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A Fixed Point Approach to the Stability of a Quadratic Quartic Functional Equation in Paranormed Spaces

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Abstract

In this paper, using fixed point method we prove the generalized Hyers-Ulam stability of a quadratic quartic functional equation for fixed integers k with $k \neq 0, \pm 1$ in paranormed spaces.

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Key words: Paranormed space, generalized Hyers-Ulam stability, fixed point, quadratic quartic functional equation.

1 Introduction and Preliminaries

A basic question in the theory of functional equations arises as follows: When is it true that a function, which approximately satisfies a functional equation, must be close to an exact solution of the equation?

If the problem accepts a unique solution, we say the equation is stable. The first stability problem concerning group homomorphisms is related to a question of Ulam [30] in 1940.

"Let G be a group and G' be a metric group with metric $d(\cdot,\cdot)$. Given $\epsilon > 0$ does there exists a $\delta > 0$ such that if a function $f : G \to G'$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G$, then there exists homomorphism $H : G \to G'$ with $d(f(x), H(x)) < \epsilon$ for all $x \in G$?"

In 1941 D.H. Hyers [11] gave the first affirmative partial answer to the question of Ulam for Banach spaces. He proved the following celebrated theorem.

Theorem 1. *([11]) Let X, Y be Banach spaces and let $f : X \to Y$ be a mapping satisfying*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon \tag{1}$$

for all $x, y \in X$. Then the limit

$$a(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \tag{2}$$

exists for all $x \in X$ and $a : X \to Y$ is the unique additive mapping satisfying

$$\|f(x) - a(x)\| \leq \epsilon \tag{3}$$

for all $x \in X$.

Aoki [2] generalized Hyers theorem for additive mappings. In 1978, a generalized version of the theorem of Hyers for approximately linear mappings was given by Th.M. Rassias [24]. He proved the following: