

OSCILLATION RESULTS FOR ODD-ORDER NONLINEAR NEUTRAL DIFFERENTIAL EQUATIONS OF MIXED TYPE

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ABSTRACT. In this paper the authors establish some new comparison theorems and Philos-type criteria for oscillation of solutions to the odd order neutral mixed type differential equation

$$(x(t) + ax(t - \tau_1) + bx(t + \tau_2))^{(n)} + p(t)x^\alpha(t - \sigma_1) + q(t)x^\beta(t + \sigma_2) = 0, \quad t \geq t_0,$$

where $n \geq 3$ is an odd integer, $\alpha \geq 1$ and $\beta \geq 1$, are ratio of odd positive integers. Examples are provided to illustrate the main results.

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1. PRELIMINARIES

This paper is concerned with the oscillation and asymptotic behavior of solutions of odd order nonlinear neutral mixed type differential equation of the form

$$(1.1) \quad (x(t) + ax(t - \tau_1) + bx(t + \tau_2))^{(n)} + p(t)x^\alpha(t - \sigma_1) + q(t)x^\beta(t + \sigma_2) = 0, \quad t \geq t_0,$$

where $n \geq 3$ is an odd integer, $\alpha \geq 1$ and $\beta \geq 1$ are the ratios of odd positive integers, $p(t)$ and $q(t)$ are continuous and positive functions for all $t \geq t_0$, and $a, b, \tau_1, \tau_2, \sigma_1, \sigma_2$ are non-negative constants. We set $z(t) = x(t) + ax(t - \tau_1) + bx(t + \tau_2)$. By a solution of equation (1.1), we mean a function $x(t) \in C([T_x, \infty), \mathbb{R})$, $T_x \geq t_0$, which has the property $z(t) \in C^n([T_x, \infty), \mathbb{R})$ and satisfies equation (1.1) on $[T_x, \infty)$. We consider only those solutions $x(t)$ of equation (1.1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$. We assume that equation (1.1) possesses such a solution. A solution of equation (1.1) is called oscillatory if it has infinitely large zeros in $[T_x, \infty)$ and otherwise, it is said to be nonoscillatory. Equation (1.1) is said to be almost oscillatory if all its solutions are either oscillatory or convergent to zero asymptotically.