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A SEMI-ANALYTICAL MODEL FOR THE NONLINEAR STABILITY ANALYSIS OF STIFFENED PLATES

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Abstract. *This paper presents a computational semi-analytical model for the post-buckling analysis and ultimate strength prediction of stiffened plates under longitudinal uniform compression. The possibility of considering fully free in-plane displacements at longitudinal edges (or unloaded edges) is the innovation of this model over existing models, where these displacements are assumed constrained to remain straight. Comparisons between the semi-analytical model results and nonlinear finite element model results are presented. This study shows that the developed semi-analytical model predicts with accuracy the elastic critical buckling stress, the buckling modes, the ultimate strength and the failure modes obtained through the nonlinear finite element method. The semi-analytical model enables versatile and comprehensive analysis of results and it is computationally efficient when compared with the nonlinear finite elements methods.*

1 INTRODUCTION

Recent work in the field of semi-analytical methods used for the nonlinear analysis and design of stiffened plates with buckling problems have shown an important and alternative tool that provide an efficient and understandable response [1,2].

Nowadays two computational programs use the semi-analytical method to analyse the post-buckling behaviour of plates and to predict their ultimate strength. The computer program ALPS/ULSAP [3], which uses a semi-analytical method previously known as the incremental Galerkin method [4] and the computer program PULS [5] developed at Det Norske Veritas (DNV) and accepted as general buckling code for ship and offshore platform structures as part of the DNV specifications [6].

The existing semi-analytical models are restricted to stiffened plates supported by rigid transverse and longitudinal girders. This type of arrangement is typical in ship, aircraft, tanks and offshore platform structures where it is assumed that the analysed stiffened plate is simply supported and the in-plane displacements perpendicular to the edges are constrained to remain straight in all edges. Generally, in bottom flanges of steel box girder bridges there are no neighbouring panels in the longitudinal edges to provide this kind of constraint and it is more conservative to consider the longitudinal edges with fully free in-plane displacements that are characteristic of edges free from stresses.

This paper presents a computational semi-analytical model for the post-buckling analysis and ultimate strength prediction of stiffened plates under longitudinal uniform compression. The possibility of considering fully free in-plane displacements at longitudinal edges (or unloaded edges) is the innovation of this model over existing models. Comparisons between the semi-analytical model results and nonlinear finite element model results are presented.

2 METHOD OF ANALYSIS

2.1 General

In order to validate the results obtained by the semi-analytical model that was developed, all nonlinear simulations were performed using the semi-analytical method and the finite element method.

The simulations with both methods were performed considering a yield stress (f_y) of 355 Nmm^{-2}, Young's modulus (E) of 2.1x10^5 Nmm^{-2} and Poisson's coefficient (ν) of 0.3.

The stiffened flanges were considered simply supported under longitudinal uniform compression (σ) with the following two cases for the in-plane displacement boundary conditions: in-plane displacements perpendicular to the edges constrained to remain straight in all edges (case CC) and in-plane displacements perpendicular to the edges constrained to remain straight at loaded edges and free at unloaded edges (case CF).

Unless otherwise indicated an equivalent geometric imperfection, including the geometric imperfections and residual stresses in accordance with European standard [7] was considered for the initial imperfection. Amplitude of 1/400 of the minimum between the length and width of the half-wave of the imperfection mode was considered for the equivalent geometric imperfection with a global imperfection mode shape. For a local imperfection mode shape the same principle was considered, with amplitude of 1/200.

2.2 Implementation of the semi-analytical method

The semi-analytical method uses the two nonlinear fourth order partial differential equations of the large deflection theory, the equilibrium and compatibility equations derived by von Kármán in 1910 [8] for perfect plates and extended to plates with initial imperfections by Marguerre [9]. The method is called semi-analytical because in a first step an analytical solution for the Airy stress function (F) is obtained solving the compatibility equation. Trigonometric series for the out-of-plane displacements (w) and for the initial imperfections (w_0) are adopted, which satisfy the boundary conditions.

In a second step approximate solutions for the unknown amplitudes (q) of the out-of-plane displacements are obtained solving the equilibrium equation using a variational method. Based on the approximate solutions for the unknown amplitudes of the out-of-plane displacements, the kinematic and constitutive relations and the yield criterion it is possible to analyse the post-buckling behaviour and to predict the ultimate strength of plates.

The computational implementation of the semi-analytical model developed in the framework of this study uses the Rayleigh-Ritz method to solve the equilibrium equation and it was implemented using the programming language of Mathematica6 [10]. The initial imperfections (w_0) and out-of-plane displacements (w) considered in the computational semi-analytical model, which satisfy the boundary conditions for simply supported plates, are given as follow

$$w_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{0,mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{1}$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{2}$$

where m is the number of half-waves in the longitudinal direction, n is the number of half-waves in the transverse direction, q_0 is the prescribed amplitudes of the initial imperfection, q is the unknown amplitudes of the out-of-plane displacement, a is the plate length, b is the plate width,

x is the longitudinal coordinate and y is the transverse coordinate. The xy plane coincides with the plate mid-surface. Figure 1 illustrates an overview of the semi-analytical model developed.

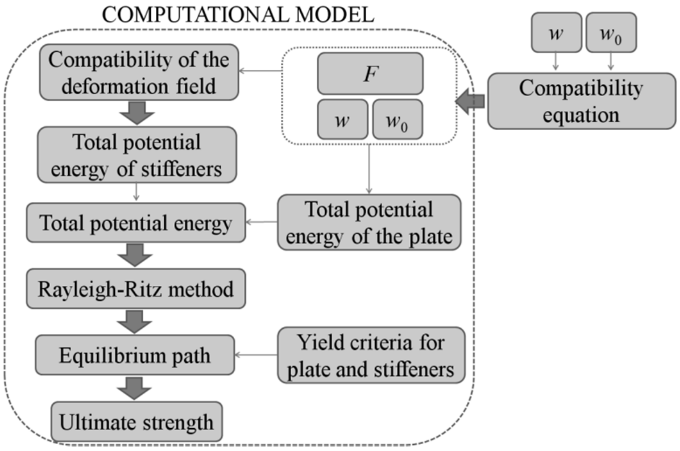

Figure 1: Overview of the semi-analytical model developed.

The stiffened plate analysis through this semi-analytical model is achieved considering the separate analyses of the plate and the stiffeners. Plate and stiffeners are modelled as an isotropic plate and as beam elements, respectively.

The compatibility equation for the isotropic plate used in the first step of the semi-analytical model to obtain the analytical solution for the Airy stress function (F) is expressed as

$$\nabla^4 F = E\left(w_{,xy}^2 - w_{,xx}w_{,yy} + 2w_{0,xy}w_{,xy} - w_{0,xx}w_{,yy} - w_{0,yy}w_{,xx}\right) \tag{3}$$

The stiffener deformations are defined through the compatibility of the deformation field between the isotropic plate and the beam elements considering the stiffener curvature (κ_{sl}) equal to the longitudinal plate curvature for bending ($-w_{,xx}$) and the stiffeners strain (ε_{sl}) equal the longitudinal plate membrane strain (ε_x^m) at their intersection. The strain of the stiffener is evaluated at transverse stiffener location (y_{sl}) and the general expression is given by

$$\varepsilon_{sl}(z) = \left[\varepsilon_x^m - (z - z_{st})w_{,xx}\right]_{y=y_{sl}} \tag{4}$$

where z is the height coordinate measured from the plate mid-surface, z_{st} is the geometric centre of the cross-section composed of the plate and the stiffeners.

The transverse (or lateral) displacements and the torsional rotation of the stiffeners are accounted for in the total potential energy of stiffeners. By compatibility the angle of twist (θ) and the transverse displacement at the shear centre of the single stiffener (v_{sc}) are defined as

$$\theta = w_{,y} \tag{5}$$

$$v_{sc} = -w_{,y}z_{sc} \tag{6}$$

where z_{sc} is the coordinate z of the shear centre of the single stiffener.

The local buckling effect of the stiffener elements is not considered in the response. This assumption is reasonable if the local buckling problems in the stiffener elements are prevented by satisfying minimum requirements for the stiffener geometric properties as established in the European standard [7]. These minimum requirements are used in the standards because the collapse of stiffened plates by local buckling of the stiffeners (lateral torsional buckling of the stiffener or local buckling of the stiffener elements) may exhibit a very unstable response.

This semi-analytical model enables to take into account the initial imperfections, cases CF and CC for the in-plane displacement boundary conditions, the interaction between local and global buckling, the geometric nonlinearity and in an approximate way, the material nonlinearity associated with the stiffness reduction after reaching the yield in the extreme fibre of the plate. The von Mises yield criterion was assumed for the plate and the stiffeners.

The plate yield criterion was defined for the sum of membrane and bending stresses at one-quarter of the plate thickness. This criterion allows for the formation of plasticity through the possibility of an additional strength after yielding in outer fibres in order to take into account in an approximate way for the material nonlinearity.

The stiffener yield criterion was defined by the yield stress, in tension or compression, at the location of the maximum stiffener stress. This criterion gives conservative ultimate strength predictions for global buckling cases when the stiffener extreme fibres are compressed.

The computational semi-analytical model developed has the following assumptions: the plate has a width b, length a and uniform thickness t, it is possible to consider one or more longitudinal stiffeners positioned symmetrically or eccentrically about the middle plane of the plate, the material is considered isotropic and linear elastic, characterized by Young's modulus E, Poisson's coefficient ν and yield stress f_y and the stiffened plate is subject to uniform compression σ in the longitudinal direction.

The complete definition of each step and the procedures of the semi-analytical model developed can be found in Ferreira [11].

2.3 Finite element modelling

The commercial program ADINA [12] was used to perform the nonlinear simulations by the finite element method. The program uses a large displacement/small strain with a Total Lagrangian formulation. The stiffened flanges were modelled using a four-node shell element for the plate and the stiffeners. This type of element allows for finite strains due to in-plane displacements (membrane) and out-of-plane displacements (bending). A collapse analysis was performed in the full range behaviour of the stiffened flanges, including the post-buckling regime and assuming a von Mises yield criterion.

The nodes of each stiffener at loaded edges were connected with rigid links in order to assure their planar rotation about the supported edge. For the plate edges with in-plane displacements constrained to remain straight it was used a constraint for all nodes of the edge to ensure a uniform in-plane displacement.

The material behaviour for all elements was modelled assuming elasto-plastic behaviour with linear strain hardening. A value of Young's modulus/100 was adopted for the yield branch, in accordance with European standard [7].

3 RESULTS AND DISCUSSIONS

To compare the results obtained by the semi-analytical model with the results obtained by the finite element program ADINA a stiffened flange with five longitudinal equally spaced symmetric stiffeners, relative cross-sectional area δ (cross-sectional area of the stiffeners without any contribution of the plate A_{sl} to the cross-sectional area of the plate (bt) ratio) of 0.5 and relative flexural stiffness γ (second moment of area of the stiffened plate I_{st} to the second moment of area for bending of the plate corrected by the effect of the Poisson coefficient ($bt^3/[12(1-\nu^2)]$) ratio) of 65 was considered. The stiffened flange has a plate aspect ratio ϕ (ratio between the length a and the width b of the plate) of 5, plate slenderness λ_{plt} (ratio between the width b and the thickness t of the plate) of 150 and panel slenderness λ_p (ratio between the width of the panels between stiffeners b_p and the thickness t of the plate) of 25.

The reasons for choosing this stiffened flange are due to the fact that it is possible to validate the present model for the simplest case of stiffened plates with symmetric stiffeners, slender plates have a considerable post-critical resistance, which enables to study the effect of boundary conditions for the in-plane displacements on post-buckling and the panels between the stiffeners do not have problems of local buckling which reduces the number unknown amplitudes in the semi-analytical model.

The q_{21} was the unknown amplitude for the out-of-plane displacements assumed to be non-zero in the semi-analytical model. Two cases of initial imperfections were considered in the analysis: (i) an ideally perfect flange (w_0=0) and (ii) an equivalent geometric imperfection with the shape of the critical buckling mode (m=2 and n=1) with an amplitude ($q_{0,21}$) of b/400.

Figure 2 presents the three dimensional view of the critical buckling mode obtained from the semi-analytical model.

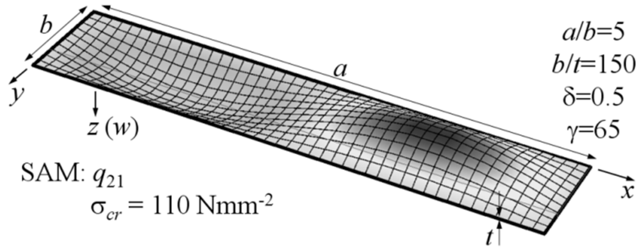

Figure 2: Three dimensional view of the critical buckling mode obtained from the semi-analytical model developed.

The elastic critical buckling stress (σ_{cr}) and the critical buckling mode obtained by the semi-analytical model illustrated in Figure 2 are in exact agreement with the theoretical analysis exposed in reference works [8,13]. Although Figure 2 presents an apparent mesh of elements the critical buckling mode of the semi-analytical was generated through a three-dimensional plot based on single unknown amplitude (q_{21}).

Figures 3-4 presents the comparison between the equilibrium paths obtained by the semi-analytical model and nonlinear finite element simulations for the stiffened flange considering the cases CC and CF for the in-plane displacement boundary conditions. Figure 3 shows the equilibrium path for the perfect flange and Figure 4 shows the equilibrium path for the imperfect flange. The equilibrium paths are presented in terms of normalized load (longitudinal compression σ to the yield stress f_y ratio) as a function of the normalized total deflection (ratio between the sum of out-of-plane displacement w and initial imperfection w_0 with the plate thickness t).

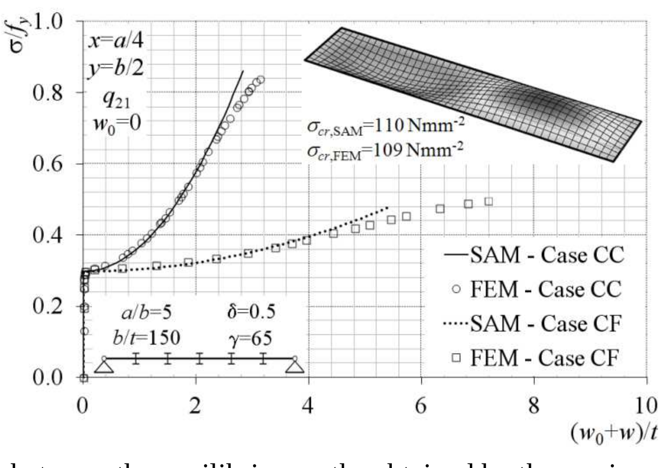

Figure 3: Comparison between the equilibrium paths obtained by the semi-analytical model (SAM) and nonlinear finite element simulations (FEM) for the perfect flange.

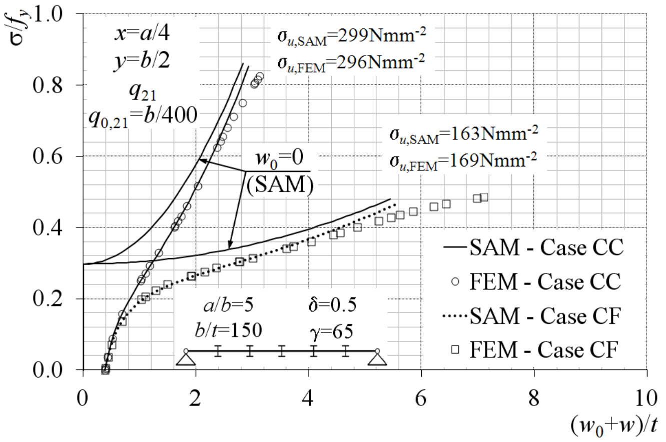

Figure 4: Comparison between the equilibrium paths obtained by the semi-analytical model (SAM) and nonlinear finite element simulations (FEM) for the imperfect flange.

The load-deflection responses and the ultimate strength (σ_u) obtained by the semi-analytical model shown in Figures 3-4 are in close agreement with the finite element results for both cases of modelling the in-plane displacements at unloaded edges.

The main differences between the results obtained with the semi-analytical model and the nonlinear finite element simulations occur in large out-of-plane displacements at the end of the load-deflection response. This can be explained by the facts that the nonlinear finite element simulations allow an increase in load and the progressive extension of plastification after local yielding of the stiffened plate, while in the semi-analytical model the plastification is accounted for through the yield criterion in an approximate way and the nonlinear finite element simulations use a large number of degrees of freedom, which allow to consider the interaction between displacement modes while in the semi-analytical model only single unknown amplitude for the out-of-plane displacements was used.

Nevertheless, the differences between the semi-analytical model and the nonlinear finite element simulations for the ultimate strength are less than 4% in the flange with fully free in-plane displacements at unloaded edges (case CF) and about 1% in the flange with uniform in-plane displacements at unloaded edges (case CC). These small differences can be reduced with an increase in the number of unknown amplitudes for the out-of-plane displacements.

Based on these results it can be verified that the post-buckling behaviour and the ultimate strength differs considerably according to the in-plane displacement boundary conditions at unloaded edges. In fact with the condition of fully free in-plane displacements at the unloaded edges, the transverse stresses are significantly reduced and consequently the stiffness of the plate also decreases. This point is particularly important for plates with higher post-critical resistance.

As expected, the elastic critical buckling stress (σ_{cr}) does not depend on the in-plane displacement boundary conditions at unloaded edges.

The evaluation and definition of the type of constraint for the in-plane displacements at unloaded edges of stiffened plates is a very important issue to estimate a correct value for the ultimate strength. In European standard [7] the ultimate strength of webs and internal flanges with longitudinal stiffeners used in steel box girder bridges is obtained through the same condition and this condition is independent of the in-plane displacement boundary conditions of the plate. Just apparently both elements may have the same in-plane displacement boundary conditions. Indeed, the webs are usually connected to flanges that possess sufficient rigidity to consider uniform the in-plane displacements at longitudinal edges, while the flanges are connected to webs that usually do not provide this type of constraint and it is more conservative to consider free the in-plane displacements at longitudinal edges, as shown in the work of Ferreira & Virtuoso [14] through a comparison between numerical and experimental results.

4 CONCLUSIONS

From this study the main conclusions can be summarised as follows: (i) the results obtained by the computational semi-analytical model are in close agreement with the nonlinear finite element results for both cases of modelling the in-plane displacements at unloaded edges, (ii) the semi-analytical model prove to have a clear potential to provide reasonably accurate solutions require only a short computer time, (iii) the boundary conditions for the in-plane displacements at unloaded edges significantly influence the post-buckling behaviour and the ultimate strength of plates and (iv) the design rules of stiffened plates cannot neglect the in-plane displacement boundary conditions of the plate.

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