Optimal Design of Thin-Walled Laminated Beams with Geometrically Nonlinear Behaviour

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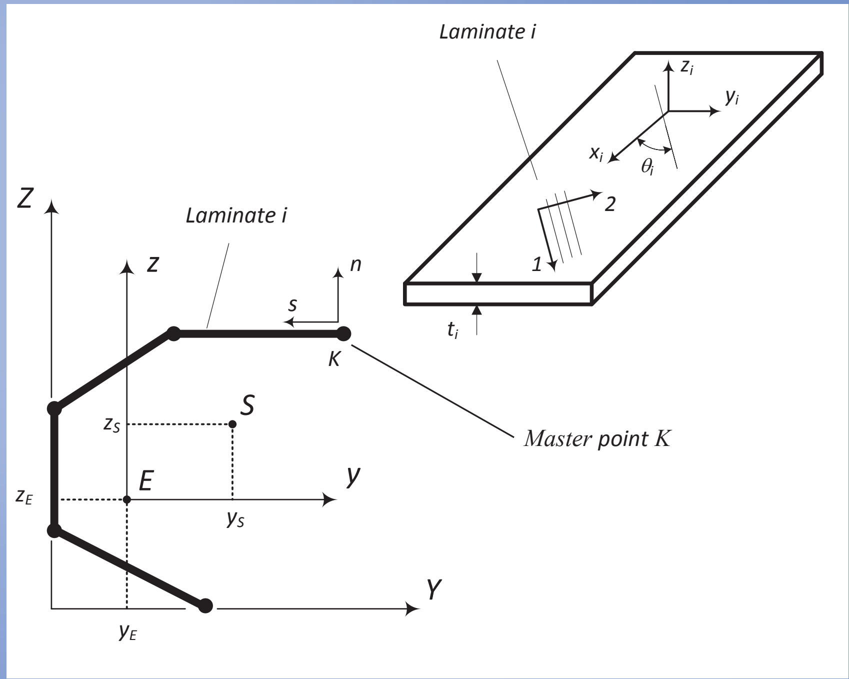
• *Objective*

- ***Finite element model for design optimization of composite laminated thin-walled beam structures with geometrically nonlinear behaviour accounting warping and post-critical behaviour;***

- ***Thin flat panels*** *corresponding to a* ***symmetric laminates;***
- ***Eight node quadratic isoparametric finite elements*** *in order to determine its bending-torsion properties;*
- *The structural response is obtained with basis on the* ***general thin-walled beam*** *theory;*
- *For sensitivity analysis the cross-section is modeled throughout* ***design elements that geometrically coincide with the laminates;***
  - ***Adjoint variable method***
  - ***First forward finite difference method***



- *Design optimization performed throughout* ***nonlinear programming techniques;***
- ***Lamina orientation*** *and* ***cross-section global geometry*** *are considered as design variables.*

• *Bending-torsion cross-section properties*

- ***Eight node quadratic isoparametric finite elements*** *in order to determine its bending-torsion properties;*

- *Are given as* ***integrals based on the cross-section geometry****, on the* ***warping functions*** *and on the* ***individual stiffness of the panels****.*

$$\overline{EA} = \int E\, dA \qquad \overline{ES}_y = \int E\, z\, dA \qquad \overline{ES}_z = \int E\, y\, dA \qquad dA = dy\, dz$$

$$\overline{EI}_{yy} = \int E\, z^2\, dA \qquad \overline{EI}_{yz} = \int E\, yz\, dA \qquad \overline{EI}_{zz} = \int E\, y^2\, dA$$

$$\overline{GA} = \int G\, dA \qquad \overline{GS}_y = \int G\, z\, dA \qquad \overline{GS}_z = \int G\, y\, dA$$

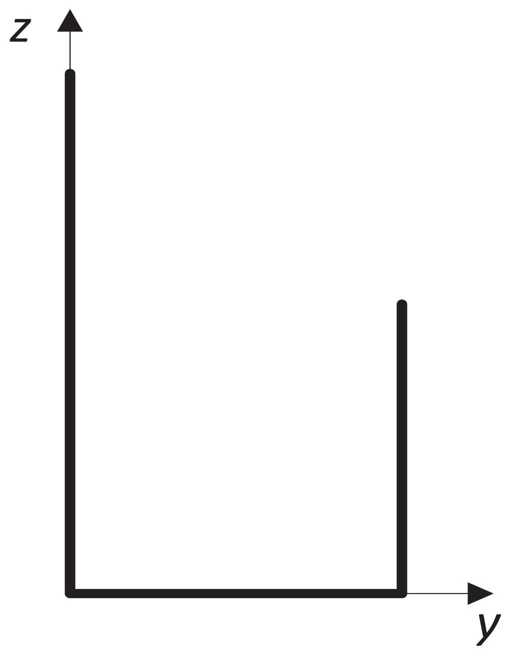
$$\overline{GI}_{yy} = \int G\, z^2\, dA \qquad \overline{GI}_{zz} = \int G\, y^2\, dA \qquad \overline{EJ}_{\phi} = \int E\, \phi\, dA$$

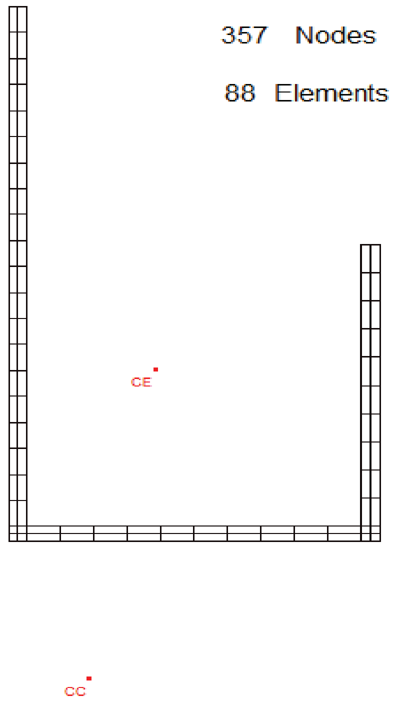
$$\overline{EJ}_{y\phi} = \int E\, y\phi\, dA \qquad \overline{EJ}_{z\phi} = \int E\, z\phi\, dA \qquad \overline{EJ}_{\phi\phi} = \int E\, \phi^2\, dA$$

$$\overline{GJ}_{\phi,_y} = \int G\, \phi,_y\, dA \qquad \overline{GJ}_{\phi,_z} = \int G\, \phi,_z\, dA \qquad \overline{GJ}_{y\phi,_z} = \int G\, y\phi,_z\, dA$$

$$\overline{GJ}_{z\phi,_y} = \int G\, z\phi,_y\, dA \qquad \overline{GJ}_{\phi,_y\phi,_y} = \int G\left(\phi,_y\right)^2 dA \qquad \overline{GJ}_{\phi,_z\phi,_z} = \int G\left(\phi,_z\right)^2 dA$$

$$\overline{Gk}_y = \overline{GA} \int G\, \nabla\phi_y \bullet \nabla\phi_y\, dA \qquad \overline{Gk}_z = \overline{GA} \int G\, \nabla\phi_z \bullet \nabla\phi_z\, dA \qquad \overline{Gk}_\omega = \overline{GI}_p \int G\, \nabla\phi_\omega \bullet \nabla\phi_\omega\, dA$$





For ***bending properties*** $\rightarrow \quad E \equiv E_x^m = \dfrac{1}{ta_{11}}$

For ***torsion properties***

open part $\rightarrow \quad G \equiv G_{xy}^b = \dfrac{12}{t^3 d_{66}}$

closed part $\rightarrow \quad G \equiv G_{xy}^m = \dfrac{1}{ta_{66}}$

E_x^m : G_{xy}^b G_{xy}^m ***are the equivalent elastic constants of the laminate****, where the compliance coefficients aij and dij are obtained by inverting the classical laminate constitutive equations. The superscripts "m" and "b" indicate the membrane and bending modes, respectively.*

Saint-Venant torsional stiffness

$$\overline{GJ} = \int G\left(y^2 + z^2 + y\, {}^E\phi,_z - z\, {}^E\phi,_y\right) dA$$

$$\phi = {}^E\phi + y_S z - z_S y$$

Location of shear center

$$y_S = -\frac{\overline{EI}_{zz}\,\overline{EJ}_{z\phi} - \overline{EI}_{yz}\,\overline{EJ}_{y\phi}}{\overline{EI}_{yy}\,\overline{EI}_{zz} - \overline{EI}_{yz}^2}$$

$$z_S = \frac{\overline{EI}_{yy}\,\overline{EJ}_{y\phi} - \overline{EI}_{yz}\,\overline{EJ}_{z\phi}}{\overline{EI}_{yy}\,\overline{EI}_{zz} - \overline{EI}_{yz}^2}$$

Location of elastic center

$$y_E = \frac{\overline{ES}_z}{\overline{EA}} \qquad z_E = \frac{\overline{ES}_y}{\overline{EA}}$$

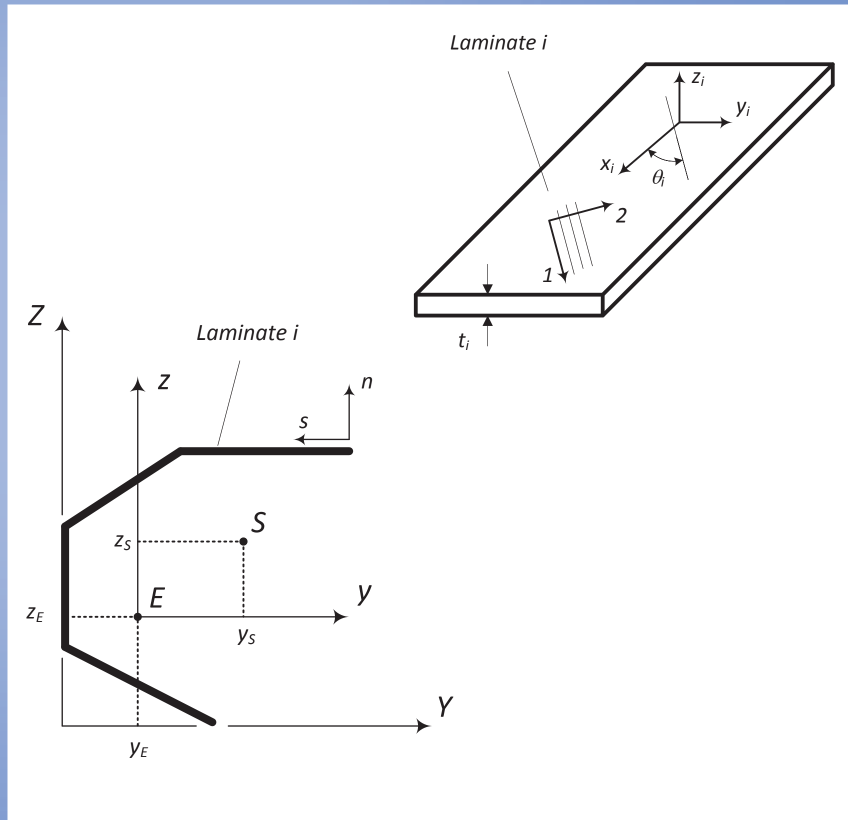
• Assumptions and kinematics

The beams are made from an assembly of thin flat-layered panels, each panel corresponding to a symmetric laminate

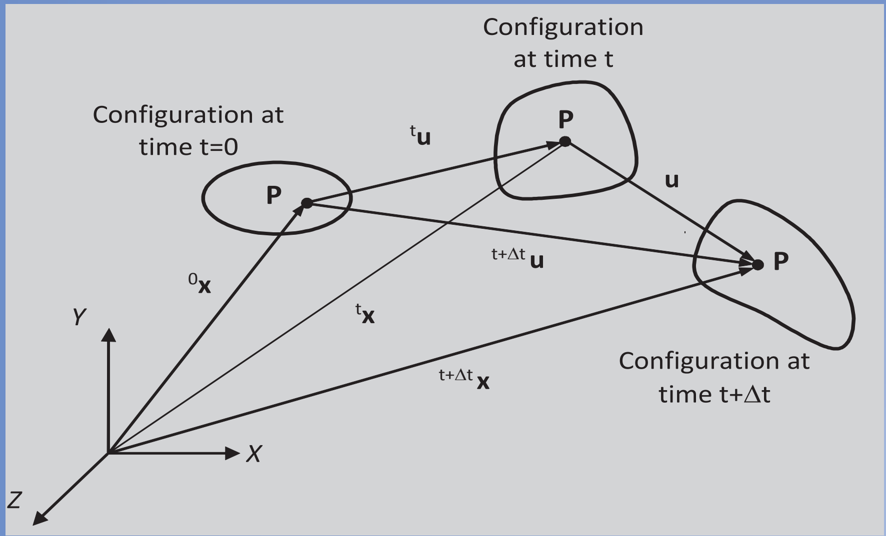
- ***The contour of the cross section does not deform in its own plane;***
- ***The shearing deformation of the middle surface is zero in each panel;***
- ***Each panel behaves as thin plate. This implies that the Kirchoff hypothesis is valid for each plate element;***
- ***Strains are small but large displacements and rotations are allowed.***

Displacement at a generic point:

$$u = u_0 - y\theta_z + z\theta_y - \phi\theta'_x$$
$$v = v_S - (z - z_S)\theta_x$$
$$w = w_S + (y - y_S)\theta_x$$



• *Nonlinear structural analysis*



Updated Lagrangean formulation

$$\int_{t} {}^{t+\Delta t}\mathbf{S}\cdot\delta_{t}^{t+\Delta t}\boldsymbol{\varepsilon}\, d^{t}V = \int_{t} {}^{t+\Delta t}\mathbf{f}\cdot\delta^{t+\Delta t}\mathbf{u}\, d^{t}V + \int_{t} {}^{t+\Delta t}\mathbf{T}^{0}\cdot\delta^{t+\Delta t}\mathbf{u}\, d^{t}\Gamma_{T}$$

Incremental decompositions

$$^{t+\Delta t}\mathbf{u} = {}^{t}\mathbf{u} + \mathbf{u}\ ,\quad {}^{t+\Delta t}_{t}\mathbf{S} = {}^{t}\boldsymbol{\sigma} + {}_{t}\mathbf{S}\ ,\quad {}^{t+\Delta t}_{t}\boldsymbol{\varepsilon} = {}^{t}\boldsymbol{\varepsilon} + {}_{t}\boldsymbol{\varepsilon} = {}^{t}\boldsymbol{\varepsilon} + {}_{t}\boldsymbol{\varepsilon}_{\mathrm{L}} + {}_{t}\boldsymbol{\varepsilon}_{\mathrm{N}}\ ,\quad {}^{t+\Delta t}_{t}\mathbf{f} = {}^{t}\mathbf{f} + {}_{t}\mathbf{f}\ ,\quad {}^{t+\Delta t}_{t}\mathbf{T}^{0} = {}^{t}\mathbf{T}^{0} + {}_{t}\mathbf{T}^{0}$$

Linearization

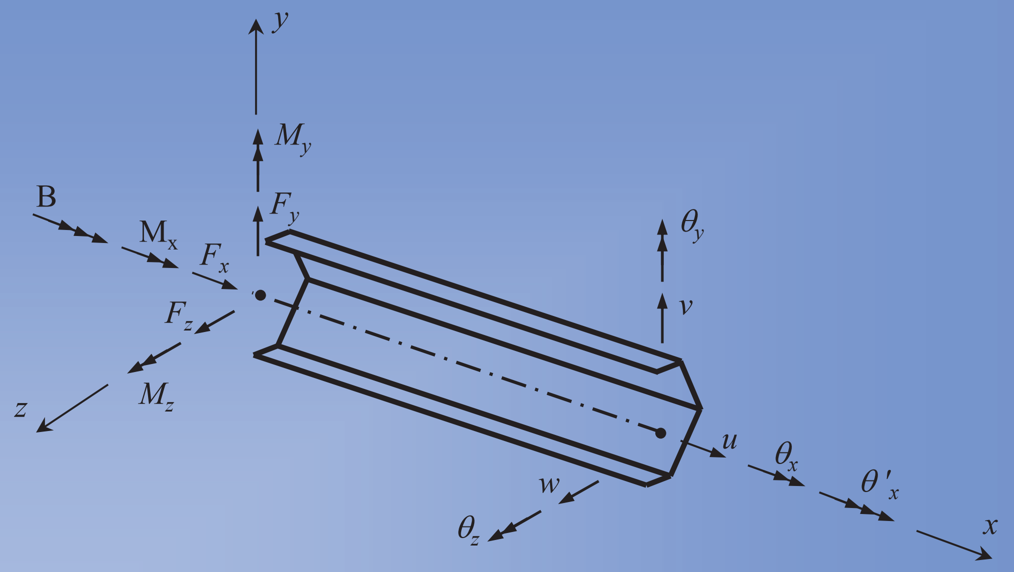
$${}_{t}\mathbf{S}\cdot\delta_{t}\boldsymbol{\varepsilon} = {}_{t}\mathbf{S}\cdot\delta_{t}\boldsymbol{\varepsilon}_{L} \quad ; \quad {}_{t}\mathbf{S} = \mathbf{E}\cdot{}_{t}\boldsymbol{\varepsilon}_{L}$$

Linearized incremental equilibrium equation

$$\int\left({}_{t}\mathbf{S}\cdot\delta_{t}\boldsymbol{\varepsilon}_{L} + {}^{t}\boldsymbol{\sigma}\cdot\delta_{t}\boldsymbol{\varepsilon}_{N}\right)d^{t}V = \int {}_{t}\mathbf{f}\cdot\delta\mathbf{u}\, d^{t}V + \int {}_{t}\mathbf{T}^{0}\cdot\delta\mathbf{u}\, d^{t}\Gamma_{T}$$

• *Beam element model*

- *Two-node Hermitean finite beam element*
- *Seven degrees of freedom per node*
- *Linear displacement field is adopted for u and a cubic displacement field is adopted for the other displacements*



Incremental vectors of nodal displacements and nodal forces

$$\mathbf{U} = \{ u_1 \quad v_1 \quad w_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad \theta'_{x1} \quad u_2 \quad v_2 \quad w_2 \quad \theta_{x2} \quad \theta_{y2} \quad \theta_{z2} \quad \theta'_{x2} \}^T$$

$$\mathbf{F} = \{ F_{x1} \quad F_{y1} \quad F_{z1} \quad M_{x1} \quad M_{y1} \quad M_{z1} \quad B_1 \quad F_{x2} \quad F_{y2} \quad F_{z2} \quad M_{x2} \quad M_{y2} \quad M_{z2} \quad B_2 \}^T$$

$$\int \left({}_t\mathbf{S} \cdot \delta_t \varepsilon_L + {}^t\sigma \cdot \delta_t \varepsilon_N \right) d^tV = \int {}_t\mathbf{f} \cdot \delta \mathbf{u} \, d^tV + \int {}_t\mathbf{T}^0 \cdot \delta \mathbf{u} \, d^t\Gamma_T$$

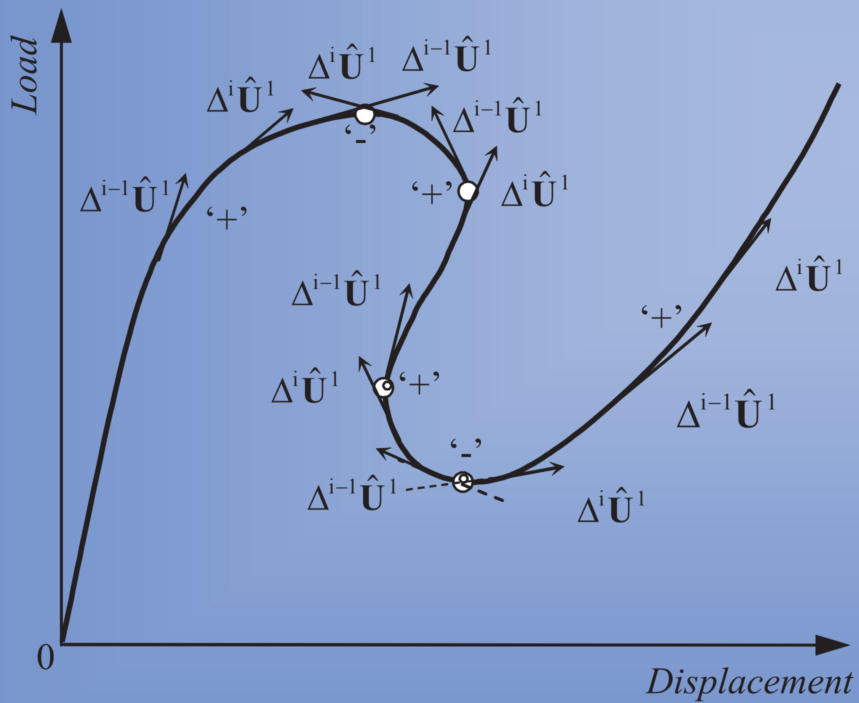
discretization →

$$\left({}^t\mathbf{K}_L + {}^t\mathbf{K}_{NL} \right) \mathbf{U} = {}_t\mathbf{P}$$

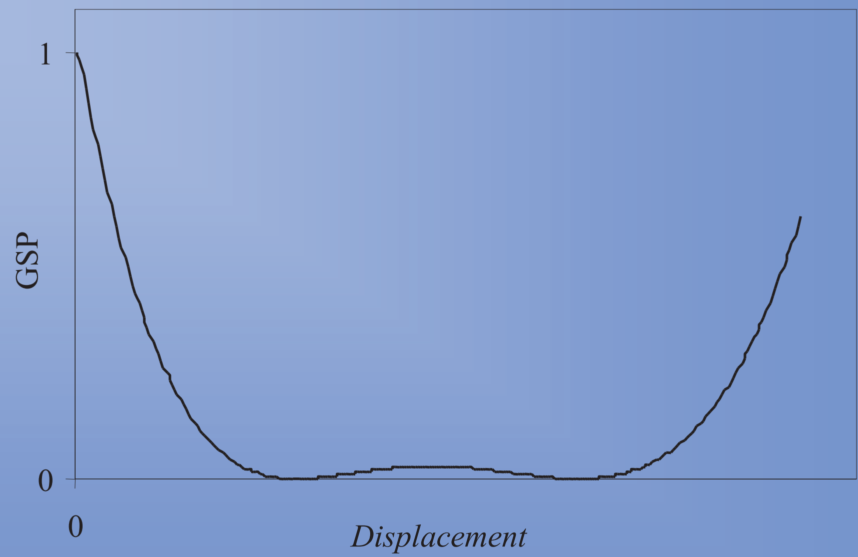
- **Generalized displacement control method** (Yang and Shieh, 1990)

Generalized stiffness parameter:

$$GSP = \frac{\Delta^1 \hat{\mathbf{U}}^{1T} \Delta^1 \hat{\mathbf{U}}^1}{\Delta^{i-1} \hat{\mathbf{U}}^{1T} \Delta^i \hat{\mathbf{U}}^1}$$

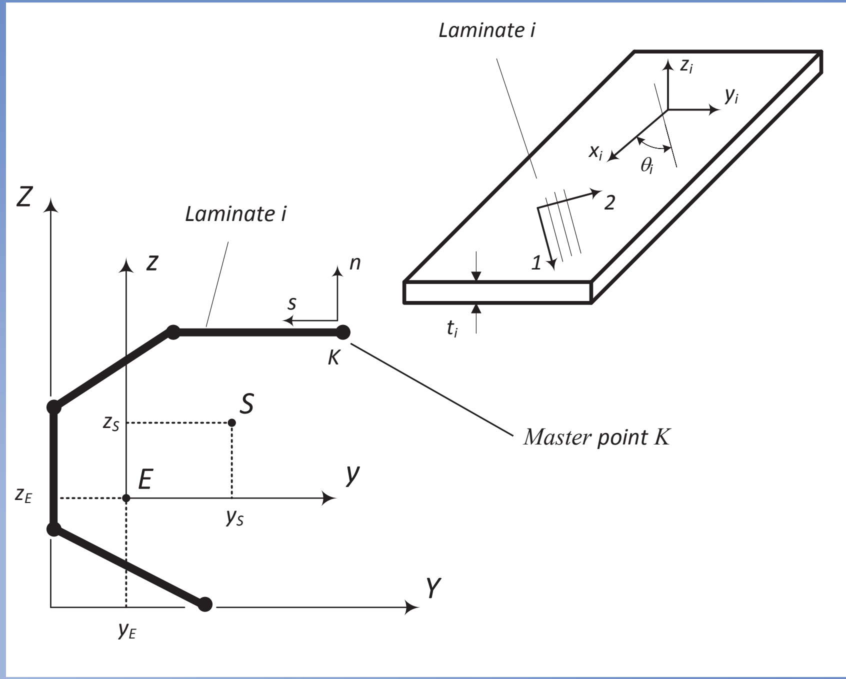


$$\Delta^i \lambda^1 = \Delta^1 \lambda^1 \left| GSP \right|^{1/2}$$



• *Design sensitivity analysis*

- *For sensitivity analysis the cross-section is modeled throughout* ***design elements that geometrically coincide with the laminates;***



An objective or a constraint may be represented by a functional Ψ,

Total design variation $\bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \tilde{\delta}\Psi$

Adjoint variable method, *may be derived by stating:*

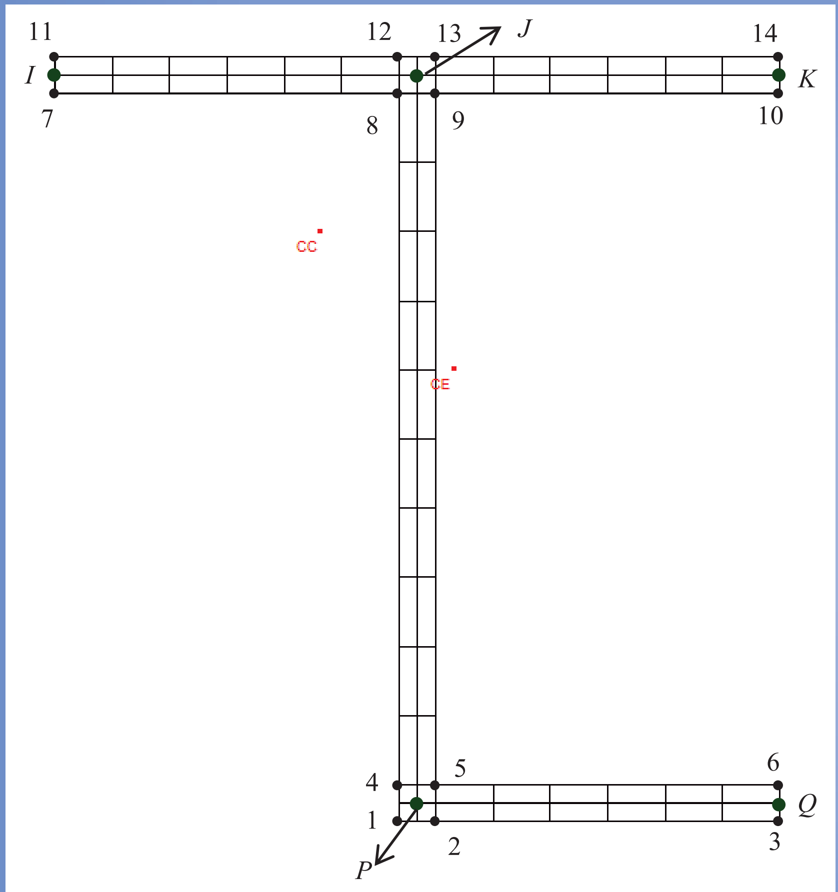
$L = \Psi - \Omega^a$ $\rightarrow$ Ω^a is the **state equation** where the **virtual fields** are **replaced** by **adjoint fields**

- *To calculate the sensitivities w.r.t to lamina orientation.*

First forward finite difference method

- *To calculate the sensitivities w.r.t. to cross section geometry.*

• *Changing cross-sectional geometry*



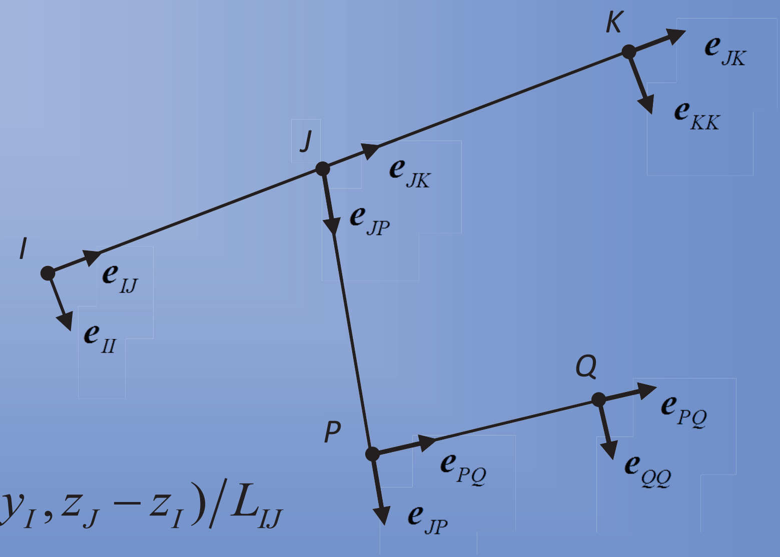
1, 2, …, 14 – to define the cross-section

I, J, K, P, Q – Master points

The position of the master points is treated as dv

A general design arbitrary configuration defined by:

- *Master points*
- *Medium lines orientation unit vectors*

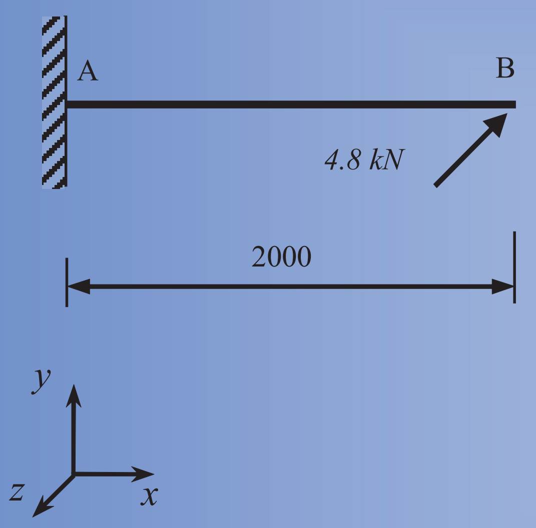


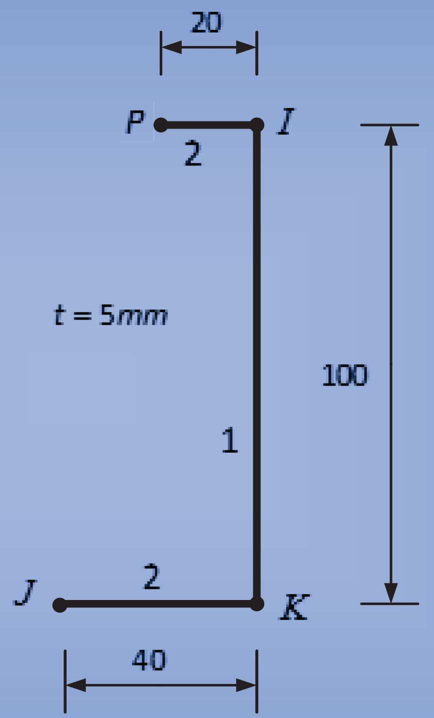
$$\mathbf{e}_{IJ} = (y_J - y_I, z_J - z_I)/L_{IJ}$$

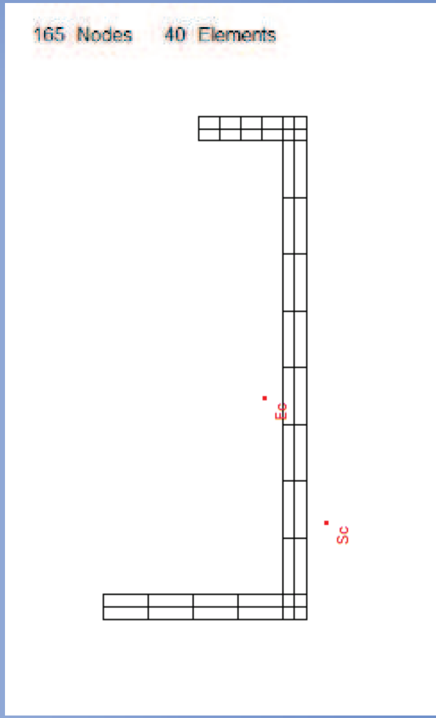
$$L_{IJ} = \sqrt{(y_J - y_I)^2 + (z_J - z_I)^2}$$

• *Numerical Examples*

Example 1 – Beam with asymmetric channel section







- *The cross-section is discretized by 40 eight node quadratic isoparametric finite elements.*
- *The beam is discretized by 8 finite elements of equal length.*
- *Two symmetric laminates with 8 layers of equal thickness h=0. 625 mm,* $[0/45/-45/0]_S$

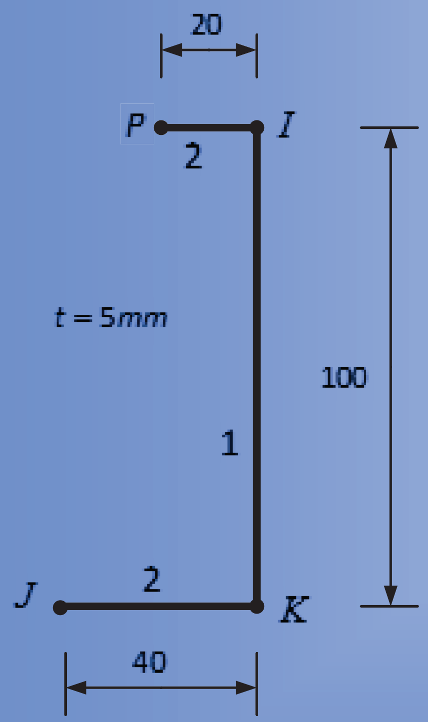
  (total thickness t=5 mm) $E_1 = 140$ *GPa;* $E_2 = 10$ *GPa;* $\nu_{12} = 0.3$; $G_{12} = 5$ *GPa*

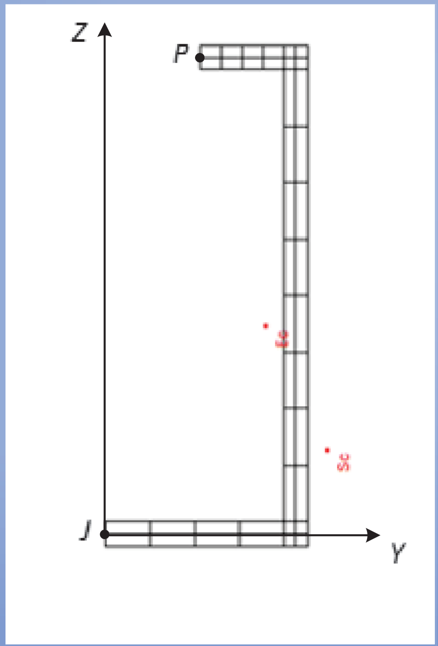
Problem:

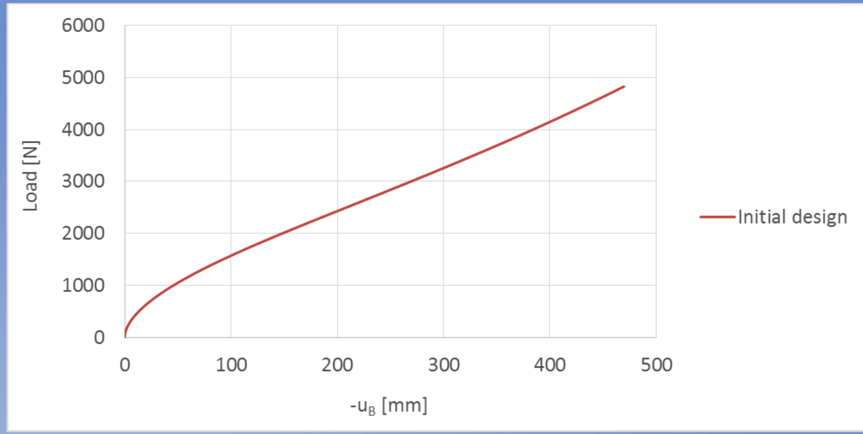
$$\min A$$

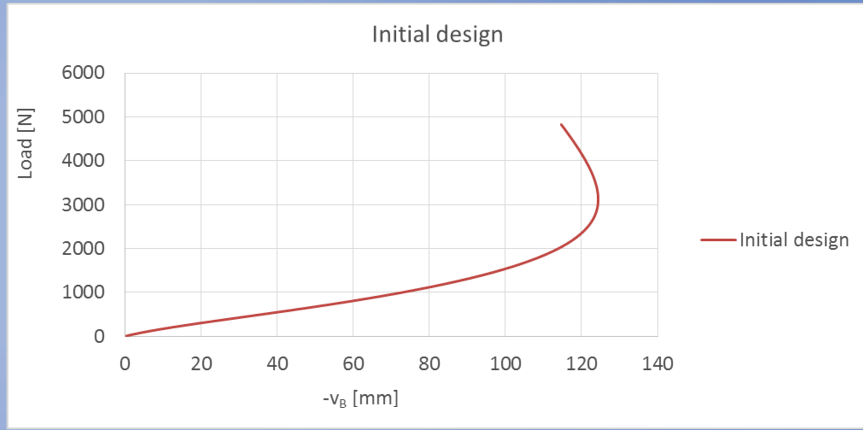
$$s.t.\ w_B \leq 500\,mm$$

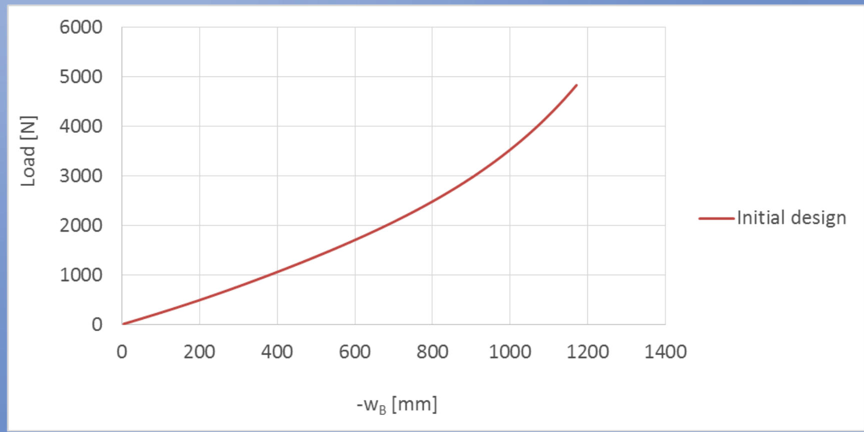
$$d.v. - Y_P, Z_P, Y_J, Z_J$$





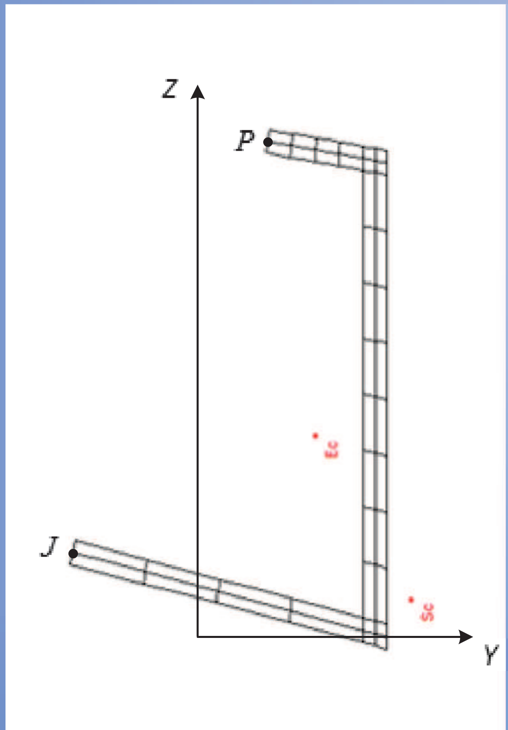


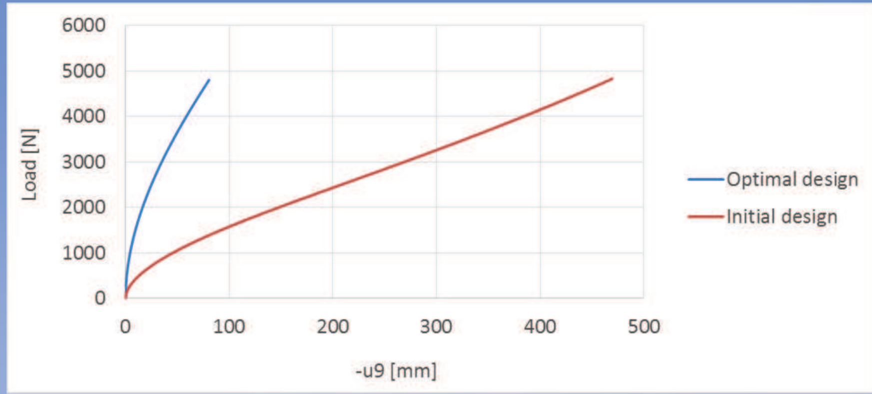


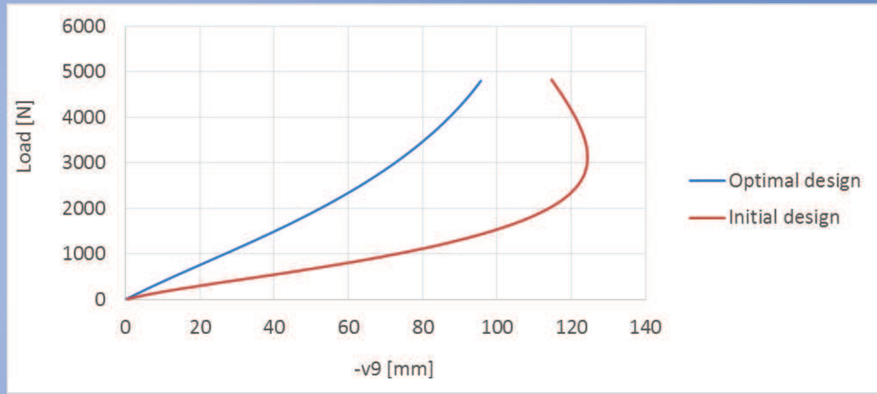


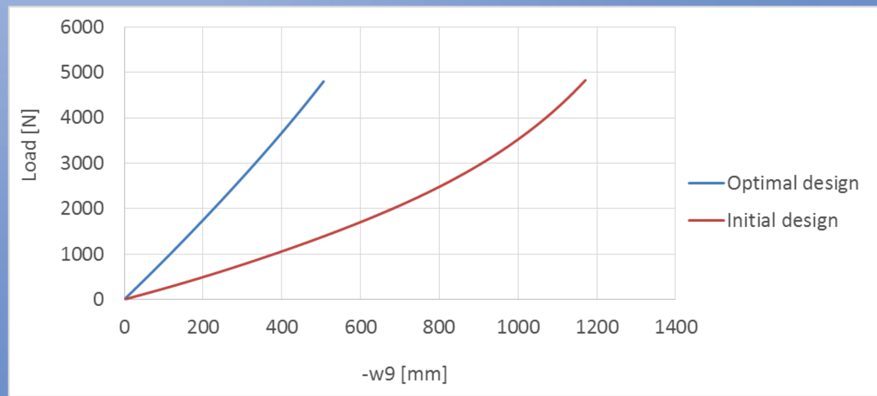
	Initial design	Optimal design
$Y_J; Z_J$ [mm]	0; 2.5	-24.2; 19.6
$Y_P; Z_P$ [mm]	20; 102.5	13.5; 105.3
A [mm^2]	800	947.8
w_B [mm]	1170	501 *

(*) – Active constraint

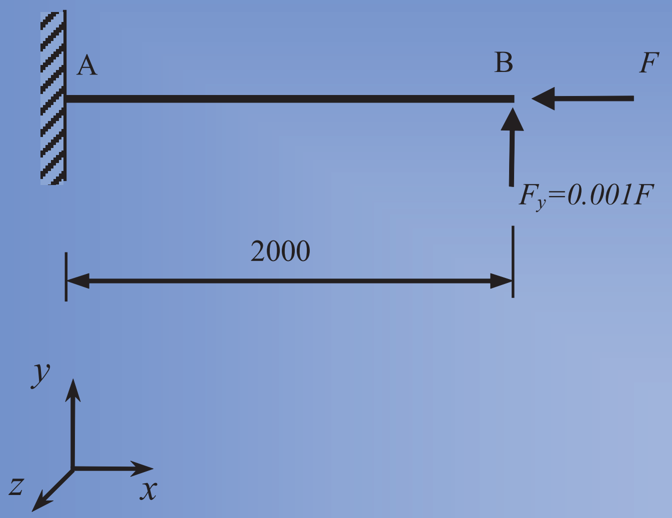


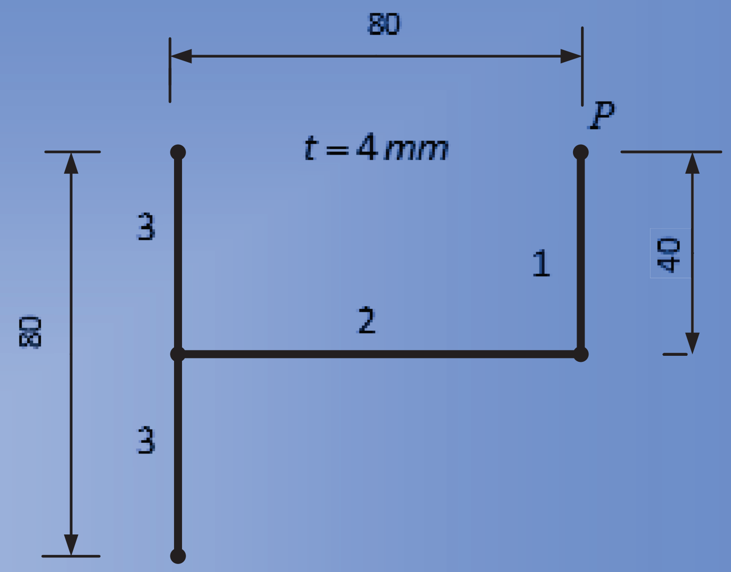






Example 2 – Column with asymmetric thin-walled open section

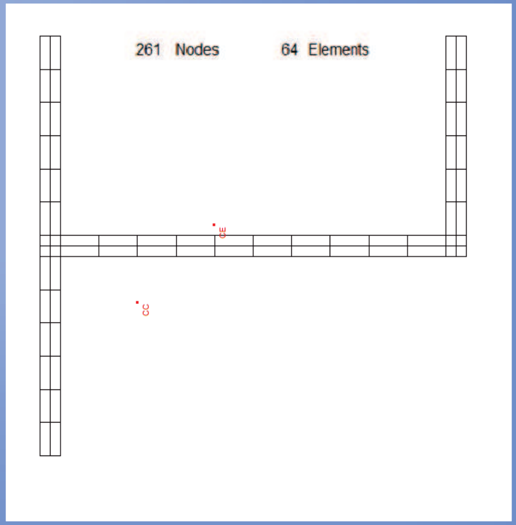


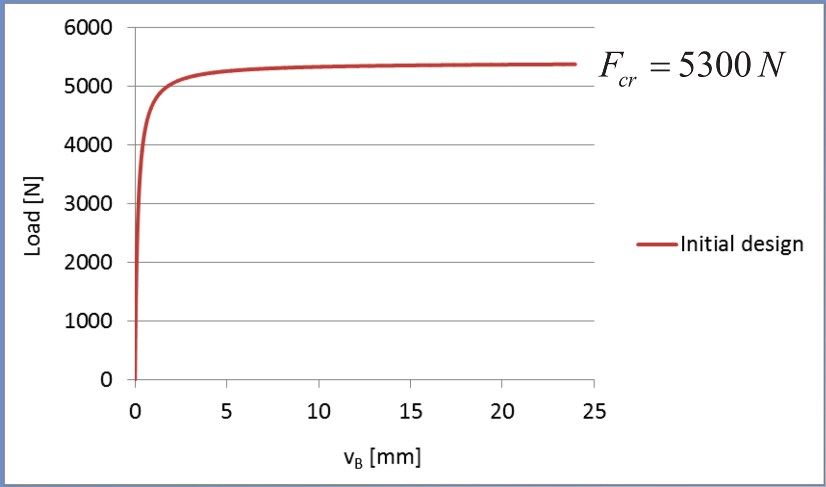


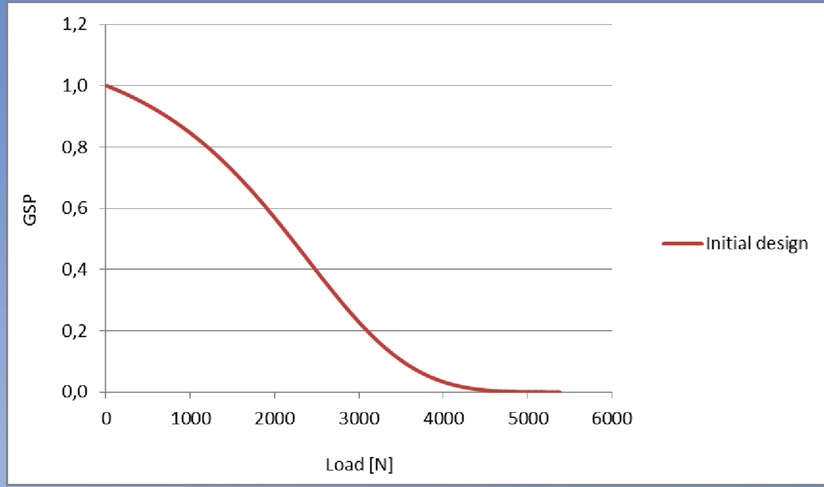
- *The cross-section is discretized by 64 isoparametric quadratic 8 node finite elements.*
- *The beam is discretized by 8 finite elements of equal length.*
- *Three symmetric laminates with ten layers of equal thickness h=0.4 mm.*

$$[0/45/-45/45/-45]_S$$

$E_1 = 140$ GPa; $E_2 = 10$ GPa; $\nu_{12} = 0.3$; $G_{12} = 5$ GPa





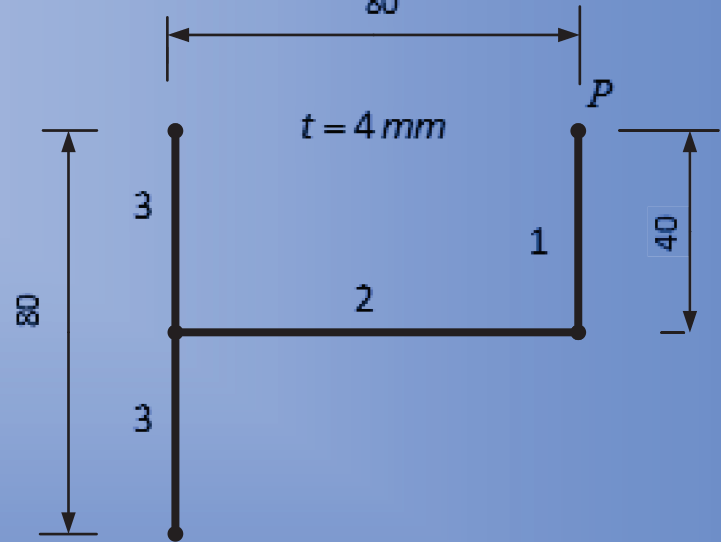


Problem 1:

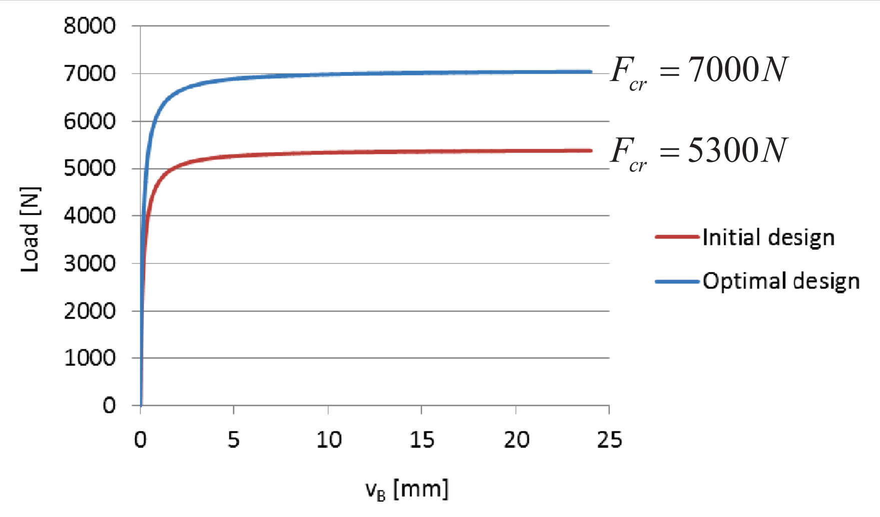
$$\min A$$

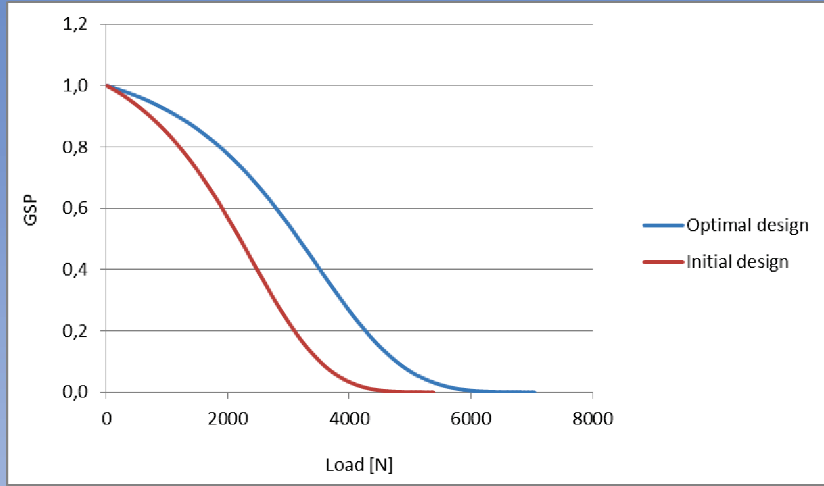
$$s.t.\ F_{cr} = 7000\,N \rightarrow v_B \le 12\,mm$$

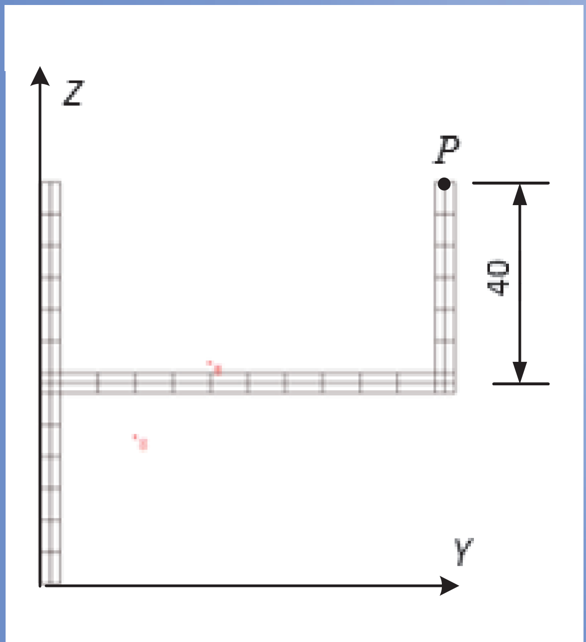
$$d.v. - Y_P, Z_P$$

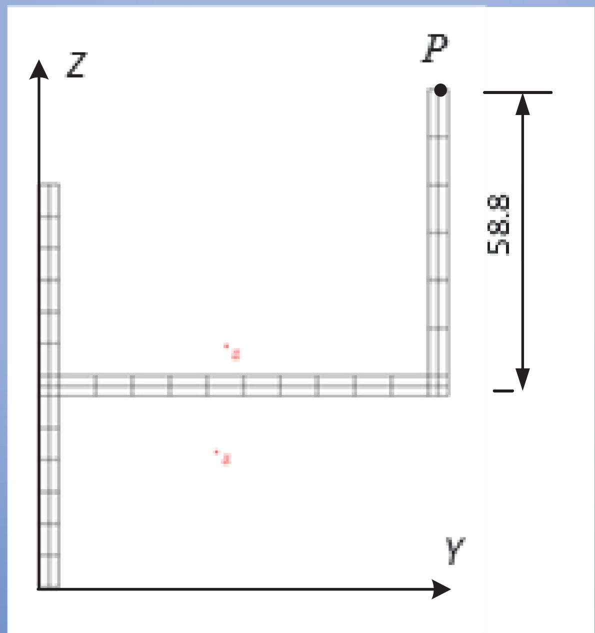


- ***The nonlinear critical load constraint is replaced by a displacement constraint;***









	Initial design	Optimal design
$Y_P;Z_P$ [mm]	82; 80	82; 98.8
A [mm^2]	792	862
F_{cr} [N]	5300	7000 *

(*) – Active constraint

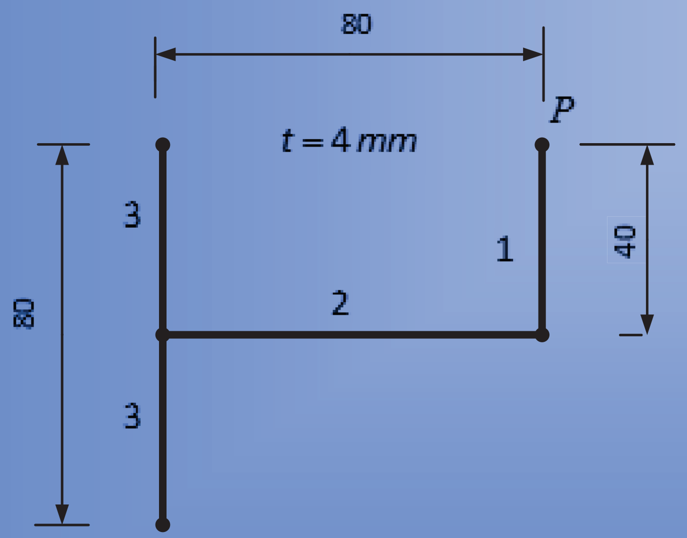
Problem 2:

$$\max \overline{GJ}$$

$$s.t.\ F_{cr} = 7000\,N \;\rightarrow\; v_B \leq 12\,mm$$

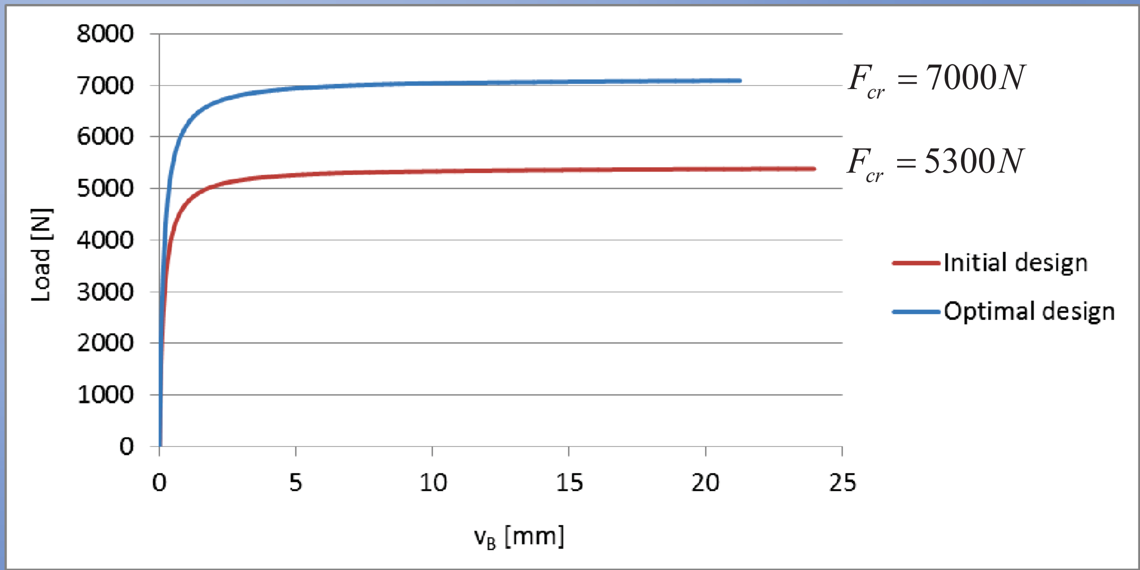
$$d.v. - \theta^1_{2,3}, \theta^1_{4,5}, \theta^2_{2,3}, \theta^2_{4,5}, \theta^3_{2,3}, \theta^3_{4,5}$$

- ***The nonlinear critical load constraint is replaced by a displacement constraint;***



	Initial design	Optimal design
Laminate 1	$[0/45/-45/45/-45]_s$	$[0/44.2/-44.2/0/0]_s$
Laminate 2	$[0/45/-45/45/-45]_s$	$[0/44.4/-44.4/0/0]_s$
Laminate 3	$[0/45/-45/45/-45]_s$	$[0/43.7/-43.7/77.6/-77.6]_s$
$\overline{GJ}$ [N.mm²]	7.72E7	7.72E7
F_{cr} [N]	5300	7000 *

(*) – Active constraint



• *Final Comments*

- ***A finite element model for optimal design of composite laminated thin-walled beam structures, with geometrically nonlinear behavior, including post critical behavior and accounting for Warping deformation;***

- ***Based on an Updated Lagrangean formulation;***

- ***To define the load-deflection path, a generalized displacement control method has been implemented;***

- ***The thin-walled cross-sections are modeled as assemblies of flat symmetric laminated panels and their bending-torsion properties are defined in terms of the cross-section geometry, warping function and properties of the corresponding laminate at each point;***

- ***An eight-node quadratic two-dimensional isoparametric finite element modeling is used in order to determine the bending-torsion properties;***

- ***The structural discretization is formulated throughout three-dimensional two-node Hermitean finite beam elements, with 7 d.o.f. per node;***

- ***Design elements are used for design sensitivity analysis and optimization. Geometrically these elements coincide with the laminates;***

- ***Lamina orientations and cross-section global geometry are considered as design variables;***

- ***The nonlinear critical load constraint was replaced by a displacement constraint;***

- ***The last example shows that the critical load of laminate composite beam is strongly dependent on the lamina orientation, hence this orientation is a fundamental parameter to these structures.***