions. For instance, the block m depends on the source signal m

$$\sum_{k=1}^{M} E_{mm}(k,j)s_m(n-k+1) \quad (5)$$

Equation (5) defines the convolution operation between column j of E_{mm} and source signal s_m. Then the columns of the matrix E_{mm} represent impulse responses of finite impulse response (FIR) filters. Considering that all the columns of E_{mm} are different its frequency response might provide different spectral densities of the source signal spectra. Then, in a total of NM output signals y there are M filtered versions of each source/unmixed signal.

III. Algorithm Implementation

There are several ways to compute the eigenvalues and eigenvectors of a matrix pencil if one of the matrices is symmetric positive definite [8]. One of those methods is based on a standard eigendecomposition applied in two consecutive steps. One of the correlation matrices of the pair is computed for delay zero to meet the symmetric positive definite condition. So, considering the pencil $(R_x(0), R_x(k_2))$ the following steps are proposed:

- Compute a standard eigendecomposition of $R_x(0) = \vee\wedge\vee^T$, *i.e*, the eigenvectors (ν_i) and eigenvalues (λ_i). As the matrix is symmetric positive definite the eigenvalues can be organized in descending order $(\lambda_1 > \lambda_2 > ... > \lambda_{NM})$. In AMUSE (and other algorithms) this procedure is used to estimate the number of sources but it can also be considered a strategy to reduce noise. In this work, we consider a variance criterium to choose the most significant eigenvalues according to

$$\frac{\lambda_1 + \lambda_2 + ... + \lambda_l}{\lambda_1 + \lambda_2 + ... \lambda_{NM}} \geq TH \quad (6)$$

  If we are interested in the eigenvectors corresponding to directions of high variance of the signals, the threshold (TH) should be chosen so that the maximum energy of the signals is preserved.
- The transformation matrix can then be computed using either the l most significant eigenvalues or all the eigenvalues and respective eigenvectors. If the following decomposition is chosen ,

$$Q = \wedge^{-\frac{1}{2}}\vee^T \quad (7)$$

  then Q is a $l \times NM$ matrix if a reduction in dimension is considered.
- Compute the matrix $C = QR_x(k_2)Q^T$ and its standard eigendecomposition: the eigenvector matrix (U) and eigenvalue matrix (D_x).

The eigenvectors (which are not normalized) of the pencil $(R_x(0), R_x(k_2))$ form the columns of the eigenvector matrix

$$E_x = Q^T U = \vee \wedge^{-\frac{1}{2}} U \tag{8}$$

Note that if $TH = 1$, the corresponding result is obtained with $eig(R_x(0), R_x(k_2))$ in the recent version of MATLAB.

In what concerns the independent components of the sensor signals and their delayed versions they can be estimated via the transformation given below and l (or NM) signals are obtained,

$$y^{(n)} = E_x^T x^{(n)} = U^T Q x^{(n)} = U^T \wedge^{-\frac{1}{2}} \vee^T x^{(n)} \tag{9}$$

IV. Results

The proposed algorithm was applied to artificially mixed signals and biological signals (RR and QT sequences of Electrocardiograms - ECG). The artificial mixtures allow the illustration of the method once every parameter of the linear mixture model is known.

The Heart Rate Variability (HRV) spectrum is currently separated into three frequency bands: Very Low Frequency band (VLF range below 0.04Hz), Low Frequency band (LF range 0.04-0.15HZ) and High Frequency band (HF range 0.15- 0.4HZ). Hence it is interesting to know if having as input a vector sequence, whose components are RR and QT intervals, the extracted signals can be related with LF and HF fluctuations of cardiovascular signals.

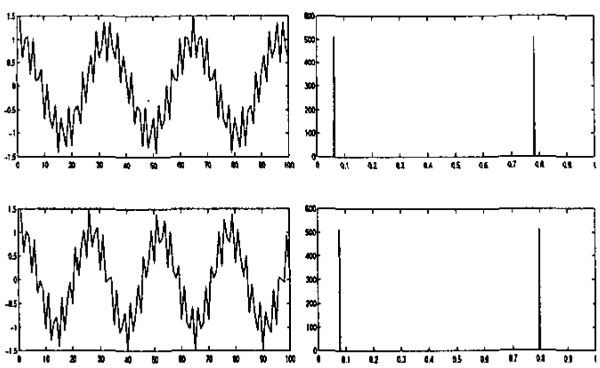

Fig. 1. Artificial Signals comprising a superposition of a high-frequency and a low-frequency sinusoidal signal: (a)-source signals; (b)-frequency contents of the source signals

A. Artificially mixed signals

The simulations were designed to illustrate the method and to study the influence of its parameters (especially M and TH) on the respective performance. As the separation process involves also a linear filtering operation, each computed independent component has its maximum correlation with one of the source signals for a non-zero delay, besides the usual indeterminacy in order and amplitude.

In the first experiment, two source signals with very similar frequency content were considered (see figure 1). The signals were randomly mixed and the algorithm was applied for different values of $M, TH = 1, k_1 = 0$ and $k_2 = 1$. The figure (2) shows the results obtained when $M = 3$. In that case six independent components are computed: three (figure 2-a) are related with the first source signal while the other three (figure 2-b) were related with the second source signal. We can see that these estimated components represent different frequency contents of each of the corresponding source signals. For instance figure 2-a) shows that: the first component isolates the high frequency component, the third component isolates the low frequency component and the second component contains both frequencies of the source signal. The figures 2-(c) and (d) represent the frequency response amplitude of the filters considering the columns of the transformed mixing matrix $E_x^T H$ as coefficients of a filter. The filters are then paired with the respective independent source signals and we can verify that the frequency response confirms the frequency contents of each of the independent signals as we have a low-pass, a band-pass and high-pass filter (see fig. (2d). Increasing the number

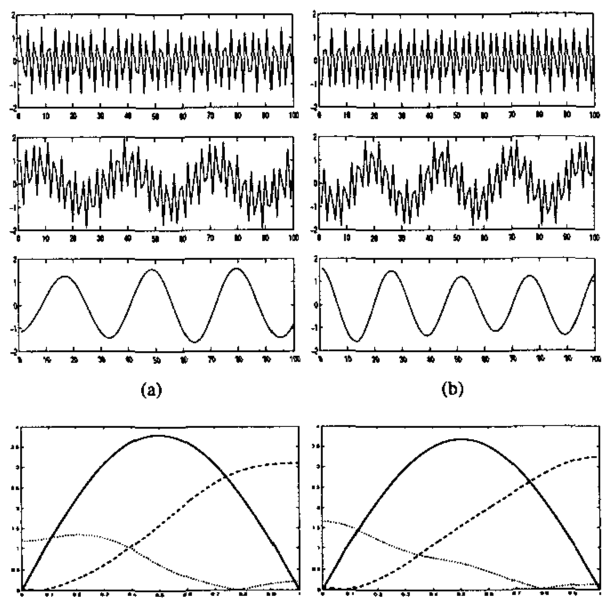

Fig. 2. (a)-Independent Components correlated with source signal 1 , (b)-Independent Components correlated with source signal 2 , (c)-Frequency response amplitude of filters related with components represented in (a), (d)-Frequency response amplitude of filters related with components represented in (b)

of delays (M) the independent components show either the low or high frequency contents of the source signals. In this case, the frequency response of the columns of $E_x^T H$ will not have a pass-band bandwidth to include all frequency contents of each source signal. Another problem is that the eigenvalues of the first standard eigendecomposition start to have very low values. In that case one should consider $TH < 1$ and reduce (from NM to l) the number of signals at the output of the first step.

The second simulation considers the influence of noise on the performance of the algorithm. Considering three narrow-band linearly mixed source signals (sinusoids), random noise was added resulting in a signal-to-noise ratio in the range of $[0, 20]dB$. The parameters of the algorithm were chosen as: $M = 8, TH = 0.95, k_1 = 0$ and $k_2 = 1$. After selecting the three estimated independent components that had the highest correlation with each of the source signals their mean square error (MSE) was computed. Figure (3) shows the results as well as a comparison with AMUSE (with the pencil computed with the same delays) for identical noise levels. It can be verified that the proposed algorithm is always better than AMUSE and that the difference increases with the increase in noise energy.

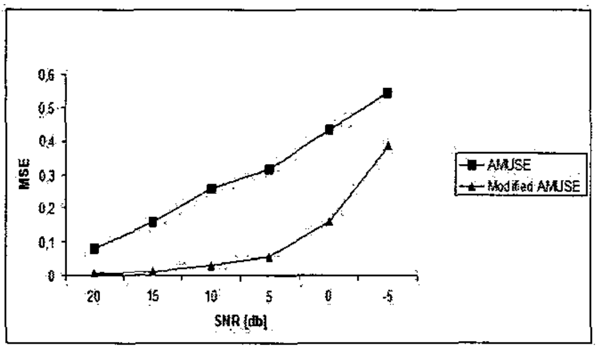

Fig. 3. Comparing AMUSE and dAmuse(modified AMUSE) using MSE error

Another important aspect of this experiment is related with the influence of the threshold parameter TH when noise is considered. The parameter has always the same value, and as the noise energy increases the number of signals (l) after the application of the first step also increases (second column of table I). Consequently the number of independent signals computed at the output is also l (last column of table I). Nevertheless, computing the correlation coefficients (using the frequency information of the signals) between output signals and noise or source signals we confirm that some components are related with the sources and others with noise. The maximum values of the correlation coefficients are distributed between the groups, in table I the third column shows the number of independent components (IC) that are related with sources while the fourth column shows the number of ICs that are correlated with noise.

These results show that for a low noise level the first step (which is mainly a principal component analysis in a space of dimension NM) achieves good solutions. But when the noise level increases the additional second step improves results considerably. This was verified by adding high energy noise and taking only the five most significant eigenvalues, and corresponding eigenvectors, at the output of the fist step. In these cases the source signals were not recovered.

TABLE I

NUMBER OF SIGNALS AT THE OUTPUTS OF THE 2 MAIN STEPS OF THE ALGORITHM FOR DIFFERENT LEVELS OF NOISE

		Output correlated with		
Noise	1st step	Sources	Noise	Total
20dB	5	5	0	5
15dB	7	6	1	7
10dB	10	6	4	10
5dB	18	14	4	18
0dB	19	10	9	19

B. Heart Rate Variability study

We use ECG signals of two young normal subjects from POLY/MEDLAV database [11]. One lead of the stored ECG signal was processed by a wavelet transform based automatic delineation system [12] and the RR and QT sequences measured over the obtained marks. The ECG signals of these two subjects were chosen because the corresponding respiration signals have spectral contents in distinct bands: (1)-ECG1 corresponds to a respiratory signal in a low frequency band $(0.05 - 0.1Hz)$; (2)-ECG2 to a corresponding signal in a high frequency band $(0.2 - 0.4Hz)$.

The proposed algorithm was applied to $[RR(k), QT(k)]^T$ sequences. In all the experiments the free parameters of the algorithm were chosen as: $M = 8$, $k_1 = 0$, $k_2 = 1$ and $TH = 0.95$. The estimated independent component signals are not correlated in the time domain and suffer from an indeterminacy on its amplitude as it is expected from these methods. Then in order to achieve a direct comparison the ICs were normalized to unit variance and amplitudes in the range [-1,1]. However, in HRV studies, the frequency contents in distinct frequency bands are the relevant information. Hence an analysis in frequency space is performed for each independent signal using the Welch method [13].

The results in frequency domain are compared graphically and the correlation coefficients are also computed. Three different independent components ($S_i, i = 1...3$) are computed having frequency contents in distinct frequency ranges as shown in figure 4.

TABLE II

CORRELATION COEFFICIENTS BETWEEN THE SPECTRA OF THE INDEPENDENT COMPONENTS

	ECG1			ECG2		
	S_1	S_2	S_3	S_1	S_2	S_3
S_1	1	0.10	0.09	1	0.38	0.14
S_2	0.10	1	0.03	0.38	1	0.13
S_3	0.09	0.03	1	0.14	0.13	1

In table II we can also see that the correlation between the frequency contents of the estimated independent signals is very low. And we can also verify that two of those signals have frequency contents in the range of the LF and HF fluctuations considered in the HRV studies.

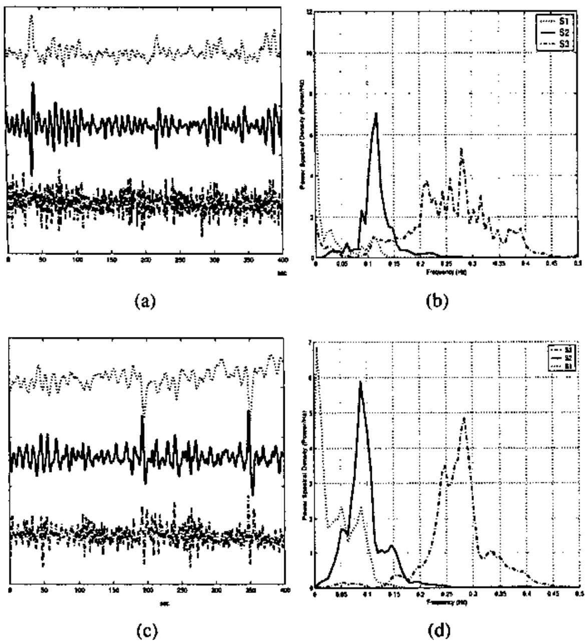

Fig. 4. ECG1: (a)-Independent Components, (b)-Spectral densities of independent components; ECG2: (a)-Independent Components,(b)-Spectral densities of independent components

V. Conclusions

In this work we propose dAMUSE, a modified version of the algorithm AMUSE. The new algorithm is based on the generalized eigendecomposition of a matrix pencil computed with time-delayed sensor signals. It was shown that the independent components obtained at the output represent filtered versions of the source signals. An implementation was also proposed which differs from the algorithm AMUSE (and other blind source separation algorithms) because it uses a different criterium for choosing the number of signals at the output of the first step (which is very similar to a principal component decomposition). The algorithm has a set of parameters whose proper choice must be further studied, particularly the number of delays(M) used to build the data set. Its choice naturally constrains the linear filtering operation that characterizes the method as it was shown in the simulations. The simulations also reveal that noise reduction cannot completely be achieved by PCA alone. Then having at the output a high number of signals it is also important to find an automatic procedure to choose the relevant signals as some might be related with noise only. The choice of time-delayed matrices to compute the pencil was done according to the algorithm AMUSE as well as the values for time-delays (k_1 and k_2)[1]. Nevertheless other second-order statistics algorithms [14] [15] might be considered as well as they achieved better results when compared with AMUSE [14].

In what concerns the HRV study, blind source separation techniques were used to study RR and QT sequences and to discuss the relation of the independent components with the LF and HF fluctuations [16]. This preliminary study achieves similar results in spite of the use of different processing steps concerning the pre-processing of the temporal sequences and the method to estimate the independent signals. The new algorithm seems to provide a promising tool to HRV studies. Nevertheless a validation against a conventional method as well as a large data set should be used to verify the reliability and the performance of the method.

Acknowledgements

R. Almeida acknowledges the grant SFRH/BD/5484/2001 supported by FCT and ESF (III CSF), K. Stadlthanner acknowledges financial support by the DFG as a member of the graduate college GRK 638 and E.W.Lang acknowledges financial support by the BMBF, project ModKog.

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