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Oscillations of difference equations with several deviated arguments

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Abstract. We establish sufficient conditions for the oscillation of all solutions to the retarded difference equation

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)x(\tau_i(n)) = 0, \quad n \geq 0,$$

and the (dual) advanced difference equation

$$\nabla x(n) - \sum_{i=1}^{m} p_i(n)x(\sigma_i(n)) = 0, \quad n \geq 1,$$

where $(p_i(n)), 1 \leq i \leq m$ are sequences of nonnegative real numbers, $(\tau_i(n))$, $1 \leq i \leq m$ are sequences of integers such that

$$\tau_i(n) \leq n - 1 \ \ \forall n \geq 0, \quad \text{and} \quad \lim_{n\to\infty} \tau_i(n) = \infty, \quad 1 \leq i \leq m,$$

$(\sigma_i(n)), 1 \leq i \leq m$ are sequences of integers such that

$$\sigma_i(n) \geq n + 1 \ \ \forall n \geq 1, \quad 1 \leq i \leq m,$$

Δ denotes the forward difference operator $\Delta x(n) = x(n+1) - x(n)$ and ∇ denotes the backward difference operator $\nabla x(n) = x(n) - x(n-1)$. Examples illustrating the results are also given.

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1. Introduction

In this paper we study the oscillation of all solutions of the difference equation with several variable retarded arguments of the form

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