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on the History of Mathematics Education
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Maximum and minimum: Approaches to these concepts in Portuguese textbooks

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Abstract

The approach in Portuguese textbooks to the concepts of maximum and minimum has undergone alterations over the years. These changes were influenced by the official curriculum and more recently by the introduction of the graphic calculator in high schools. The concept of the derivative was first mentioned in the national curriculum in 1905 (Aires, 2006; Santiago, 2008). However, not until 1954, with the change of the program, the official curriculum began to make reference to the application of derivatives in the study of the variation of functions. In this study we will begin by examining the first national curriculum, which makes reference to the concepts of maximum and minimum of a function. We will, then, continue analyzing how the concept was shown in Portuguese textbooks, comparing two textbooks and identifying differences and similarities.

Introduction

The analysis of official programs and textbooks is an important part for researchers in Mathematics Education. According to González (2005) "For Mathematics Education researchers, textbooks are a privileged source of information because they can find information about the development of a specific content and the conceptual aspects, activities, problems, exercises sequenced in them" (p. 34).

In this communication we will analyze the textbooks' approach to the maximum and minimum concepts just as an application of the derivative concept.

We will start analyzing the first national curriculum that makes reference to maximum and minimum concepts. Then, we will present the analysis of two

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textbooks one from 1955 and another one from 1958. Both of these textbooks were edited in the period of unique book[1] regime, during which the Ministry of Education selected a textbook that must be used in all secondary schools in Portugal. For each one we will explain where and how maximum and minimum concepts appear, and the definitions and examples/applications they show.

Method and theory

According to Schubring (2005) analyzing official programs is a mean to understand the aim of a group in the educational community or the ministry policy; however, by analyzing textbooks we can see the teaching reality because they determine it decisively.

As historical researchers, we use the method defined by Ruiz Berrio (1997), which consists of four phases: heuristic, criticism, hermeneutic and exposition. The heuristic phase consists of searching and collecting documents essential for the research. After the selection of documents we proceed to its criticism analysis, with intern criticism (reliability) and extern criticism (authenticity). In the hermeneutic phase the interpretation of the data is done and, finally, in the exposition phase the description of the results are done.

The official programs references to maximum and minimum

The derivative concept was introduced in Portugal official programs in 1905 (Aires & Santiago, 2014). It made part of the 7th year from *liceu*[2] (the last year before University studies) and was included in the algebra chapter. Let us see the reference in the official program to the derivative:

> Derivative notion; its geometrical interpretation. Derivative of a sum, product, quotient, potency, square. Derivative of circular functions. Revisions. (Decreto-Lei[3] Nº 3 from 03/11/1905)

As we can see, this program did not make any reference to maximum and minimum concepts; it just referred to the derivative notion, its geometrical interpretation and the derivative rules.

Between 1905 and 1950, except between 1936 and 1948, the derivative concept made part of 6th or 7th year of the *liceu* studies (15 and 17 years old

[1] Translation of "Livro Único"
[2] High school
[3] Law

students) and was included in an algebra chapter or a calculus chapter inside the textbooks.

The applications of the derivative only appeared in national curriculum in 1954, as part of the 6th year of *liceu* in an algebra chapter. Let us see the reference in the official program of the derivative:

> Derivative of a function at a point; derivative function. Derivative of algebraic functions. Application to the study of variation in simple cases. (Decreto-Lei Nº39 807 from 07/09/1954)

We can verify that this program, from 1954, deepens the study about the derivative concept, considering not only the derivative notion and its rules; but also the application to functions variation in simple cases.

As in this period we had the Unique Book regime, in the end of this official program we found out information about the textbook: "*Compêndio de Álgebra, num volume"*.

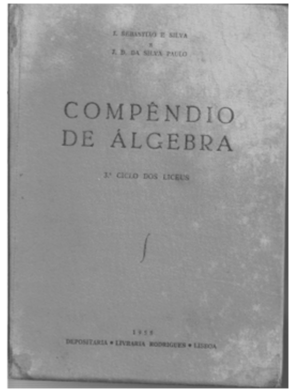
Textbooks approach to the concept of maximum and minimum

According to Aires and Santiago (2011), in 1947 the Unique Book regime started over again and in 1950 was approved as Mathematics textbook "Compêndio de Álgebra" from António Augusto Lopes (Diário de Governo nº 145, II Série, from 24/06/1950). The law foresaw that in 1954 another competition should choose over again a textbook. In this competition António Augusto Lopes' textbook and two other textbooks were selected. This competition had some problems and in 1955 a new competition was opened with another textbook by J. D. da Silva Paulo (as one of the authors of this textbook) competing. This was the person that should choose the textbook and so this was the chosen textbook. However, it was only chosen in 1958 (Almeida, 2014).

In the following we present an analysis of both textbooks: the first one from António Augusto Lopes "*Compêndio de Álgebra para o 6º ano dos liceus"* published in 1955 from Porto Editora in Porto (Fig. 1), and the second one from J. Sebastião e Silva e J. D. da Silva Paulo "*Compêndio de Álgebrapara o 3º ciclo dos liceus"* published in 1958 from Livraria Rodrigues in Lisbon and approved as Unique Book in 22/1/1958 (Fig. 2).

Figure 1: Book cover from 1955

Figure 2: Book cover from 1958



Both of these books have hard covers. The first one is just for the 6th grade of *liceu* and the second one is for 6th and 7th grade of *liceu*.

In the textbook from António Augusto Lopes, we can see that it has seven chapters and the 5th is dedicated to the derivative of functions with one variable and functions variation. This chapter has three parts, the first one entitled "*Definition*", the second one entitled "*Derivative of algebraic functions*" and the last one entitled "*Functions variation*".

It is in the last one that we find the maximum and minimum concepts. This part starts explaining the relation between derivative and direction of variation:

> I. *If a function is increasing in (a, b), it's derivative is positive or null.*
>
> II. *If a function is decreasing in (a, b), it's derivative is negative or null.*
>
> III. *If a function is constant in (a, b), it's derivative is null for all values of x in this interval.*

And reciprocally:

> I. *If the derivative is positive for all values of x in (a, b), the function is increasing in this interval.*
>
> II. *If the derivative is negative for all values of x in (a, b), the function is decreasing in this interval.*
>
> III. *If the derivative is null for all values in (a, b), the function is constant in this interval.* (p. 169–171)

The first three were followed by the explanation and a graphical representation. In the following we present the explanation from the first one:

Let x_0 *and* $x_0 + h$ *be two values of* x, y_0 *and* $y_0 + k$ *the corresponding values of the function. As the function is increasing, k e h has the same sign;* $\frac{k}{h}$ *is positive: when h tends to zero, the limit of* $\frac{k}{h}$ *exists, for hypothesis, and can't be neither thing except positive nor null.*

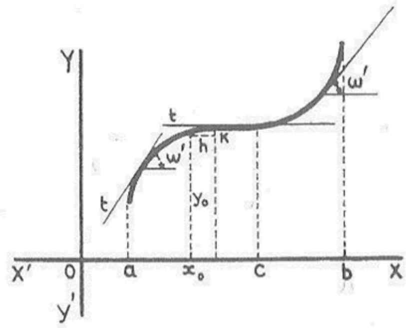


Fig. 52

So the function $y = f(x)$ *represented on fig. 52 is increasing in (a, b); the image has an unique tangent in each one of its points: the gradient of the tangent, that is to say, the derivative, is positive (because h and k has the same signal), for all of the values of x in (a, b), except for x=c; the tangent is parallel to X'X.* (p. 169)

After that, the author presents "*Local maximum and minimum (elementary notion)"*. In this part, the author starts establishing a relation between the signal of the derivative and the monotony of the function, then the definitions of the maximum and minimum concepts are presented. The definition of maximum in this book is:

Referring the neighbourhood of the value x = c, we can recognize that the function is increasing on the left side of c, in other words, in an interval (c-h, c) and decreasing on the right side of c, in other words, in an interval (c, c+h). The value of the function for x = c, is bigger than its values for x = c-h and x = c+h:

$$f(c) > f(c-h); f(c) > f(c+h)$$

We can say that the function has a maximum f(c), in x = c.(p. 171)

This definition relates the monotony of the function and the extreme of the function.

After that, the author presents the relation between extremes of the function and the signal of the derivative:

For the values c and c', were the function has maximums or minimums, the derivative is null changing the signal:

a) If it's a maximum, the function moves from increasing to decreasing and, for that, the derivative changes from positive to negative;

b) If it's a minimum, the function moves from decreasing to increasing and, for that, the derivative changes from negative to positive;

It's indispensable the change of signal; if it doesn't happened, the sense of variation of the function doesn't change (p. 172)

The author notes that the maximum or minimum is the biggest/lowest value in an interval and a maximum could be smaller than a minimum. This note is illustrated with a graphical representation of a function *f(x)* in an interval $[a, b]$where the maximum *f(c)* is smaller than the minimum *f(c')* (Fig. 3).

Figure 3: Graph representation

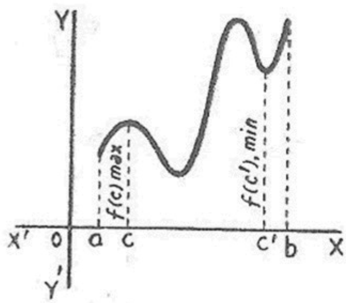


Fig. 56

Then, we find examples of the study of the variation in two different kinds of functions: linear and quadratic. In each example the first step to find the extreme is to calculate the domain, the second step is to differentiate the function, the third step is to make the study of the signal of the derivative and, finally, the conclusion relates the signal of the derivative with the monotony of the function. Those examples are accompanied by the graphical representation of the function, as we can see in the next excerpt:

Study the variation of the function $y = x^2 - 3x + 2$.

1. *Field of existence: (- ∞, + ∞).The function is continuous for all the real values of x.*
2. *Derivative function:* $y' = 2x - 3$. *It's a continuous function in x. Writing it as*

$$y^{\wedge\prime} = 2(x - 3/2)$$

we can conclude: it is negative, null and positive depending on

$x < \frac{3}{2}; x = \frac{3}{2}; x > \frac{3}{2}$; *So that:*

1^{st}. *For* $x = \frac{3}{2}$, *the function admits a minimum* $f\left(\frac{3}{2}\right) = -\frac{1}{4}$, *because, when x changes from* $\frac{3}{2} - h$ *to* $\frac{3}{2} + h$, *the derivative changes from negative to positive;*

2^{nd}. *As the derivative is null just for* $x = \frac{3}{2}$, *the function is decreasing in the interval* $\left(-\infty; \frac{3}{2}\right)$ *and is increasing in the interval* $\left(\frac{3}{2}; +\infty\right)$.

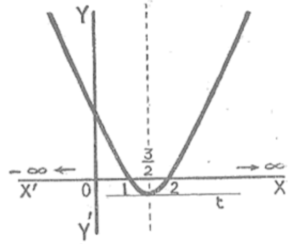


Fig. 59

> *3rd. The geometrical representation of the function (fig. 59) shows these conclusions and let us know the signal of y when x goes from* $-\infty$ *to* $+\infty$*. Particulary, the function is null when* $x=1$ *and* $x=2$*.* (p. 174–175)

After the examples the author explains that in this year it is just considered the study of the variation of a function in the simplest cases: linear functions and quadratic functions, but in the 7th year it will be considered functions with a 2nd grade polynomial as the derivative.

At the end of this chapter, as well as in the other chapters, we find eight exercises and the bibliography:

> *V. Herbiet, Compléments d'Algébre, Namur, 1947;*
> *Cours d´Algébre (par une réunion de professeurs), Paris, 1936.*

In this bibliography there are clear French and Belgian influences.

The other textbook by J. Sebastião e Silva and J. D. da Silva Paulo "*Compêndio de Álgebrapara o 3º ciclo dos liceus"* was published in 1958 from Livraria Rodrigues in Lisbon and was approved as a unique book in 22/1/1958.

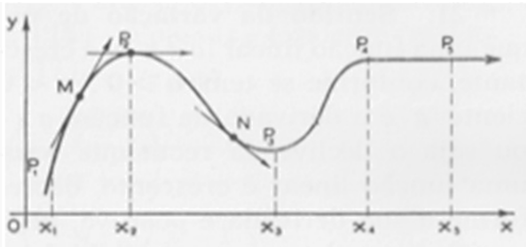
This textbook has twenty-two chapters: nine for the 6th grade of *liceu* and thirteen for the 7th grade of *liceu*. In 6th grade part, the 7th chapter is dedicated to the derivative. This chapter has four parts, the first one entitled "*Introduction*", the second one entitled "*Derivative concept*", the third one "*Derivative rules*" and the last one entitled "*Derivative Applications*". After that we find exercises and a historical note.

The part called "*Derivative Applications*" starts explaining the direction of variation of a function, establishing the relation between the signal of the derivative and the direction of variation:

> *I. If the derivative is positive in all point of an interval, the function is increasing in this interval.*
>
> *II. If the derivative is negative in all point of an interval, the function is decreasing in this interval.*
>
> *III. If the derivative is null in all point of an interval, the function is constant in this interval.* (p. 225)

This is followed by the explanation with a graphical representation (fig. 4).

Figure 4: Graph representation



Then begins a part called "*Application of the theorems stated*" that starts explaining how to make the table of monotony with these following two rules and its explanation:

> *Point where the derivative of f(x) change signal, passing from positive to negative, is a point of maximum relative of f(x) (supposing the function continuous in these point).*
>
> *Point where the derivative of f(x) change signal, passing from negative to positive, is a point of minimum relative of f(x).* (p. 227)

These rules are followed by some examples: quadratic function, cubic function and homographic function.

The author solves some examples in the same way: first he calculates the derivative and its zeros, then he explains the signal of the derivative before and after each zero and builds the table of monotony and finally he establishes the conclusion that the zeros are a maximum or a minimum point. After that the authors suggest the student to draw the graph of the function in an interval.

Let be the cubic function:

$$f(x) = x^3 - 12x + 7$$

we have:

$$f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x - 2)(x + 2)$$

The function $f(x)$ *is null at the points 2 and -2. And:*

for x<-2, x-2<0, x+2<0, so f'(x)>0;

for -2<x<2, x-2>0, x+2<0, so f'(x)<0;

for x>2, x-2>0, x+2>0, so f'(x)>0;

We have the table:

	$-\infty$		-2		2		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$		$\nearrow$	23 máx	$\searrow$	-9 mín	$\nearrow$	

So, at the point -2, the function has a relative maximum, equal to 23, and, at the point 2, a relative minimum, equal to -9.

We recommend the student to draw the graph of this function in the interval $[-5, 4]$*, taking as unit 1 cm in the x axis and 1 mm in the y axis.* (pp. 228–229)

In the part called "*Concrete Applications*" the authors refers that this theory is applied in various questions (geometry, physics, etc) and two examples were presented:

1. *Among the triangles rectangles that the hypotenuse measures 6 cm, determine those with maximum area.*(p. 230)

2. *It is intended to construct a cylindrical boiler, closed, with volume V, so that total area was minimum. Determinate the radius of the basis, r, and the height, h, of the boiler with that conditions.* (p. 231)

Finally the authors set a point about "*Successive derivatives"* about the second and third derivative, relating it with physics and with inflection points, with exercises followed by their solutions and, after that an historical note.

Conclusion

Comparing both textbooks we can identify some differences and similarities. In both textbooks we can find the relation between the signal of the derivative and the monotony of the function and the definition of maximum and minimum, however, the textbook from 1955 presents the results in a more descriptive way.

Both textbooks show graphs of functions with the tangent line to some points. The textbook from 1955 has three graphics: one for an increasing function, one for a decreasing function and one for a constant function. The textbook from 1958 shows just one graph with a part that increases, another that decreases and another where it is constant. However, the explanation of maximum and minimum definitions is accompanied by a graph only in the textbook from 1955.