

NUMERICAL MODEL OF PROPELLER VORTEX WAKES FOR CALCULATION OF INDUCED VELOCITIES

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ABSTRACT

A numerical model for partial alignment of propeller vortex wakes with the flow is investigated. A lifting line representation of the propeller blades is used combined with the vortex lattice discretization method. The model takes into account radial contraction and axial variation of the pitch of the trailing vortices.

The convergence of the iterative pitch alignment procedure and the influence of the number of vortex elements is studied in the case of an optimum moderately loaded propeller. The effects of axial variation of pitch and contraction are investigated for the same optimum circulation distribution case.

The model is applied to the vortex wake of propeller DTRC 4119 and results are compared with experimental data.

1. INTRODUCTION

In aero and hydrodynamic applications the flow around lifting surfaces is characterised by the presence of vortex wakes trailing behind the lifting surface. Under the influence of the disturbance velocity field produced by the lifting surface, the vortex wake deforms when being convected downstream and the vorticity tends to become more concentrated. A well-known example of this phenomenon is the formation of concentrated vortex cores shed from wing tips.

In potential flow models for lifting surfaces the vortex wakes are modelled by infinitely thin vortex sheets, usually attached to the lifting surface sharp trailing edge. Since no pressure discontinuity may occur across the vortex sheets, the vortex lines on the vortex sheet surface must be aligned with the local flow velocity. In turn, the disturbance velocities induced by the vorticity distribution on the vortex sheets depend on the geometric configuration of the vortex lines on the surface. This fact, which is a consequence of the nonlinearity of the Euler's equation of motion in regions with vorticity, makes it difficult to determine the exact shape of the vortex sheets by solving the flow equations in the wake.

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A rather complete overview of the physical, mathematical and computational modelling aspects of this kind of vortical flows has been given by Hoeijmakers (1989).

Propeller vortex wakes follow a general helicoidal pattern resulting from the combined translation and rotation motion of the blades. In classical propeller lifting line theory, in which the propeller blades are represented by radial bound vortices of variable circulation distribution, a convenient distinction has been made regarding the shape of the vortex lines trailing behind the lifting line. For lightly loaded propellers the vortices are assumed to be helical lines of constant radius and pitch aligned with the undisturbed flow in a reference frame rotating with the blades; in the case of a propeller in a uniform flow, the vortex lines form a helicoidal surface of constant pitch. For moderately loaded propellers, Lerbs (1952) also assumed the vortex lines to be helices of constant radius and pitch, but the pitch of each vortex line is determined by aligning the vortex line with the disturbed flow velocity at the lifting line, neglecting radial velocities. For heavily loaded propellers the variation of radius or contraction and the variation of pitch in axial direction of the vortex lines has to be taken into account. The assumption of helical vortices with constant radius and pitch makes it possible to compute the velocities at the lifting line with the induction factor method (Lerbs, 1952). For more general vortex shapes this is no longer possible.

The use of numerical methods to discretize the vorticity distribution on the vortex sheets allows the calculation of the induced velocities with the law of Biot-Savart for more general shapes of the vortex sheets, provided the geometry of the vortex lines is known. A number of models have been used to specify the geometry of the trailing vortices in a propeller wake. Kerwin and Lee (1978) divided the propeller wake into two parts: a transition wake where the vortex sheet roll-up and the contraction occur and an ultimate wake region where the vorticity is concentrated on a set of concentrated helical vortices, one for each blade and a hub vortex. The wake is fully defined by four parameters that can be empirically specified by comparison with experimental observations. Greeley and Kerwin (1982) introduced a model in which the trailing vortices are assumed to lie on axisymmetric surfaces of varying radius with prescribed shape in the transition wake. In the ultimate wake the vorticity is assumed to be concentrated in a single hub vortex and a set of tip vortices. The axial variation of pitch in the transition wake is obtained by aligning each vortex with the local total axial and tangential velocity components at two downstream planes and using an interpolation formula for the pitch variation between them. The partial alignment achieved by this model requires an iterative procedure since the local velocities depend on the geometry of the vortex lines. A model with a similar specification of the geometry of the vortex lines has been used by Hoshino (1989), with the pitch on the two downstream stations being determined from experimental data. These models can be used in combination with the different representations of the propeller blades: lifting line, lifting surface (Kerwin and Lee, 1978 and Greeley and Kerwin, 1982) or the panel method (Hoshino, 1989).

More theoretically based methods have been proposed to attain a more complete alignment of the propeller blade vortex wakes with the flow. Early attempts using the discrete vortex model combined with the lifting line model were not very successful due to lack of computational resources or lack of convergence of the iterative schemes set to align the discrete vortices with the local flow velocity (see Cummings, 1976). More recent attempts by Moulijn and Kuiper (1995) appear to be successful in converging the iterative procedure for full vortex alignment for particular discretizations and moderate propeller loading. More recently, the complete vortex sheet roll-up problem for wings and propeller blades has been approached with the panel method by Pyo (1995). He obtained, in a number of cases, converged shapes of the vortex sheet exhibiting the typical roll-up behaviour at the edge. Still, non-smooth results of the wake shape are apparent in the tip vortex region.

The purpose of this paper is to investigate a numerical model of a propeller vortex wake to be used in the steady flow analysis of propellers with a boundary element method. To be compatible with the present implementation of the boundary element method (Falcão de Campos *et al*, 1996) a discrete vortex model is adopted. In particular, we would like to investigate the effects of contraction and axial variation of pitch of the vortex lines in the induced velocities at the propeller. To simplify the computations as much as possible, the semi-empirical model of Hoshino (1989) was selected for the wake geometry. In order to examine the convergence properties of alignment procedures, the pitch of the line vortices is aligned iteratively with the local flow velocity in two wake stations, similarly to the model of Greeley and Kerwin (1982). At this stage and to enable the validation of the numerical model, a simple lifting line representation of the propeller is retained. In this way, a comparison with the results of the induction factor method of Lerbs (1952) for a moderately loaded propeller can be made. The integration of the present wake model in the boundary element representation of the propeller will be dealt with later.

Section 2 describes the numerical model. In section 3, first the convergence of the numerical method is examined by comparison with the results of Lerbs for a moderately loaded optimum propeller. Next, the effects of axial pitch variation and contraction are studied for the same case. Finally, an example of the deformed vortex wake obtained with the present method for the circulation distribution of propeller DTRC 4119 at design condition is presented. Section 4 presents the conclusions.

2. NUMERICAL MODEL

2.1 Numerical Lifting Line Model

Consider a propeller of radius R (diameter $D = 2R$) with Z blades rotating with an angular speed Ω (rotational speed $n = 2\pi\Omega$) in a uniform inflow with velocity U . The fluid is assumed to be inviscid and incompressible. We introduce a cartesian coordinate system (x, y, z) , with unit vectors $(\bar{e}_x, \bar{e}_y, \bar{e}_z)$, and a cylindrical coordinate system (x, r, θ) , with unit vectors $(\bar{e}_x, \bar{e}_r, \bar{e}_\theta)$, rotating with the propeller blades. The coordinate system convention is shown in Fig. 1. In a rotating reference frame the flow is steady.

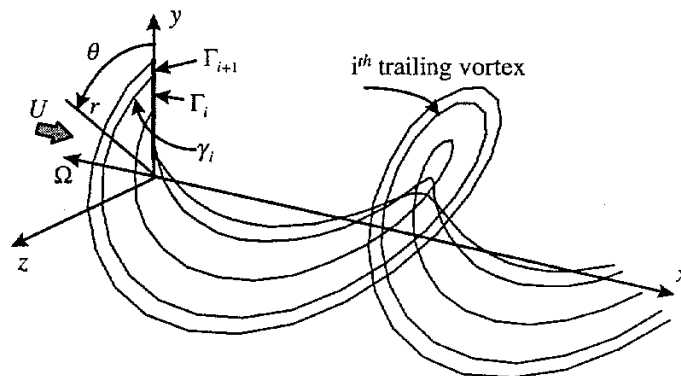


Fig. 1 - Numerical lifting line model: vortex lattice with trailing vortices. Coordinate system convention.

In the lifting line model each propeller blade is represented by a radial lifting line $\theta = \theta_k \equiv 2(k-1)\pi/Z$, $k = 1, \dots, Z$, with variable circulation distribution along the radius $\Gamma(r)$, extending from the hub radius r_h to the propeller radius R . In the numerical lifting line model (Kerwin *et al*, 1986) the continuous distribution of circulation is discretized by a lattice of M discrete straight-line vortex elements of constant strength (Fig. 1)

$$\Gamma(r) = \Gamma_i, \quad r_i < r < r_{i+1}, i = 1, \dots, M. \quad (1)$$

From each endpoint of the element a trailing vortex is shed with strength

$$\gamma_i = \Gamma_i - \Gamma_{i-1}, \quad i = 2, \dots, M, \quad \gamma_1 = \Gamma_1, \gamma_{M+1} = \Gamma_M \quad (2)$$

equal to the discontinuity of the circulation values of adjacent elements.

The velocity induced by the system of discrete vortices (1-2) for the k^{th} blade can be obtained by the law of Biot-Savart, summing the contributions of the vortex lattice elements on the lifting line and the trailing vortices in the form

$$\bar{v}_k(x, r, \theta) = -\frac{1}{4\pi} \left(\sum_{i=1}^M \Gamma_i \int_{r_i}^{r_{i+1}} \frac{\bar{S}_1 \times \bar{e}_{r_k}}{S_1^3} dr' + \sum_{i=1}^{M+1} \int_{l_{ik}} \frac{\bar{S} \times \bar{\gamma}_{ik}(x', r', \theta')}{S^3} dl \right), \quad (3)$$

where

$$\bar{e}_{r_k} = \cos \theta_k \bar{e}_y + \sin \theta_k \bar{e}_z \quad (4)$$

is the unit vector in the radial direction.

$$\bar{S}_1 = (x, y - y'_{ik}, z - z'_{ik}) = (x, r \cos \theta - r'_i \cos \theta_k, r \sin \theta - r'_i \sin \theta_k), \quad (5)$$

is the vector radius from the integration point $(0, y'_{ik}, z'_{ik}) \equiv (0, r'_i \cos \theta_k, r'_i \sin \theta_k)$ on the lifting line element to the field point (x, r, θ) ; $\bar{S}$ is the vector radius from the point (x', r', θ') on the trailing vortex to the field point (x, r, θ) , with cartesian components

$$\bar{S} = (x - x', y - y', z - z') = (x - x', r \cos \theta - r' \cos \theta', r \sin \theta - r' \sin \theta'). \quad (6)$$

$$S_1 = [x^2 + r^2 + r'^2 - 2rr' \cos(\theta - \theta_k)]^{\frac{1}{2}}, \quad (7)$$

$$S = [(x - x')^2 + r^2 + r'^2 - 2rr' \cos(\theta - \theta')]^{\frac{1}{2}}, \quad (8)$$

are the modules of $\bar{S}_1$ and $\bar{S}$, respectively. $\bar{\gamma}_{ik} = \gamma_{ik} \bar{e}_l$, is the trailing vortex strength, with $\bar{e}_l$ and $dl = (dx'^2 + dy'^2 + dz'^2)^{1/2}$ being the unit vector and the arc length along the trailing line vortex l_{ik} , respectively.

The total induced velocity is obtained by summing the contributions from all propeller blades

$$\bar{v}(x, r, \theta) = \sum_{k=1}^Z \bar{v}_k(x, r, \theta). \quad (9)$$

In the second integral of eq. (3) the integration has to be carried out along the trailing line vortex l_{ik} , which extends from the point $(0, r_i, \theta_k)$ on the lifting line to infinity. The integration requires the shape of the trailing line vortex to be known. The geometry of the trailing vortices can be given parametrically in the form

$$x = x^{(ik)}(\theta), r = r^{(ik)}(\theta), i = 1, \dots, M, k = 1, \dots, Z. \quad (10)$$

The numerical model used to determine the equations (10) iteratively for partial alignment with the local flow velocity is dealt with in the next section.

2.2 Wake Alignment Model

The propeller wake model considers two distinct regions, the transition wake region and the ultimate wake region as shown in Fig. 2. Variations of pitch and contraction of the trailing line vortices occur only in the transition wake region. The ultimate wake region preserves pitch and contraction constant with values identical to those of the ultimate station.

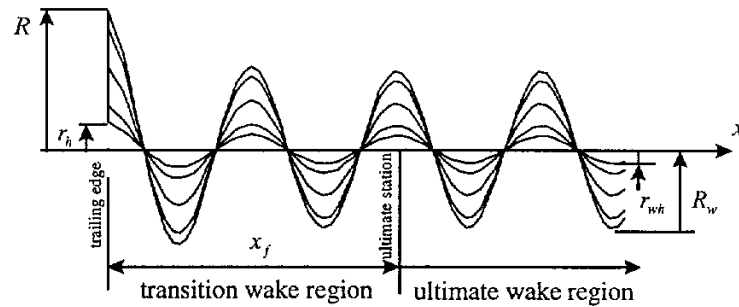


Fig. 2 - Propeller wake model.

The variation of the radius of the discrete vortices in the transition wake region is determined by:

$$r = r_{te} - (r_{te} - r_f) f(\xi), \quad (11)$$

where $f(\xi)$ is a polynomial function given by Hoshino (1989) as

$$f(\xi) = \sqrt{\xi} + 1.013\xi - 1.920\xi^2 + 1.288\xi^3 - 0.321\xi^4, \quad (12)$$

with

$$\xi = \frac{x^{(\gamma)} - x_{te}^{(\gamma)}}{x_f^{(\gamma)} - x_{te}^{(\gamma)}} \quad (13)$$

and $r_{te,f}$, $x_{te,f}$ representing, respectively, the radial and axial coordinates of the vortex at the trailing edge of the blade and at the ultimate station.

Hoshino defines the axial coordinate of the ultimate station to be fixed at $x_f = 2.0R$. The hub vortex radius at that station is kept constant, $r_{wh} = 0.1R$ and the tip vortex radius is given as a function of the slip ratio s , by $R_w/R = 0.887 - 0.125s$, where $s = 1 - J/p$, $J = U/nD$ is the advance coefficient and $p = P_{0.7R}/D$ is the pitch ratio at $r = 0.7R$.

For the purpose of application of the method to propeller blades the lifting line is assumed to coincide with the trailing edge of the blade. The radial positions of the discrete vortices at the trailing edge are determined by the cosine distribution between tip radius R and hub radius r_h . For the ultimate station the radial positions of the vortices are determined similarly between R_w and r_{wh}

$$r_{te,f} = \frac{1}{2}(R_w + r_{h,wh}) - \frac{1}{2}(R_w - r_{h,wh})\cos\frac{(j-1)\pi}{M}, \quad j = 1, \dots, M+1. \quad (14)$$

An iterative procedure is used to determine the pitch along the transition wake region. Calculations of the induced velocities at control points at the trailing edge and ultimate station are used to determine pitch at those stations

$$P_{te,f} = 2\pi r_{te,f} \frac{U + v_{a_{te,f}}}{\Omega r_{te,f} - v_{t_{te,f}}}, \quad (15)$$

where $P_{te,f}$ represents pitch at the trailing edge or ultimate station and $v_{a_{te,f}}$, $v_{t_{te,f}}$ are the axial and tangential velocities, respectively at the trailing edge and ultimate station. The axial variation of pitch between the trailing edge and the ultimate station is given by an expression similar to (11).

A new trailing wake vortex geometry can be defined by the following recursive formulas. In the initial step induced velocities are made zero and $r_1 = r_{te}$, $\theta_1 = \theta_{te}$, $x_1 = x_{te}$, $P_1 = P_{te}$. With N_L being the number of elements that discretize each vortex in the transition wake region, Δx defines the axial step

$$\Delta x = \frac{x_f}{N_L}. \quad (16)$$

The endpoint positions of the transition wake vortex elements can be determined, with $l = 2, \dots, N_L + 1$, by

$$x_l = x_{l-1} + \Delta x, \quad r_l = F_r(x_l), \quad P_l = F_p(x_l), \quad \theta_l = \theta_{l-1} + \frac{2\pi\Delta x}{\frac{P_l + P_{l-1}}{2}}, \quad (17)$$

where F_r and F_p are radius and pitch functions of the form (11-13).

This same recursive scheme is used to determine the endpoint positions of the ultimate wake vortex elements, but with $F_r(x_l) = r_f$, $F_p(x_l) = P_f$ constant and $l > N_L + 1$.

With the new trailing vortex geometry new induced velocities are determined from eq. (3) by discretizing the vortex with straight-line elements. The wake alignment iterative procedure is stopped when convergence in the pitch at the trailing edge and ultimate stations is achieved.

3. RESULTS

Results of the present numerical method for wake alignment are presented for the case of a moderately loaded optimum propeller and compared with the results of Lerbs (1952), induction factor method. This test case enables the validation of the numerical procedures of the method. Results are also presented for the case of the non-optimum circulation distribution of propeller DTRC 4119 studied by Jessup (1989).

3.1 Optimum Propeller

To test the wake alignment method we consider the case of an optimum propeller with 4 blades, $r_h = 0.2$, advance coefficient $J = 0.628$, power coefficient $C_P = P/(1/2\rho U^3\pi R^2) = 1.214$ and nondimensional circulation $G(r) = \Gamma(r)/\pi DU$ given as a function of radius in Table. 1. The results of the analysis of this propeller with the induction factor method including the induced velocities at the lifting line are presented by Lerbs (1952). For the linear helicoidal wake geometry of constant pitch the values of the induced velocities far downstream are twice the values of the induced velocities at the lifting line.

Table. 1 – Radial distribution of nondimensional circulation. Optimum propeller. $Z=4$, $J=0.628$, $C_P=1.214$, $r_h=0.2$.

r/R	0.2	0.254	0.4	0.6	0.8	0.946	1.0
G	0.0	0.01430	0.02543	0.03102	0.02986	0.01918	0.0

To test the wake alignment method and to investigate the effects of axial variation of pitch and of contraction, the induced velocities are compared with results of Lerbs at the lifting line and the theoretical results far downstream in the wake in the following cases:

- i) No wake alignment; constant pitch and no wake contraction;
- ii) Wake alignment only at the lifting line; constant pitch and no wake contraction;
- iii) Wake alignment only at the ultimate station; constant pitch and no wake contraction;
- iv) Wake alignment at the two stations; axial variation of pitch and no wake contraction;
- v) Wake alignment at the two stations; axial variation of pitch and wake contraction.

Case i) reproduces the case of Lerbs. Fig. 3 shows the numerical results for an increasing number of lifting line elements M at the lifting line and ultimate station and the comparison with the results of Lerbs at the lifting line and far downstream.

Good agreement with Lerbs' results is obtained at the lifting line with increasing lifting line element discretization. At the ultimate station the results of the tangential induced velocities are also in good agreement, but axial induced velocities stay below the theoretical values. This is due to the small value of the axial coordinate of the ultimate station. Fig. 4 shows that the axial induced velocities tend to the theoretical value when the ultimate station is moved downstream. It can be seen that for an axial position at a distance of $4R$ from the lifting line the results of the induced velocities are very close to the theoretical results far downstream. The convergence of the results with the number of elements used to discretize each trailing vortex line was examined and with $N_L = 1000$ results had already converged.

For case ii) the iterative alignment is introduced at the lifting line. The wake alignment model converged to the results of Lerbs at the lifting line and to the ones presented in the ultimate station in case i) of Fig. 3.

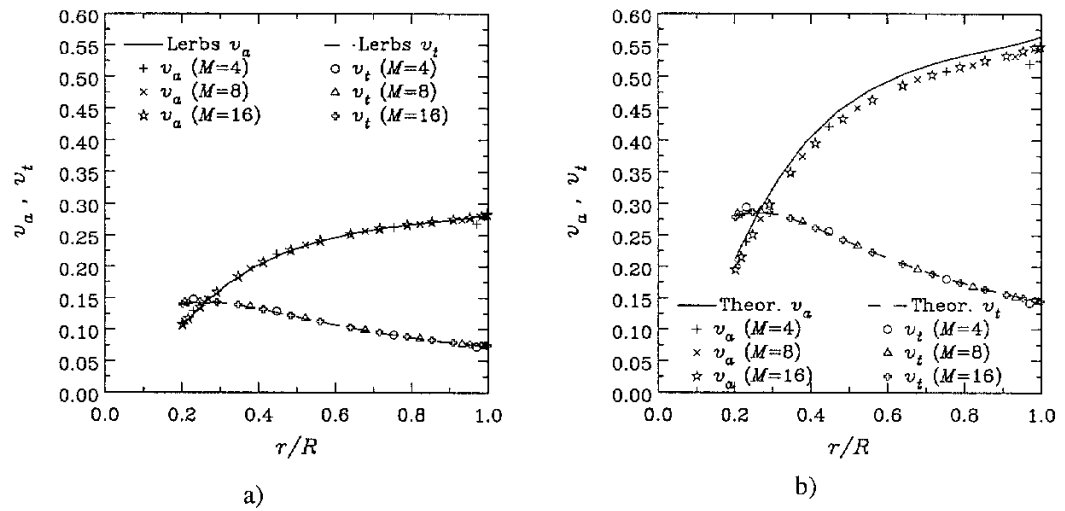


Fig. 3 - Calculated induced velocities with 4, 8 and 16 lifting line elements for case i). Comparison with Lerbs (1952) and theoretical results at a) the lifting line and b) the ultimate station.

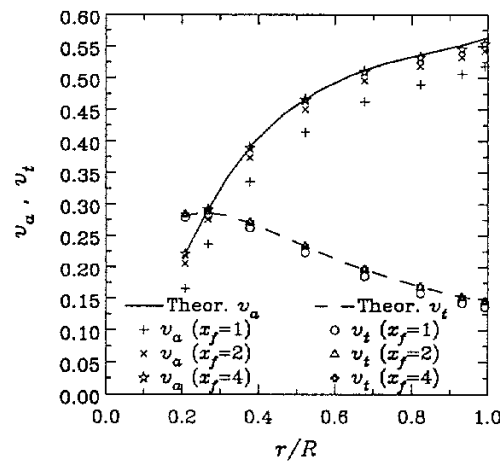


Fig. 4 - Effect of moving the ultimate station axial coordinate (x_f) downstream. Case i) with 8 lifting line elements.

Fig. 5 shows the results for the case iii) when iterative alignment is made at the ultimate station. The induced velocities were calculated at the lifting line and in the ultimate station for an increasing number of lifting line elements M .

Results did not converge with increasing number of lifting line elements. In fact, it was found that there is a maximum number of vortices for which the alignment model will converge. Greeley and Kerwin (1982) already drew this conclusion for their method. This is probably due to the strong singular behaviour of the induced velocities near the hub and tip when the discrete vortices become too close to each other.

Having this limitation in mind, the maximum number of lifting line elements for convergence was investigated and found to be 8 for case iv), with pitch alignment at the two stations and axial variation of pitch. Fig. 6 shows the converged results for this case after pitch alignment.

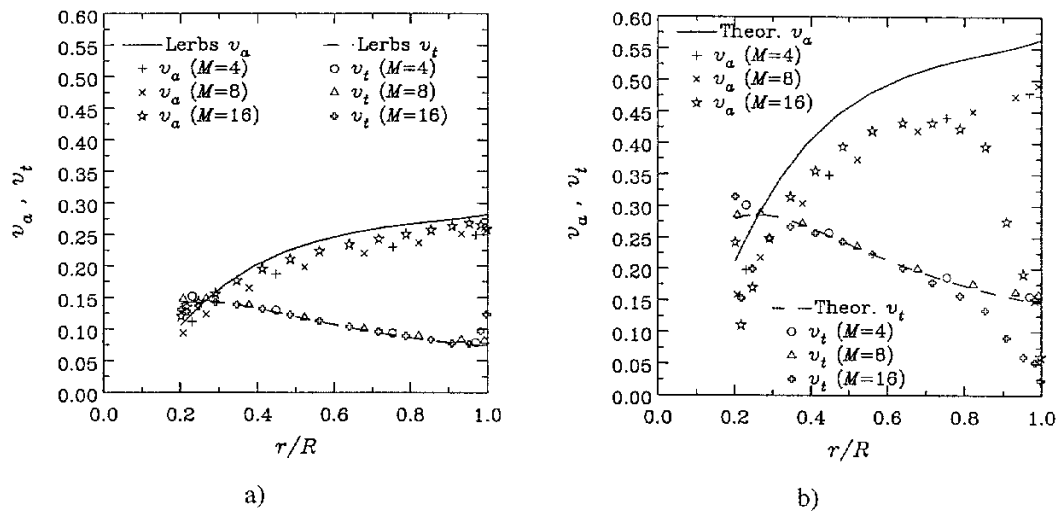


Fig. 5 – Calculated induced velocities with 4, 8 and 16 lifting line elements for alignment case iii). Comparison with Lerbs and theoretical results at a) the lifting line and b) the ultimate station.

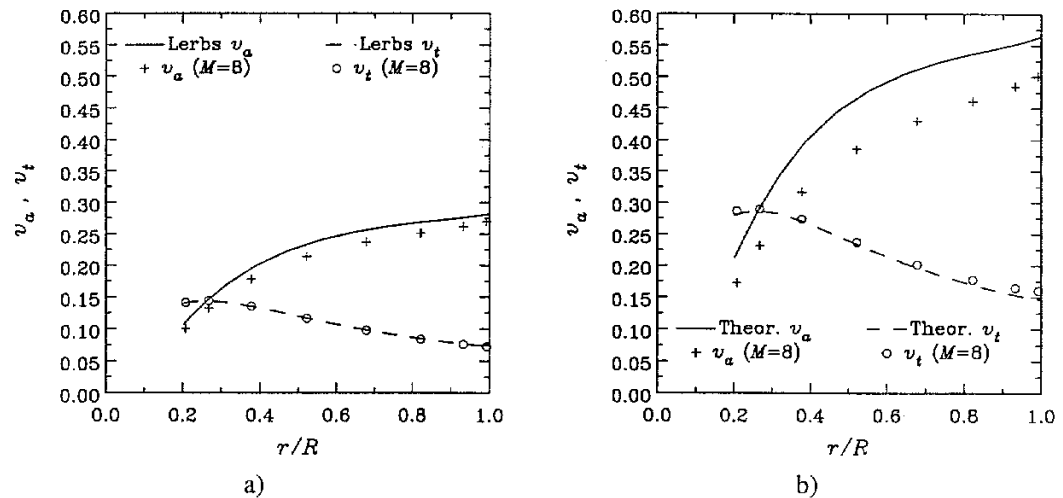


Fig. 6 – Calculated induced velocities with 8 lifting line elements and alignment case iv). Comparison with Lerbs and theoretical results at a) the lifting line and b) the ultimate station.

The tangential induced velocities in both stations are close to Lerbs' and theoretical results. Axial induced velocities converge to smaller values, especially in the ultimate station. This results in an aligned wake with increasing pitch from the trailing edge to the ultimate station. Comparison of this case with the results of Figs. 3 and 5 shows intermediate results between cases ii) and iii), as expected. The results are, however, closer to case iii), suggesting that the alignment at the ultimate station has the dominant effect.

For case v) the maximum number of lifting line elements to which converged results could be obtained was only 5. Results of the induced velocities for this case are presented in Fig. 7.

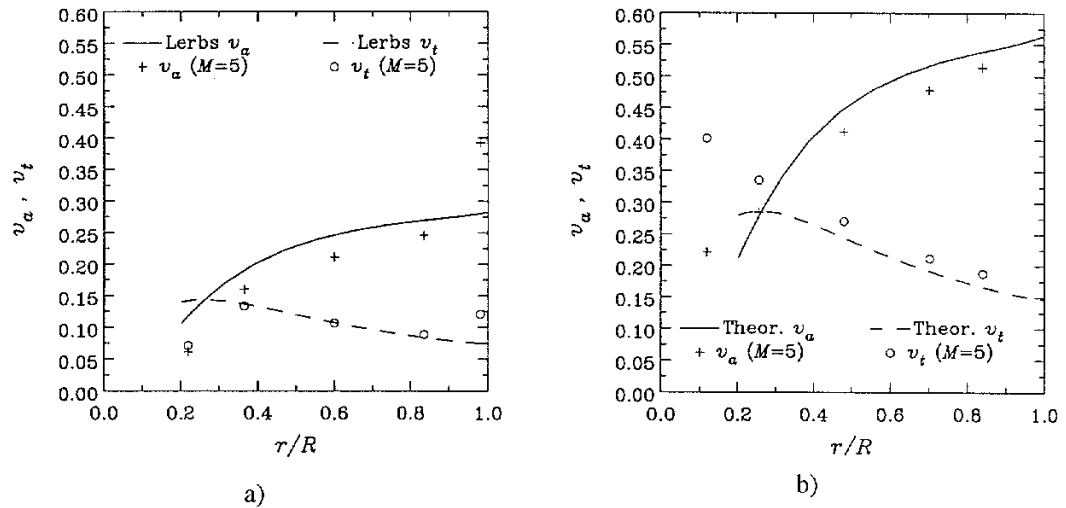


Fig. 7 - Calculated induced velocities with 5 lifting line elements and alignment case v). Comparison with Lerbs and theoretical results at a) the lifting line and b) the ultimate station.

It can be seen that introducing the wake contraction significantly changes the induced velocities, especially in the ultimate station where contraction is greater. However, the limiting number of 5 lifting line elements constitute a severe limitation to obtain accurate induced velocity results, as it can be seen in Fig. 3, which shows insufficient precision for this level of discretization.

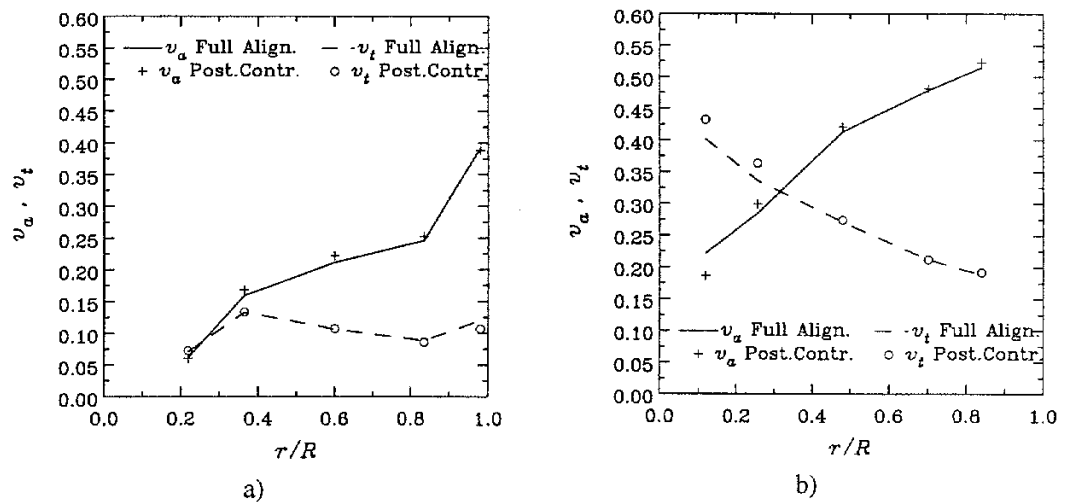


Fig. 8 - Calculated induced velocities with 5 lifting line elements at a) lifting line and b) ultimate station. Comparison of results obtained with full alignment, case v), and case iv) with contraction added *a posteriori*.

Fig. 8 compares the induced velocities at the trailing edge and ultimate station for the case where pitch alignment and contraction are carried out simultaneously in each iteration, with the case of performing only pitch alignment iteration (case iv) and adding contraction *a posteriori*. It can be seen that similar results are obtained at the lifting line with both methods. For the ultimate station a fair approximation is achieved especially for larger radii. We conclude that, for the present case, the wake pitch alignment and the contraction of the wake can be treated independently in the calculation of the induced velocities at the lifting line. This separation enables the use of greater discretizations of the lifting line and, hopefully, better results.

Fig. 9 compares a linear helicoidal wake and the wake obtained after pitch alignment (case iv) with 8 lifting line elements with contraction added *a posteriori*. Greater differences are noticeable downstream for smaller radii with a significant increase of pitch.

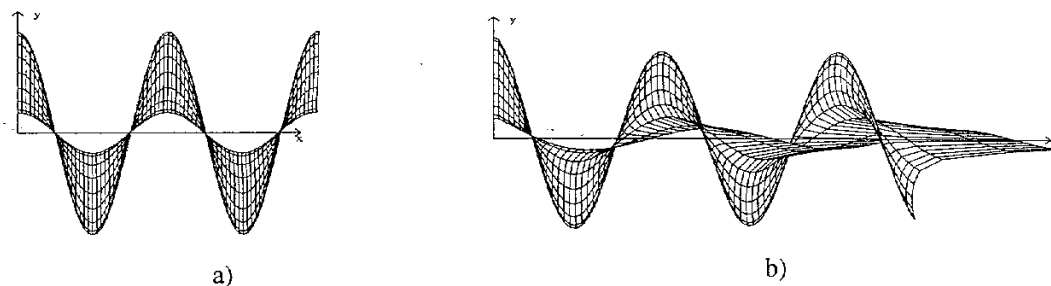


Fig. 9 – Comparison between a) linear helicoidal wake and b) the wake obtained after alignment with the present method.

3.2 DTRC 4119 Propeller

The wake alignment model was used in conjunction with a boundary element method for the analysis of marine propellers, Falcão de Campos *et al* (1996), to calculate the wake of the DTRC 4119 propeller at design condition. The propeller is described by Jessup (1989). It is a 3 bladed propeller with hub radius $r_h=0.2$ and experimental thrust loading coefficient $K_T = T / \rho n^2 D^4 = 0.146$ at the design advance coefficient $J=0.833$. Experimental results by Jessup for this propeller are available, and serve as a benchmark for the comparison between different existing numerical methods for propeller analysis.

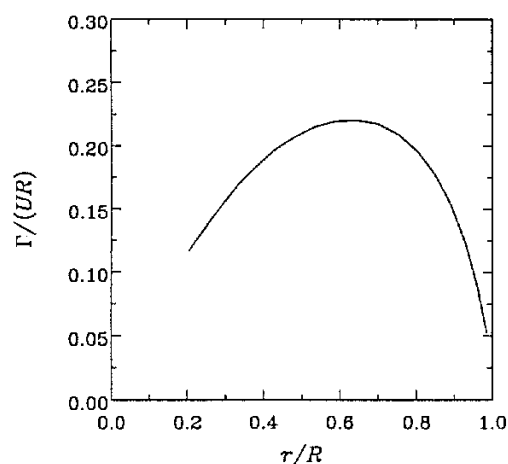


Fig. 10 – Circulation distribution predicted with a boundary element method. Propeller DTRC 4119, $J=0.833$.

The nondimensional circulation distribution computed by the boundary element method is presented in Fig. 10. The calculation was carried out with a linear helicoidal wake with the geometric pitch distribution and after iteration of the Kutta condition.

Fig. 11 compares the wake geometry obtained after convergence of the present wake alignment method with the linear helicoidal wake.

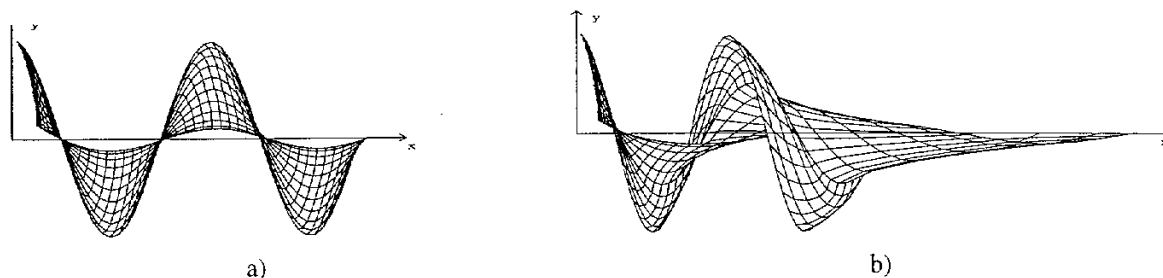


Fig. 11 - DTRC 4119 propeller wake, a) linear helicoidal wake and b) aligned wake.

Fig. 12 compares the pitch distribution at the trailing edge and at the ultimate station for both cases of linear wake and aligned wake. A significant increase of pitch is observed near the hub for the ultimate station. Near the tip a decrease in pitch is observed. These results are in qualitative agreement with experimental ones.

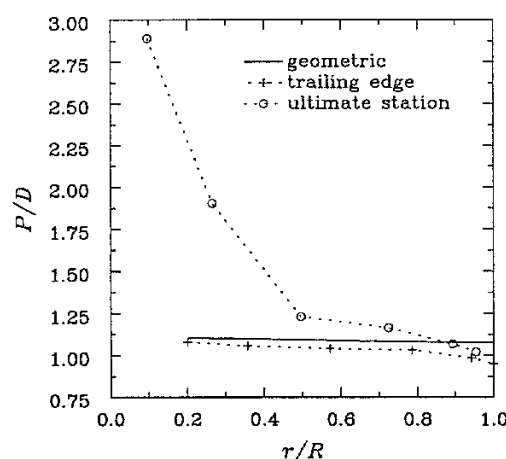


Fig. 12 - Pitch distributions at the trailing edge and at the ultimate station for propeller DTRC 4119, $J=0.833$. Linear wake is based on the geometric pitch for both stations.

Fig. 13 compares the numerical results for two stations in the transition wake region with the experimental data of Jessup (1989). Near the hub the pitch calculated by the numerical method overestimates the experimental results. At the station closest to the trailing edge the pitch increase tendency contradicts the experimental result. This is due to the high value of circulation near the hub (Fig. 10). For this circulation distribution the discrete hub vortex has a very strong singularity which causes high induced tangential velocities. An increase of pitch in the axial direction is expected. However, considering the pitch distribution at the trailing edge in Fig. 12, the large values found near the hub for the $x/R = 0.328$ station may be due to the rapid approach to the pitch distribution of the ultimate station given by the interpolation formulas.

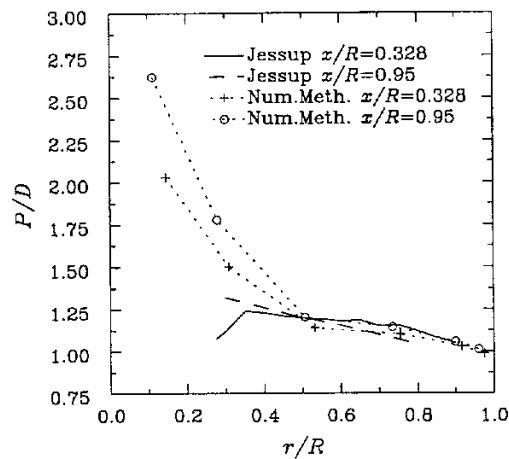


Fig. 13 - Pitch distributions at the stations $x/R = 0.328$ and $x/R = 0.95$ for propeller DTRC 4119, $J=0.833$. Comparison between numerical results and the experimental data of Jessup (1989).

4. CONCLUSIONS

From the investigation on the numerical method used to obtain partial alignment of propeller vortex wakes described in this paper, the following conclusions can be drawn:

- The method converges with the number of vortex elements on the lifting line only when the vortex wake alignment is carried out at the lifting line. For the case of an optimum moderately loaded propeller, with constant vortex pitch and without contraction, the converged results coincide with the results of Lerbs (1952) induction factor method.
- For the optimum moderately loaded propeller, the method does not converge with the number of vortex elements on the lifting line if the alignment is carried out at the ultimate station. In this case, there is a maximum number of vortex elements for which the iterative alignment converges. This conclusion is extensive to all cases where alignment at the ultimate station was required (with axial variation of pitch and with both axial variations of pitch and contraction).
- The effect of the vortex pitch on the induced velocities is rather large, as it may be seen from the comparison of the case with alignment at the lifting line and the case with alignment at the ultimate station. However, the effect of axial variation of vortex pitch without contraction is small if the comparison is made with the case where the alignment is carried out at the ultimate station.
- The effect of contraction on the induced velocities is large, especially near the hub. For the case with pitch alignment and contraction, the maximum number of vortex elements for convergent results reduces further. The difference on the induced velocities at the lifting line between the case of adding contraction *a posteriori* to a converged solution of vortex pitch alignment and the case of alignment with contraction on each iteration was found to be small.
- For the wake of propeller DTRC 4119 the present method is in fair agreement with the experimental data of Jessup (1989) for $r/R > 0.5$, but considerably overestimates the pitch close to the hub. The influence of the high finite circulation value at the hub radius predicted by the boundary element method needs to be further investigated.

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