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Early Years' Trainee Teachers' Geometric Knowledge: the case of defining a rectangle

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ABSTRACT
Teachers' knowledge is one crucial aspect on students learning. Such knowledge can be perceived in a wide range of perspectives, being one of the most prominent the conceptualization of the Mathematical Knowledge for Teaching, and its specificities linked with the labor of teaching. In mathematical activity, definitions can be perceive as essential as they are transversal to the different topics and they constitute a basic component of the mathematical structure. Being Geometry one of the areas in which both students and teachers reveal difficulties, and polygons, in particular rectangles, one core domain in which many other topics are sustained, improving students and teachers knowledge on definitions is of fundamental importance.
This paper discusses some problematic aspects on prospective teachers' knowledge on definitions, discussing the particular case of defining a rectangle. It intends to devise some insights on the reasons behind the main difficulties in order to allow conceptualize ways for improving training.

INTRODUCTION
Teachers' knowledge is assuming a growing importance as focus of research. Such importance is linked with the recognition that teachers have a greater impact than any other factor when it comes to student achievement (e.g., Nye, Konstantopoulos & Hedges, 2004), and also with the fact that

teachers' play a key role in students' learning at all educational levels (e.g., Rowan, Correnti & Miller, 2002). The case of elementary teachers' should deserve an even more special attention, as they are responsible for the introduction of basic but fundamental mathematics (Ma, 1999).

Research focusing on teachers' knowledge, on accessing and getting a deeper understanding of it and on developing conceptualizations to describe and analyzing is mostly grounded on Shulman's (1986) ideas concerning teachers' knowledge. One of such conceptualizations corresponds to the Mathematical Knowledge for Teaching – MKT (Ball, Thames & Phelps, 2008).

Literature shows that in all mathematic areas we can find some problematic aspects not only on students' knowledge, but also, and associated with teachers' knowledge (e.g., Gomes, 2012; Pinto, 2011). Geometry is one of such areas (e.g., Fujita & Jones, 2006), and, in particular definitions are perceived as a problematic topic (being the case of squares a particular example – e.g., Zazkis & Leikin, 2008). Although they are included in the Portuguese National Curriculum (Ponte *et al*., 2007), and it is considered important that students (since early years) have contact with them, teachers' training seem to have left definitions, their role and importance aside. The fact that students and teachers difficulties in defining is not a new problem that has been long addressed in research makes it even more problematic as it leads us to think on the (effective) impact of (such) research in training. As MKT for different topics is perceived has having different contents, as well as for different school levels (e.g., Jakobsen, Thames & Ribeiro, 2013), there is the need to effectively focus training in where it is most needed allowing to develop teachers' and students' knowledge (obviously at different levels). Being definitions transversal to all educational levels, it should be, thus, a core aspect to focus on.

This paper analyses and discusses early years' trainee teachers' knowledge while answering a questionnaire focusing on geometrical concepts, here in particular, on the definition of rectangle. With such an approach it's also aimed to develop a deeper understanding on the problematic aspects that would work as starting points for designing tasks specifically for teacher training that would allow developing their MKT and awareness of powerful situations for practice and on the possibilities to explore it.

THEORETICAL FRAMEWORK

Concerning teachers' knowledge, the work of Ball, Thames and Phelps (2008), with their refinement of Shulman's (1986) work, is perceived as one

of the most influential when addressing specifically the mathematical knowledge that teachers need to carry out the work of teaching – which they term as Mathematical Knowledge for Teaching. Such conceptualization considers Shulman's subject matter knowledge (SMK) and pedagogical matter knowledge (PCK) divided in three sub-domains each. Here we will focus only on the SMK and particularly on Common and Specialized Content Knowledge (CCK and SCK)[1]. The reason for our focus is linked with our aim and, necessarily with the fact that SMK is determinant in what is done in class, as it influences the development of other knowledge, namely, PCK (Baumert *et al*., 2010). The sub-domains are perceived as a relevant starting point for designing tasks for the mathematical preparation of teachers, and for doing research on what inputs to teachers' training and teachers' knowledge show effects on students and practices.

One of the contents of teachers' MKT (should) concern, necessarily, definitions. It seems unquestionable that definitions play a fundamental role in the teaching and learning of mathematics, thus, when intending allowing students a deeper understanding and knowledge on the mathematical topics and its connectedness, it's of core importance to approach definitions – both explicitly as well as implicitly. Definitions and teachers' knowledge on definitions are of fundamental importance as mathematical concepts are defined grounded on primitive terms and axioms (deductive theory). Thus it's important not only a knowledge on definitions but also (mainly, we say) a knowledge leading to learn to define, being such lack of knowledge one basic problem of mathematical education (Mariotti & Fischbein, 1997). It's thus fundamental that teachers have clear notions on definitions either as fundamental component of mathematical activity as well as part of the educative process (both as starting point as well as final product).

Such knowledge includes recognizing some basic principles (e.g., Poincaré, 1927; Gomes & Ralha, 2003), which a definition should comply, such as: provide the means to name objects; establish necessary and sufficient conditions for the object/concept (which differs from an extended list of properties and/or characteristics, and thus they should be minimal); and arbitrariness (different, but equivalent, definitions for one same object/concept). Arbitrariness in definitions is a core element of teachers' knowledge allowing exploring equivalent definitions for the same concept,

[1] The other sub-domain of the SCK accordingly with the MKT conceptualization concerns the Horizon Content Knowledge. For a discussion and examples focusing on HCK see, for example, Jakobsen, Thames, Ribeiro and Delaney (2012).

to keep its mathematical validity along schooling and allowing exploring the connections within one same topic and between topics – in the sense of HCK as connections as perceived by Fernández, Figueiras, Deulofeu and Martínez (2011). Also the fact that the choice of a definition has implications for the subsequent path to follow – in terms of its didactical approaches (need of accepting the logical consequences) must be a focus of attention concerning teachers' knowledge and awareness. One essential aspect to have into account when looking at definitions and on teachers and/or students knowledge on definitions concerns also the mismatch between what Tall and Vinner (1981) distinguish as concept image and concept definition. Concept image concerns the meaning/subjective interpretation each individual attributes to a concept, including mental figures as well as the properties and processes associated with the concept. On the other hand, concept definition is perceived as a set of words allowing specifying the concept. Thus, knowledge of definitions and of the process of defining is an essential element of teachers' subject matter knowledge (Zazkis & Leikin, 2008).

CONTEXT, DESIGN AND METHOD

This paper is part of a broader research project aiming at obtaining a deeper understanding on (early years' trainee) teachers' MKT, allowing designing tasks for developing such knowledge in geometry. In such a broader project we aim to identify, discuss and reflect upon some mathematical critical situations identified in (early years' trainee) teachers knowledge in different mathematical topics. This will provide an effective starting point to focus training on where it is most needed, and the conceptualization of tasks to promote the development of teachers' MKT. These tasks for teachers' training must be somehow different from tasks for pupils – not in nature but in focus – taking in consideration the specificities of teachers mathematical knowledge (e.g., Ribeiro, Mellone & Jakobsen, 2013).

In this paper we focus on early years' trainee teachers' knowledge on definitions and in particular on definitions of rectangle. Data was gathered in five different Higher Education Institutions in Portugal using a set of tasks, but for the purpose of this paper we consider only data from two of them (67 early years' trainee teachers). In one of the Institutions the tasks were discussed afterwards with the trainee teachers' as part of a course content – aimed at developing their MKT from discussing their difficulties – being such exploration recorded in audio and video. Although data was gathered in different Institutions, our aim here is not to make a comparative

study but rather to use the diversity of contexts as one more element contributing to a richer understanding of early years' trainee teachers' knowledge. It takes an instrumental case study approach (Stake, 2005) combining a qualitative and quantitative methodology. The quantitative approach serves mainly to identify the most critical situations where to focus on our analysis.

The questionnaire was build from a selection of questions of two different natures. Some of them were taken from the national examinations for pupils in years 4 and 6 (from different academic years) – in order to access and discuss trainee teachers' knowledge on the kind of tasks they are supposed to be teaching in some time (at the level of primary students CCK). Another set of questions was more focused on teachers' knowledge on definitions, justification and argumentation. Here we will address one of the questions focused on teachers' knowledge on definitions, and in particular concerning rectangles: give a mathematically valid definition of rectangle.

On the analysis and discussion we focus on the critical situations in terms of early years' trainee teachers' knowledge in answering this task, as they are perceived as an opportunity to learn, and a starting point for allowing teachers training to focus on where it is most needed (e.g., Ribeiro & Carrillo, 2011). We present a quantitative analysis of early years' trainee teachers' answers and afterwards we focus on discussing some particular aspects of such answers. Such an approach aims to allow a deeper understanding on the content of (their) CCK and SCK (whenever the case).

SOME PRELIMINARY RESULTS AND DISCUSSION

Concerning the question where early years' trainee teachers were asked to give a mathematically valid definition of rectangle, none of them reveal a knowledge allowing them to give such definition – in order to establish necessary and sufficient conditions for the object/concept, being minimal. Some of the trainee teachers assume as equivalent define and give a (exhaustive) list of characteristics, and in all of the answers the critical elements in such "pseudo-definitions" are sides and/or angles. One of the typical answers was: "*A rectangle is a figure with 4 sides, equal two by two*". By considering the rectangle the only quadrilateral with four sides they reveals a slightly limited knowledge on quadrilaterals in general, and in rectangles in particular.

One other cluster of answers is grounded on a disjoint classification of the quadrilaterals, considering squares and rectangles belonging to disjoint sets (Figure 1). This same conceptualization was also previously identified in

primary teachers' practices (e.g., Ribeiro *et al*, 2009), being one of reasons for the inclusion of this question, expressed in these terms, as one of the questions in the task.

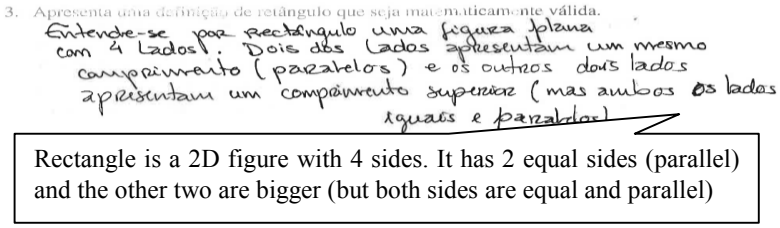

Figure 1: Rectangles and squares as part of disjoint sets

Although some of the trainee teachers consider also the (possible) role and importance of the angles for defining a rectangle, complementary to the sides – both explicitly or implicitly – in their perception (and thus part of their knowledge on rectangles), there is the need of mentioning that in order to be a rectangle a figure must have (four) equal right angles. The fact of naming it "a figure" does not clarify it they consider it as including or not the interior of the lines. Examples of answers belonging to this set are: "*It is a geometrical figure with four sides, equal two by two and with four right angles*"; "*rectangle is a figure with the sides corresponding to length equal, as well as the two corresponding to the width and the angles are right angles*".

Trainee teachers do not consider the need for including a limitation to the number of sides (only four – quadrilateral), and include more information than needed (revealing assuming that *the more information the more correct it is*)[2].

Some other trainee teachers use their concept image (Tall & Vinner, 1981) in order to try to build a definition (Figure 2). A description of such concept image is perceived as a definition itself, and it considers also rectangles and squares as part of disjoint sets.

[2] This was one of the comments while discussing the tasks with trainee teachers. Such discussion focused mainly on finding examples of possible different objects, polygons and figures represented from their "pseudo-definitions". None of the trainee teachers' took in consideration the importance of mention explicitly the fact that a rectangle is a quadrilateral – or in any other mathematically valid and equivalent way – or the differences between defining (minimal) and enunciating a (extensive) list of properties (Ribeiro, in preparation).

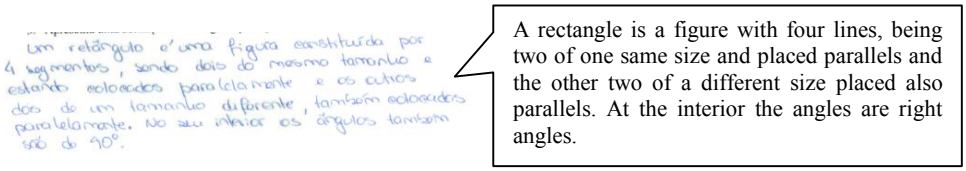

Figure 2: Definition as a description of the concept image

Such an approach to definitions, describing what seems to be their mental image, allows for several variations in what could be perceived, for example, as curve lines, or as a set of lines without vertices – there is no reference to the fact that the four segments must joint.

SOME FINAL COMMENTS

These results show evidence of different ways trainee teachers perceive definitions and in particular on how they consider what comprises to define a rectangle – one of the geometrical element they will have to teach, since kindergarten. These different ways are sustained on their knowledge on the mathematical concept they are to define and in on what is perceived as defining. Their knowledge is only at the CCK level, and even at such level they reveal some incoherencies, showing evidence of being less knowledgeable (on this specific aspect) than what is required (nowadays) to a student in early years. Such weak knowledge calls attention to the need for focusing our efforts on developing (trainee) teachers MKT on what is understood as a mathematic valid definition, also in order to amplify and bring together their concept image and concept definition. For such increase/development of teachers' MKT its essential that these trainee teachers experience similar kind of situations they are expected to allow their students (e.g., Magiera, van den Kieboom & Moyer, 2011), and in such the proposed tasks – nature, kind and focus – assume a major role. Such experiences can be grounded in identifying examples and non-examples of a concept, in relating concept, in problem solving and in proof (Vinner, 1991). In that sense, it's of fundamental importance that trainee teachers discuss situations they feel identified with (Tichá & Hošpesová, 2006) in order to allow its discussion in the first person, and increase their knowledge on the differences amongst defining and enumerating, on the need of considering quadrilateral, and on the possibility of defining rectangles having critical elements others than sides and/or angles (e.g., diagonals, symmetry axis).
Thus, these preliminary results call attention to a need of a change in focus of teachers training, in order to allow developing their knowledge specifically linked with the tasks of teaching. It's also evident the need for

further research on the mathematical reasons in which trainee teachers knowledge difficulties on definitions sustain – which correspond to one of the next steps of our research.

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