5–10 July 2004, Antalya, Turkey — Dynamical Systems and Applications, Proceedings, pp. 563–571

Nonoscillations in Retarded Equations

Sandra Pinelas
Universidade dos Açores
R. da Mãe de Deus
9500-321 Ponta Delgada
Portugal

1 Introduction

This note regards the existence of nonoscillatory solutions of the linear difference retarded functional equation

$$x\left(t\right)=\int_{-1}^{0}x\left(t-r\left(\theta\right)\right)dq(\theta), \tag{1}$$

where $x\left(t\right)\in\mathbb{R}$, $r\left(\theta\right)$ is a positive real continuous function on $[-1,0]$ and $q(\theta)$ is a function of bounded variation on $[-1,0]$, normalized in a manner such that $q\left(-1\right)=0$.

In the case where $q(\theta)$ is a step function, with a number p of jump points, we obtain the important class of delay difference equations

$$x\left(t\right)=\sum_{j=1}^{p}a_jx\left(t-r_j\right), \tag{2}$$

where the a_j are nonzero real numbers and each r_j is a positive real number $\left(j=1,\ldots,p\right)$.

Considering the value $\|r\|=\max\left\{r\left(\theta\right):\ -1\leqslant\theta\leqslant0\right\}$, by a *solution* of (1) we mean a continuous function $x:\ \left[-\|r\|,+\infty\right[\rightarrow\mathbb{R}$, which satisfies (1) for every $t\geq0$. A solution is said to be *oscillatory* whenever it has an infinite number of zeros; otherwise it will be said to be *nonoscillatory*. When all solutions are oscillatory, the equation (1) is called *oscillatory*. If (1) has at least one nonoscillatory solution, then the equation will be said to be *nonoscillatory*.

We will say that a function $\phi:\ [-1,0]\rightarrow\mathbb{R}$ is increasing (decreasing) on $J\subset[-1,0]$, if ϕ is nonconstant on J and for every $\theta_1,\theta_2\in J$ such that $\theta_1<\theta_2$, one has $\phi(\theta_1)\leq\phi(\theta_2)$ (respectively, $\phi(\theta_2)\leq\phi(\theta_1)$). For a given $\theta\in[-1,0]$, if for every $\varepsilon>0$, sufficiently small, ϕ is increasing (decreasing) in $[\theta-\varepsilon,\theta+\varepsilon]$ ($[-\varepsilon,0]$ if $\theta=0$,