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Analysis of football player's motion in view of fractional calculus

Research Article

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Abstract: Accurately retrieving the position of football players over time may lay the foundations for a whole series of possible new performance metrics for coaches and assistants. Despite the recent developments of automatic tracking systems, the misclassification problem (*i.e.*, misleading a given player by another) still exists and requires human operators as final evaluators. This paper proposes an adaptive fractional calculus (FC) approach to improve the accuracy of tracking methods by estimating the position of players based on their trajectory so far. One half-time of an official football match was used to evaluate the accuracy of the proposed approach under different sampling periods of 250, 500 and 1000 ms. Moreover, the performance of the FC approach was compared with position-based and velocity-based methods. The experimental evaluation shows that the FC method presents a high classification accuracy for small sampling periods. Such results suggest that fractional dynamics may fit the trajectory of football players, thus being useful to increase the autonomy of tracking systems.

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Keywords: football • prediction methods • fractional calculus • fractional dynamics

1. Introduction

Football is one of the most popular sports in the world being of great interest for many scientific areas [1]. The main interest for sport sciences is to improve the teams' performance increasing the opportunities to win [2]. Therefore, many mathematical and technological approaches have been proposed and developed over the last few years [3]. One of them is the on-the-fly, i.e., online, match analysis [4]. Through tracking techniques, either manual or automatic, it has been possible to collect each player's positioning on the field at every instant [5]. Such information is considered one of the most relevant that may contribute to improve the performance understanding [6]. This spatio-

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temporal information allows many kind of analysis such as kinematical [7], technical and tactical [3]. Bearing such idea in mind, some systems have been developed to collect the players' positional data [9]. Although there are other technological approaches such as Global Positioning System (GPS) or Radio-Frequency Identification (RFID), the video analysis is, by far, the most adopted one in official football, as it does not allow the use of devices on players during the matches [10]. Therefore, for now only two main video analyses can be performed to collect the positional data from official matches: i) automatic tracking; and ii) manual tracking.

The automatic tracking of only one player was originally presented by Ohashi et al [11]. The method considered the calculation of player's position and speed through trigonometric techniques. Over the years, many other automatic tracking techniques were proposed, in which some few systems such as AMISCO Pro and ProZone already have the ability to track all players and the ball at each iteration [12]. These kind of video-based multi-player tracking systems generally require the permanent installation of several fixed cameras in optimally calculated positions to cover the whole field [12]. Systems such as AMISCO Pro or ProZone provide online information to coaches and their staff about players' movements (*e.g.*, energy spent by a player). Despite of their efficiency and autonomous properties, many problems still remain. For instance, player-to-player occlusion, similar player appearance, number of players changing over time, variability of players' motion and noises or video blur present themselves as open problems [13]. Therefore, despite being generally autonomous, these tracking systems still require some human input as well as continual online verification by an operator to make sure that players are correctly tracked by the computer program [12]. Hence, beyond their expensive devices (*e.g.*, many high-definition video cameras), the automatic tracking of multiple player still requires human operators to fix some mistakes.

In order to overcome this situation, some studies have been proposed using monocular solutions. For instance, in the approach presented in [14], the authors use a particle filter to track each player. Preliminary results suggest that to overcome occlusions, the classifier detects players and resample the centre of gravity [14]. Despite their promising results, some questions about the applicability of this technique still remain due to its computational complexity [15]. Moreover, this tracking strategy only allows to identify the players without any memory properties. Nevertheless, it is those memory properties that may provide further information about the variability, predictability and stability level of each player. Moreover, these systems are based uniquely on colour segmentation eve despite soccer matches can occur at different moments of the day with or without artificial light [15]. It is due to those reasons that many scientific studies have been using the manual tracking as a low-cost solution to overcome the expensive automatic multi-player tracking.

Many manual tracking methods only use a single high-definition video camera to collect the positional data from the players [16]. In this context, the image treatment is performed after the match, i.e., in an offline fashion. The TACTO software is one of the many manual tracking systems with accuracy levels reported as superior to 95% [17]. Similarly, many other software's designed for specific applications have been used in the literature [8]. Most of those software's use the Direct Linear Transformation (DLT) technique to relate an object point located in the object space/plane and the corresponding image point on the image plane of the camera [18]. Despite of its user-friendly technology, the manual tracking to analyse official football matches can be a massive and exhaustive work since it requires the manual tracking of 22 players and the ball at each iteration. Furthermore, as most of those use only one camera it reduces the possibility to concretely define the players' identification and position. In sum, both manual and automatic tracking systems present advantages and disadvantages. Nevertheless, they still require human operators to ensure the accuracy of players' identification. Next section formalizes our problem and presents a couple of classical initial strategies to estimate the current position of a given player.

1.1. Problem formulation

In both manual and automatic multi-player tracking systems, a matrix containing the planar position of each player n of team δ over time is generated. Let us call this as the *positioning matrix* $X_\delta[t]$ wherein row n represents the planar position of player n of team δ at time t, *i.e.*,

$$X_\delta[t] = \begin{bmatrix} x_1[t] \\ \vdots \\ x_{N_\delta}[t] \end{bmatrix}, \; x_n[t] \in \mathbb{R}^2, \tag{1}$$

wherein N_δ represents the current number of players in team δ at iteration/time t. For the 11-football match, teams will start with an initial number of 11 players, *i.e.*, $N_\delta = 11$,thus resulting in a 11×2 positioning matrix $X_\delta[t]$. Nevertheless, the row-order of the matrix may be incorrectly retrieved due to all the problems previously mentioned. Therefore, many techniques may be proposed to overcome this issue.

For instance, the most easiest way to correctly sort the positioning matrix at iteration $t+1$, *i.e.*, $X_\delta[t+1]$, may be carried out as a minimization problem of the distance between its rows, *i.e.*, $x_n[t+1]$, and the rows from the positioning matrix at iteration t, *i.e.*, $x_n[t]$. Nevertheless, the accuracy of estimating the current position based on the previous one may significantly decrease as the time between iterations, *i.e.*, *sampling period* T, increases. For instance, as the average velocity of football players may reach 3.06 m/s [19], and considering a sampling period of 1 second, *i.e.*, $T = 1$, results in an average difference between two consecutive positions of 3.06 meters. This may easily lead to the misidentification of players. Moreover, players' velocity may vary between 0 and 6.39 m/s [19], with some outstanding cases such as Cristiano Ronaldo (second FIFA world player of the 2012) that is able to achieve an average maximum velocity of 9.33 m/s, which would result in difference between two consecutive positions of 9.33 meters. Although limited, the estimation of the current position based on the previous one only requires a memory complexity of $O[N_\delta]$ as it only requires memorizing the previous position of all players in team δ. Alternatively, and considering players' dynamics, one may contemplate the velocity vector of players. Note that, as we consider discrete systems with a constant sampling period, the velocity vector of a given player may be obtained based on its consecutive planar positions, thus returning both magnitude and direction [20]. To do this, let us consider the discrete case in which the motion of player n may be defined as:

$$x_n[t+1] = x_n[t] + v_n[t+1], \qquad (2)$$

in such a way that the position of player n from team δ at iteration $t+1$ will be its previous position incremented of its current velocity vector $v_n[t+1]$. Nevertheless, as the current velocity vector is unknown, the following approximation may be carried out:

$$x_n^s[t+1] = x_n[t] + v_n[t], \qquad (3)$$

in which $x_n^s[t+1]$ is the estimated position of player n and $v_n[t]$ is the velocity vector retrieved from the previous iteration which may be calculated as:

$$v_n[t] = x_n[t] - x_n[t-1], \qquad (4)$$

This may only be accomplished for small sampling periods (*e.g.*, $T \leq 1$), as players may not be able to drastically change their velocity between two consecutive iterations. The estimation of the current position based on the previous velocity vector may overcome the non-dynamical characteristics of the previous method. Such higher accuracy is achieved by slightly increasing the memory complexity of the algorithm to $\mathcal{O}[2N_\delta]$ as the two previous positions of all players in team δ are necessary to compute the velocity vector of each player (*cf.* equation (4)).

Nevertheless, considering the unpredictable movements of football players, the previously presented strategies may be inefficient as they do not consider the whole trajectory performed by players so far. A possibility to overcome such limitations is by taking advantage of Fractional Calculus (FC) properties.

1.2. Statement of contribution

Only a few number of applications based on FC has been reported so far within sport sciences literature. One of them was the development of a correction metric for golf putting to prevent the inaccurate performance of golfers when facing the golf *lipout* phenomenon [21]. The authors extended a performance metric using the Grünwald-Letnikov approximate discrete equation to integrate a memory of the ball's trajectory.

Although FC was not fully explored in sport sciences, many other areas have been taking full use of its properties. For instance, FC has been used to describe the dynamical characteristics of robots' motion since it is well suited to describe irreversibility and chaos due to its inherent memory property [22, 23]. In this line of thought, the dynamic phenomena of a robot's trajectory configure a case where fractional calculus tools fit adequately [22].

Football has been regularly identified as a complex dynamic system wherein the motion of each player is usually chaotic and difficult to predict [24]. Under those assumptions, this paper proposes a new strategy to estimate the current position of football players benefiting from fractional calculus concepts. This will allow overcoming the automatic and manual tracking problems described above.

2. Fractional order estimation

2.1. Preliminaries

This section summarizes some preliminaries regarding fractional calculus. For a more detailed description please refer to Machado *et al* [25] or Couceiro *et al* [22]. Briefly, Fractional Calculus (FC) can be considered as a generalization of integer-order calculus, thus accomplishing what integer-order calculus cannot. As a natural extension of the integer (*i.e.*, classical) derivatives, fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of processes. The concept of Grünwald-Letnikov fractional differential is presented by the following definition.

Definition 1.

[25] Let Γ be the gamma function defined as:

$$\Gamma(k) = (k-1)! \tag{5}$$

The signal $D^{\alpha}[x[t]]$ given by

$$D^{\alpha}[x[t]] = \lim_{h \to 0}\left[\frac{1}{h^{\alpha}}\sum_{k=0}^{+\infty}\frac{(-1)^k\Gamma(\alpha+1)x(t-kh)}{\Gamma(k+1)\Gamma(\alpha-k+1)}\right], \tag{6}$$

is said to be the *Grünwald-Letnikov fractional derivative of order* α, $\alpha \in \mathbb{C}$, *of the signal* $x[t]$.

An important property revealed by (6) is that while an integer-order derivative just implies a finite series, the fractional-order derivative requires an infinite number of terms. Therefore, integer derivatives are "local" operators while fractional derivatives have, implicitly, a "memory" of all past events. However, the influence of past events decreases over time. The formulation in (6) inspires a discrete time calculation presented by the following definition.

Definition 2.

[25] The signal $D^{\alpha}[x[t]]$ given by

$$D^{\alpha}[x[t]] = \frac{1}{T^{\alpha}}\sum_{k=0}^{r}\frac{(-1)^k\Gamma(\alpha+1)x(t-kT)}{\Gamma(k+1)\Gamma(\alpha-k+1)}, \tag{7}$$

where T is the sampling period and r is the truncation order, is the *approximate discrete time Grünwald-Letnikov fractional difference of order* α, $\alpha \in \mathbb{C}$, *of the discrete signal* $x[t]$.

The series presented in (7) can be implemented by a rational fraction expansion which leads to a superior compromise in what concerns the number of terms versus the quality of the approximation. That being said, it is possible to extend an integer discrete difference, *i.e.*, classical discrete difference, to a fractional-order one, using the following definition.

Definition 3.

[26] The classical integer "direct" discrete difference of signal $x[t]$ is defined as follows:

$$\Delta^d x[t] = \begin{cases} x[t], & d=0 \\ x[t]-x[t-1], & d=1 \\ \Delta^{d-1}x[t]-\Delta^{d-1}x[t-1], & d>1 \end{cases}, \tag{8}$$

where $d \in \mathbb{N}_0$ is the order of the integer discrete difference. Hence, one can extend the integer-order $\Delta^d x[t]$ assuming that the fractional discrete difference satisfies the following inequalities:

$$d-1 < \alpha < d. \tag{9}$$

The features inherent to fractional calculus make this mathematical tool well suited to describe many phenomena, such as irreversibility and chaos, because of its inherent memory property. In this line of thought, the dynamic phenomena of a player's trajectory configure a case where fractional calculus tools may fit adequately.

2.2. Fractional calculus approach

FC will be addressed in our study based on problem formulation presented in Section 1.1. At first, let us rewrite equation (3) as:

$$x_n^s[t+1] - x_n[t] = v_n[t], \tag{10}$$

Hence, $x_n^s[t+1]-x_n[t]$ corresponds to the discrete version of the fractional difference of order $\alpha = 1$, *i.e.*, the first order integer difference. By Definition 2 it is possible to consider:

$$D^{\alpha}[x_n^s[t+1]] = v_n[t]. \tag{11}$$

Based on the FC concept and Definition 3, the order of the position derivative can be generalized to a real number $0 < \alpha < 1$, thus leading to a smoother variation and a longer memory effect. Equation (11) can then be written as:

$$x_n^s[t+1] = v_n[t] - \frac{1}{T^{\alpha}}\sum_{k=1}^{r}\frac{(-1)^k\Gamma(\alpha+1)x(t-kT)}{\Gamma(k+1)\Gamma(\alpha-k+1)}. \tag{12}$$

Replacing $v_n[t]$ in function of the position as presented in equation (4) results in:

$$x_n^s[t+1] = x_n[t]-x_n[t-1]-\frac{1}{T^{\alpha}}\sum_{k=1}^{r}\frac{(-1)^k\Gamma(\alpha+1)x(t-kT)}{\Gamma(k+1)\Gamma(\alpha-k+1)}. \tag{13}$$

It should be noted that such strategy increases the memory complexity as it requires memorizing the last r positions of each player, *i.e.*, $\mathcal{O}[rN_{\delta}]$. Nonetheless, the truncation order r does not need to be too large and will always be inferior to the current iteration/time t, *i.e.*, $r \leq t$. For instance, considering a truncation order $r = 10$, $\alpha = \frac{2}{3}$ and $T = 1$ second, *i.e.*, considering the last 10 previous seconds, results in an attenuation of players' position at time $t-9$, *i.e.*, the $x[t+1-10]$, of approximately 99.5%, *i.e.*, $\frac{(-1)^{10}\Gamma\left[\frac{2}{3}+1\right]}{\Gamma[10+1]\Gamma\left[\frac{2}{3}-10+1\right]}$. Correctly choosing the possible next position of player n may be achieved by minimizing the distance between the estimated position $x_n^s[t+1]$ and the positions on the football match matrix $X_\delta[t+1]$:

$$[d_n^{min}, i_n^{min}] = \min_{i \in N_\delta} \left(x_n^s[t+1] - x_i[t+1]\right), \tag{14}$$

where d_n^{min} is the minimal distance between the last known position at time t of player n and all new positions stored on an estimated positioning matrix $X_\delta^s[t+1]$, with $X_\delta^s[t+1] = [x_1^s[t+1] \cdots x_n^s[t+1]]^T$, retrieved by the method at time $t+1$, in such a way that the position represented by player i_n^{min} has a strong possibility of being player n.
Considering α as the fractional order derivative, then by using α values near 0, past events are not that relevant to the final result. On the opposite, using α values near 1, it means that past events have a major influence in the final result (*cf.*, [21]). In football context, analysing the fractional coefficient α of each player may imply the predictability of players' motion. Therefore, the α value can be useful information to analyze the level of unpredictability of each player.

Yet, a problem arises regarding the tuning of the fractional coefficient α that may vary from player to player and from iteration to iteration. Hence, one should find out the most fitted α for player n at time t, *i.e.*, $\alpha_n[t]$, based on its last known positions so far, *i.e.*, the $\alpha_n[t]$ that would result in a smaller d_n^{min}. This $\alpha_n[t]$ will be used to assess the next possible position and, once again, be systematically updated at each t. This may be formulated by the following minimization problem:

$$\min_{\alpha_n[t]} d_n^{min}(\alpha_n[t]) = \left| -x_n[t+1] + x_n[t] - x_n[t-1] - \frac{1}{T^\alpha} \sum_{k=0}^{r} \frac{(-1)^k \Gamma(\alpha_n[t]+1) x(t+1-kT)}{\Gamma(k+1)\Gamma(\alpha_n[t]-k+1)} \right| \tag{15}$$

$s.t\ \alpha_n[t] \in [0,1]$.
We will not focus on the most adequate type of optimization method in this paper. In this paper, the solution of equation (15) was based on golden section search and parabolic interpolation. Successive parabolic interpolation allows finding the minimum distance by successively fitting parabolas to the optimization function at three unique points and, at each iteration, replacing the "oldest" point with the minimum value of the fitted parabola. This method is alternated with the golden section search hence increasing the probability of convergence without hampering the convergence rate. For a more detailed description about this optimization methods please refer to [27, 28].
A particular case occurs for an iteration/time inferior to 2. Considering that the game starts at the 1 second and $T=1$, it is noteworthy that for an iteration/time inferior to 2, *i.e.*, $t<2$, it is impossible to compute $x_n^s[2]$ due to $x_n[0]$. In other words, it is necessary to have the first two consecutives positions of a given player. A way to overcome this problem is to consider the pregame start position equal to the first ingame position, *i.e.*, $x_n[0] = x_n[1]$. This assumption holds for a small periodic time t between iterations. Due to the human movement dynamics, and since football players are still in the first iteration/time, *i.e.*, $v_n[1] = 0$, the difference between the first two steps (immediately pre and after game start) is usually minor, *i.e.*, $v_n[0] \approx v_n[1]$. As the average velocity of football players is 4 m/s, one may considered a maximum iteration time of 1 second, *i.e.*, $T=1$, thus representing an average difference of 4 meters between two consecutive positions. In other words, for the second iteration and considering the main equation, one may approximate $x_n^s[2]$ as:

$$x_n^s[2] = \frac{3}{2}\alpha[1]x_n[1]. \tag{16}$$

Considering that a player is considerably near the same position for the first 2 seconds after the game starts, which may be assumed due to players' dynamics, then $\alpha_n[t]$ may be initially defined as $\frac{2}{3}$ for all players, $\alpha_n[1] = \frac{2}{3}, \forall n \in [1, N_\delta]$. Figure 1 depicts the predicted position of two players using the methods presented in this paper. It is noteworthy that although the position-based method is not explicitly presented, it corresponds to the exact point of the previous position of each player.
As one may observe, the crossing events between two players from the same team can be one of the most challenging and deceiving aspects for estimation methods. For instance, from Figure 1b to Figure 1c, the velocity vector method turns out to provide erroneous information since the position estimation of each player gets closer to the real position of the opposing player. This would induce the tracking system into the misclassification of players. This misinformation can be critical to analyze the behavior of a specific player over time. On the other hand, the fractional calculus method starts following a certain tendency since it takes into account the whole trajectory so far. In

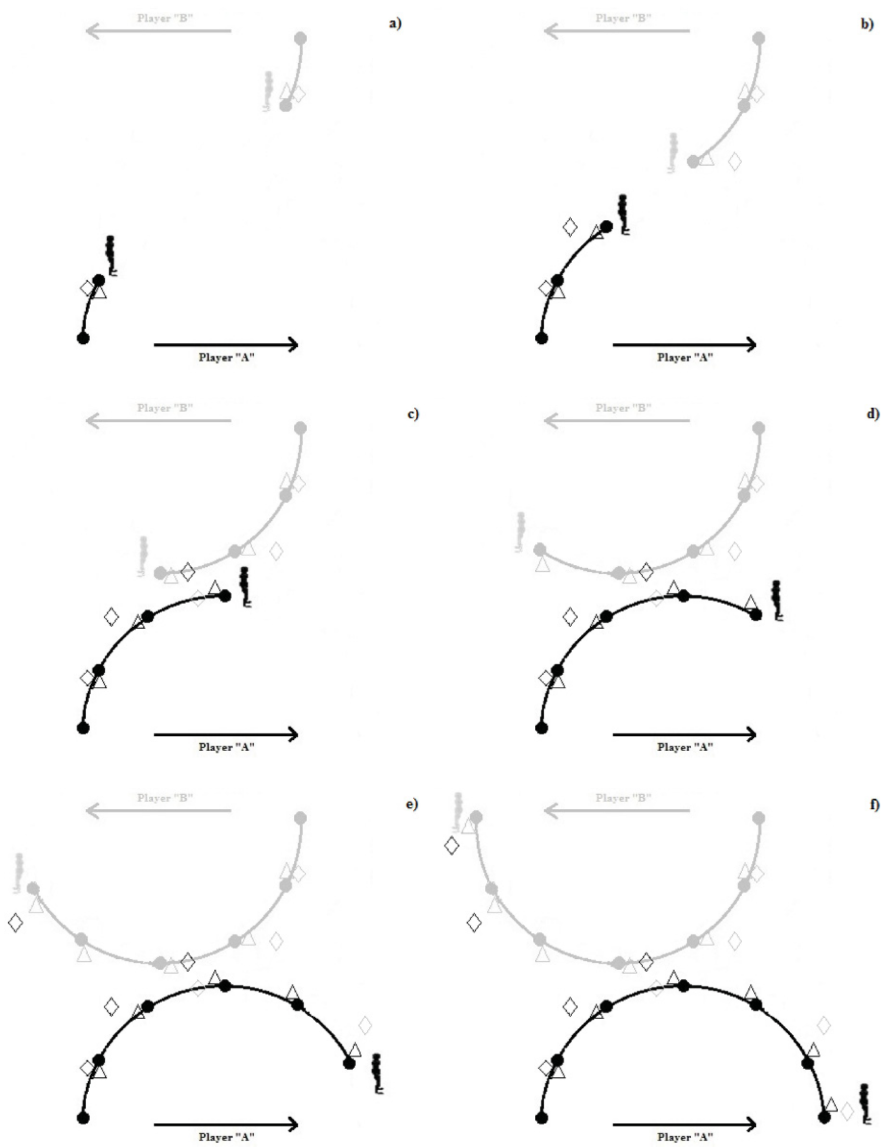

Figure 1. Estimation position of two players.

other words, the fractional calculus and its inherent memory property provide a better alternative for multi-player tracking systems.

Next section evaluates and compares the herein proposed approach on a real example of football match.

3. Results

To evaluate the accuracy of the proposed method, one half-time of an official football match was analysed. All players' position in the field was acquired using a single camera (GoPro Hero with 1280 × 960 resolution), with capacity to process images at 30 Hz (*i.e.*, 30 frames per second) and recording with a wide angle of 180°. The camera was placed on an elevated position on the stadium to capture the whole field without any changes from the beginning to the end of the match. The movements of the 22 players (goalkeepers included) from the two competing teams were recorded during the entire game. After capturing the football match, the physical space was calibrated using direct linear transformation (DLT) [29], thus obtaining the Cartesian planar positioning of all players and the ball over time. The whole process inherent

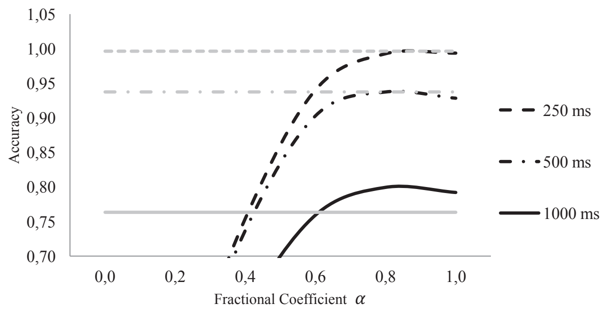

Figure 2. Comparing adaptive fractional coefficients $\alpha_n[t]$ with predefined values α_n.

to this approach, such as the detection and identification of players' trajectories, the space transformation and the computation of metrics, was handled using the high-level calculation tool MatLab.

For a matter of efficiency, only play moments were used, hence excluding all the pause moments in which the ball was not in the field (i.e., ball out-of-bounds). This resulted in 1833 seconds of useful match, *i.e.*, approximately 31 minutes. To evaluate the methods under different configurations, sampling periods of $T = \{250, 500, 1000\}$ $[ms]$ were used. Moreover, the players were afterward identified by a human operator at each iteration so as to compare the accuracy of the methods, *i.e.*, the number of times they correctly predict the right player, *i.e.*, number of true positives. The human operator performed the manual tracking at each iteration, depending on the sampling period T, following the players in such a way to correctly sort the positional matrix $X_\delta[t]$ over time. Afterwards, the human operator manual tracking was compared with the sorting performed by the estimation methods.

As the fractional methodology herein proposed have higher memory complexity than the other methods, a truncation order of $r = 10$ was settled (*cf.* Section 2.2, thus resulting in a memory complexity of $\mathcal{O}[10N_\delta]$. The first experiment was used to compare the method using adaptive fractional coefficients $\alpha_n[t]$ with predefined α_n values (Figure 2). As a performance metric, the overall accuracy of the fractional method was defined as the fraction of correctly classified players over the whole process, *i.e.*, ratio between the number of samples and the number of true positives.

The horizontal lines on Figure 2 (i.e., constant values) correspond to the accuracy of the algorithm under the adaptive mechanism (adaptive $\alpha_n[t]$ for each player). The corresponding curves (*i.e.*, with the same line style) represents the accuracy of the algorithm for each α_n between 0 and 1. Therefore, as one may observe, the adaptive $\alpha_n[t]$ generally results in a higher performance than when using a fixed α_n. For sampling periods of 250 and 500 ms, such performance may be almost attained by using a fixed α_n between 0.8 and 0.9. The only case in which this tendency is broken may be observed for a larger sampling period of 1 second. Under such sampling period, a fixed α_n between 0 and 1 results in a better accuracy when compared to the adaptive one. This may be explained by the fact that a large sampling period may not be ideal to represent the dynamic behaviour of players.

Despite this specific situation, it may be observed that the adaptive mechanism turns out to partially overcome the unpredictability of players, thus presenting an overall accuracy above 90% for smaller sampling periods. Hence, the fractional method with adaptive $\alpha_n[t]$ (FC) was compared with the traditional methods presented in the preliminaries, namely the estimation based on the velocity vector (VEL) and the estimation based on the previous position (POS).

Once again, the classification accuracy was used to evaluate the performance of the methods. Due to the number of classes (*i.e.*, players) and to allow a straightforward comparison between the three methods under different configurations (*i.e.*, sampling period T), the traditional confusion matrices were used. The confusion matrix will be represented by a $N_\delta - by - N_\delta$ matrix, where each (i, j) entry will be the number of samples whose target is the i^{th} player that was classified as the j^{th} player. For instance, Figure 3 presents the confusion matrix for a sampling period of $T = 250$ ms wherein the 3^{rd} player was misclassified as the 2^{nd} player 5 times, as the 4^{th} player 3 times, as the 5^{th} player and 7^{th} players only once and as the 6^{th} player 4 times using the proposed fractional method. Even so, the overall classification accuracy of the fractional method (FC) greatly outperforms the other methods with a success percentage of 99.65% against the 99.36% and the 97.18% from velocity vector method (VEL) and the previous position method (POS), respectively. The same conclusions may be withdrawn from a sampling period of $T = 500$ ms (Figure 4). Nevertheless, one may observe that for a higher sampling period of $T = 1000$ ms (Figure 5), the fractional method (FC) presents a higher misclassification ratio, thus failing to correctly identify the team at each second 23.67% of the time, against a misclassification percentage of 20.79% and 14.31% from the velocity vector method (VEL) and the previous position method (POS).

This is an interesting phenomenon and may be explained by the common strategy of football teams. At each match, players have a playing position wherein they spend most of their time (*e.g.*, goalkeeper, defender, etc . . .). With a large sampling rate, the position method (POS) turns out to present a better accuracy since players keep around their playing position. Although this was verified in this

Actual Player	Method	Predicted Player 1	2	3	4	5	6	7	8	9	10	11	Accuracy
1	FC	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
	VEL	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
	POS	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
2	FC	0	**7310**	5	0	0	4	3	2	4	1	1	0,9973
	VEL	0	**7304**	8	0	0	7	4	0	5	1	1	0,9965
	POS	0	**7195**	38	0	2	39	7	2	37	1	9	0,9816
3	FC	0	5	**7316**	3	1	4	1	0	0	0	0	0,9981
	VEL	0	8	**7308**	3	2	9	0	0	0	0	0	0,9970
	POS	0	27	**7214**	26	5	31	7	7	9	1	3	0,9842
4	FC	0	0	4	**7310**	9	4	1	1	0	1	0	0,9973
	VEL	0	1	5	**7304**	12	5	1	2	0	0	0	0,9965
	POS	0	1	20	**7182**	54	14	12	21	5	10	11	0,9798
5	FC	0	0	1	8	**7297**	0	3	12	1	8	0	0,9955
	VEL	0	0	1	12	**7271**	0	4	28	1	13	0	0,9920
	POS	0	3	4	58	**7118**	9	8	92	5	29	4	0,9711
6	FC	0	4	3	5	0	**7300**	9	3	6	0	0	0,9959
	VEL	0	7	4	8	0	**7269**	19	7	13	2	1	0,9917
	POS	0	35	37	12	8	**7068**	88	27	38	7	10	0,9643
7	FC	0	4	1	1	3	9	**7282**	14	2	5	9	0,9935
	VEL	0	4	3	1	5	20	**7229**	18	11	9	30	0,9862
	POS	0	10	4	7	16	95	**6901**	137	40	24	96	0,9415
8	FC	0	1	0	3	11	3	13	**7288**	4	6	1	0,9943
	VEL	0	0	0	2	26	8	16	**7258**	5	12	3	0,9902
	POS	0	2	6	34	85	33	120	**6967**	24	41	18	0,9505
9	FC	0	4	0	0	1	6	3	4	**7306**	1	5	0,9967
	VEL	0	4	1	0	0	11	15	4	**7280**	3	12	0,9932
	POS	0	53	5	4	3	33	38	22	**7128**	10	34	0,9724
10	FC	0	1	0	0	8	0	5	6	1	**7303**	6	0,9963
	VEL	0	1	0	0	14	1	10	10	4	**7284**	6	0,9937
	POS	0	2	2	4	38	2	31	38	10	**7157**	46	0,9764
11	FC	0	1	0	0	0	0	10	0	6	5	**7308**	0,9970
	VEL	0	1	0	0	0	0	32	3	11	6	**7277**	0,9928
	POS	0	2	0	3	1	6	118	17	34	50	**7099**	0,9685
									Overall Accuracy FC				**0,9965**
									Overall Accuracy VEL				**0,9936**
									Overall Accuracy POS				**0,9718**

Figure 3. Confusion matrix for the sampling periods of $T = 250$ ms.

Actual Player	Method	Predicted Player 1	2	3	4	5	6	7	8	9	10	11	Accuracy
1	FC	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
	VEL	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
	POS	**7330**	0	0	0	0	0	0	0	0	0	0	1,0000
2	FC	0	**7310**	5	0	0	4	3	2	4	1	1	0,9973
	VEL	0	**7304**	8	0	0	7	4	0	5	1	1	0,9965
	POS	0	**7195**	38	0	2	39	7	2	37	1	9	0,9816
3	FC	0	5	**7316**	3	1	4	1	0	0	0	0	0,9981
	VEL	0	8	**7308**	3	2	9	0	0	0	0	0	0,9970
	POS	0	27	**7214**	26	5	31	7	7	9	1	3	0,9842
4	FC	0	0	4	**7310**	9	4	1	1	0	1	0	0,9973
	VEL	0	1	5	**7304**	12	5	1	2	0	0	0	0,9965
	POS	0	1	20	**7182**	54	14	12	21	5	10	11	0,9798
5	FC	0	0	1	8	**7297**	0	3	12	1	8	0	0,9955
	VEL	0	0	1	12	**7271**	0	4	28	1	13	0	0,9920
	POS	0	3	4	58	**7118**	9	8	92	5	29	4	0,9711
6	FC	0	4	3	5	0	**7300**	9	3	6	0	0	0,9959
	VEL	0	7	4	8	0	**7269**	19	7	13	2	1	0,9917
	POS	0	35	37	12	8	**7068**	88	27	38	7	10	0,9643
7	FC	0	4	1	1	3	9	**7282**	14	2	5	9	0,9935
	VEL	0	4	3	1	5	20	**7229**	18	11	9	30	0,9862
	POS	0	10	4	7	16	95	**6901**	137	40	24	96	0,9415
8	FC	0	1	0	3	11	3	13	**7288**	4	6	1	0,9943
	VEL	0	0	0	2	26	8	16	**7258**	5	12	3	0,9902
	POS	0	2	6	34	85	33	120	**6967**	24	41	18	0,9505
9	FC	0	4	0	0	1	6	3	4	**7306**	1	5	0,9967
	VEL	0	4	1	0	0	11	15	4	**7280**	3	12	0,9932
	POS	0	53	5	4	3	33	38	22	**7128**	10	34	0,9724
10	FC	0	1	0	0	8	0	5	6	1	**7303**	6	0,9963
	VEL	0	1	0	0	14	1	10	10	4	**7284**	6	0,9937
	POS	0	2	2	4	38	2	31	38	10	**7157**	46	0,9764
11	FC	0	1	0	0	0	0	10	0	6	5	**7308**	0,9970
	VEL	0	1	0	0	0	0	32	3	11	6	**7277**	0,9928
	POS	0	2	0	3	1	6	118	17	34	50	**7099**	0,9685
									Overall Accuracy FC				**0,9965**
									Overall Accuracy VEL				**0,9936**
									Overall Accuracy POS				**0,9718**

Figure 4. Confusion matrix for the sampling periods of $T = 500$ ms.

Actual Player	Method	Predicted Player 1	2	3	4	5	6	7	8	9	10	11	Accuracy
1	FC	**1833**	0	0	0	0	0	0	0	0	0	0	1,0000
	VEL	**1833**	0	0	0	0	0	0	0	0	0	0	1,0000
	POS	**1833**	0	0	0	0	0	0	0	0	0	0	1,0000
2	FC	0	**1405**	78	20	10	105	59	27	91	12	26	0,7665
	VEL	0	**1457**	61	28	7	101	45	27	81	8	18	0,7949
	POS	0	**1457**	61	28	7	101	45	27	81	8	18	0,8762
3	FC	0	73	**1442**	114	20	50	31	30	34	14	25	0,7867
	VEL	0	77	**1492**	90	24	33	24	24	31	15	23	0,8140
	POS	0	77	**1492**	90	24	33	24	24	31	15	23	0,8893
4	FC	0	20	102	**1438**	74	50	39	63	14	14	19	0,7845
	VEL	0	16	97	**1483**	71	43	30	51	21	9	12	0,8091
	POS	0	16	97	**1483**	71	43	30	51	21	9	12	0,8816
5	FC	0	16	20	71	**1436**	32	35	105	21	76	21	0,7834
	VEL	0	9	23	77	**1479**	26	32	79	10	74	24	0,8069
	POS	0	9	23	77	**1479**	26	32	79	10	74	24	0,8707
6	FC	0	120	72	41	25	**1273**	101	88	61	14	38	0,6945
	VEL	0	82	54	35	31	**1356**	89	85	62	15	24	0,7398
	POS	0	82	54	35	31	**1356**	89	85	62	15	24	0,8112
7	FC	0	44	33	33	33	106	**1160**	146	90	66	122	0,6328
	VEL	0	45	28	23	22	91	**1253**	139	75	49	108	0,6836
	POS	0	45	28	23	22	91	**1253**	139	75	49	108	0,7561
8	FC	0	18	39	69	125	90	129	**1209**	33	66	55	0,6596
	VEL	0	21	31	61	105	98	113	**1280**	30	52	42	0,6983
	POS	0	21	31	61	105	98	113	**1280**	30	52	42	0,7812
9	FC	0	97	21	20	22	63	84	35	**1370**	24	97	0,7474
	VEL	0	93	26	8	13	48	79	39	**1408**	18	101	0,7681
	POS	0	93	26	8	13	48	79	39	**1408**	18	101	0,8554
10	FC	0	9	10	12	68	29	69	65	25	**1470**	76	0,8020
	VEL	0	11	10	12	53	17	54	61	21	**1522**	72	0,8303
	POS	0	11	10	12	53	17	54	61	21	**1522**	72	0,8773
11	FC	0	31	16	15	20	35	126	65	94	77	**1354**	0,7387
	VEL	0	22	11	16	28	20	114	48	94	71	**1409**	0,7687
	POS	0	22	11	16	28	20	114	48	94	71	**1409**	0,8265
									Overall Accuracy FC				**0,7633**
									Overall Accuracy VEL				**0,7921**
									Overall Accuracy POS				**0,8569**

Figure 5. Confusion matrix for the sampling periods of $T = 1000$ ms.

match, some football strategies include switching playing positions which would greatly jeopardize the position method.

It is noteworthy, however, that such misclassification is related with the distance between the estimated position $x_n^s[t+1]$ and the real one $x_n[t+1]$, *i.e.*, d_n^{min}. Therefore, and to further compare the three methods, the average accumulated error distance over time was analysed. Once again it is possible to observe that the fractional method (FC) presents a higher performance for sampling periods of 250 ms and 500 ms (Figures 2 and 3).

Also, the previous position method (POS) is considerably worse than the other methods for a small sampling period, presenting an error almost 3 times higher than the fractional method (FC) for T=250 ms. This means that the dynamical characteristics of players may be more accurately represented as the sampling period decreases, thus being able to predict their position with a higher performance using the fractional (FC) and velocity vector (VEL) methods. However, for a larger sampling period of $T = 1000$ ms, the previous position method (POS) once again presents a better classification accuracy. The explanation about this phenomenon still remains the same and allows concluding that a large sampling rate of 1 second turns out to be too high to correctly represent play-

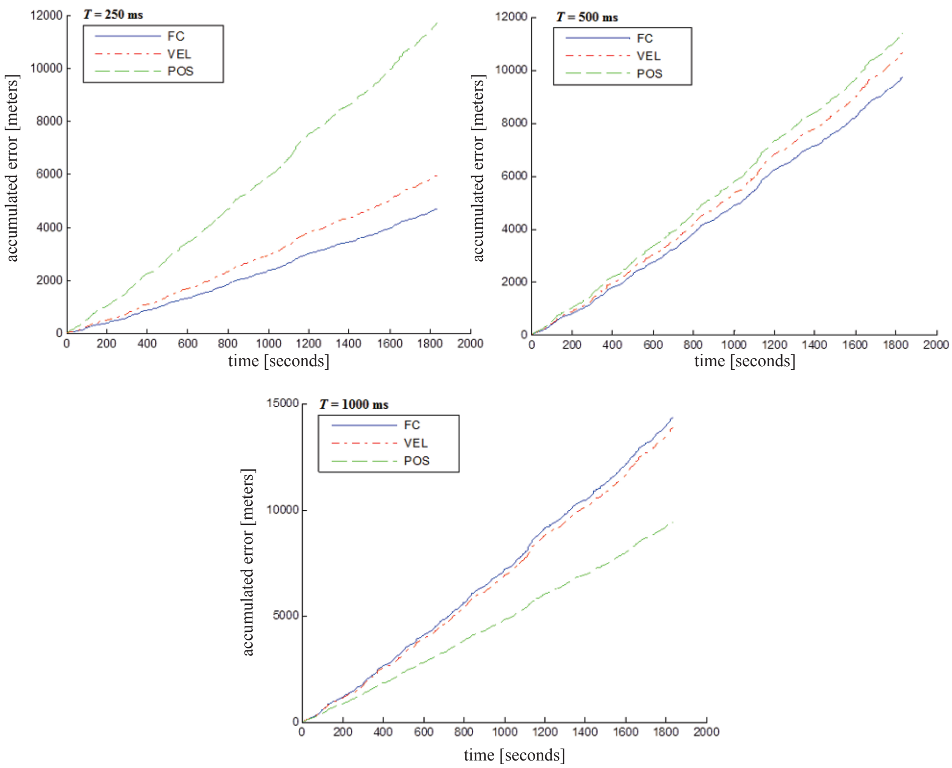

Figure 6. Comparing FC, VEL and POS methods for the sampling periods of $T = 250$ ms, $T = 500$ ms and $T = 1000$ ms.

ers' dynamics. Therefore, and as players usually play around a specific region, the previous position method (POS) present a better performance, and yet with a high accumulated error of almost 10 km.

4. Discussion and conclusion

Increase the autonomy and accuracy of multiplayer football tracking systems is one of the main goals to improve the performance analysis of football players and teams. Nevertheless, problems such as players' occlusion can produce erroneous information that may deceive the automatic tracking system, requiring the intervention of human operators to solve such situations. This paper proposed a fractional calculus method to predict the position of players over time in the field. The first step was to analyze the influence of the fractional coefficient in the performance of the method. This analysis showed that an adaptive $\alpha_n[t]$ generally results in a higher performance than when using a fixed α_n. Afterwards, the adaptive fractional calculus approach was compared with two other classical methods based on the previous position and the previous velocity of players. It was possible to observe a superior accuracy using the proposed strategy for sampling rates of $T = 250$ and $T = 500$. These results suggest that the motion of football players may be explained using fractional dynamics for small sampling periods, especially for long periods of time. Using the fractional order intrinsic memory property, one can predict the location of occluded players based on their trajectory so far. Also, a simple exclusion mechanism may be used to reinforce the robustness of the approach, i.e., the fractional method can predict the position of all visible players with high accuracy in such a way that the remaining players will be the lasts to be considered. Note that if it turns out to be impossible to detect a player due to a high level of occlusions, as the fractional algorithm still returns an estimated position based on its trajectory so far, one may consider it as the correct one and posteriorly correct it based on new acquired positional data. This is an assumption that holds as the positional error obtained on the results is still considerably inferior to other methods.

Regardless on that, the experiments fulfilled so far showed that it was possible to overcome most of occlusions occurring during the game combining the herein proposed fractional approach with a single monocular camera. Nevertheless, the fractional coefficient still has a major influence on such accuracy. Therefore, we propose to further analyze this parameter and, at some extent, evaluate the stability of players.

In sum, this study fostered the use of fractional calculus to improve the accuracy of tracking systems, thus overcoming some of the misclassification problems that still occur in most traditional methods. Moreover, using this information one may benefit from online tactical metrics, thus helping coaches to understand how players behave in a collective way.

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