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Bounds for analytic solutions to integral equations in the complex domain

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ABSTRACT

We establish the existence of analytic solutions to integral equations in the complex domain when the nonlinearity satisfies either a growth condition or a monotonic type condition.

1. Introduction

Let G be a simply connected domain with $0 \in G$. In this work we examine the integral equation

$$y(z) = \int_{C_1} k(z, w) f(w, y(w))\, dw \quad \text{for } z \in G$$

where C_1 is a contour (lying in G) between 0 and z. In this work conditions are imposed on k and f such that the line integrals of kf are independent of the contour taken. For convenience we will consider the domain to be the ball $B_T = B(0, T) = \{z \in \mathbf{C} : |z| < T\}$ (here $T > 0$ is fixed) and since the line integrals of kf are independent of path, we will just consider C_1 to be the straight line between 0 and z. As a result, in this work we only consider the integral equation

$$y(z) = \int_0^z k(z, w) f(w, y(w))\, dw \quad \text{for } z \in \overline{B_T}. \tag{1.1}$$

Of course $\int_0^z$ means $\int_{C_1}$ where C_1 is the straight line between 0 and z. We note here that only particular forms of (1.1) have been considered in the literature; see [1–5] and the references therein.

We next gather together some notation and results which will be needed in this work. For any $y = (y_1, \ldots, y_n), z = (z_1, \ldots, z_n) \in \mathbf{C}^n$ let

$$|y| = \left(\sum_{i=1}^n |y_i|^2\right)^{\frac{1}{2}} \quad \text{and} \quad \langle y, z\rangle = \sum_{i=1}^n y_i \overline{z_i}.$$

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