

Asymptotic Behavior of Solutions of Certain Nonlinear Differential and Integro-differential Equations

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Abstract: We establish several growth theorems for second order nonlinear differential and integro-differential equations. We also give necessary and sufficient conditions for solutions of second order non-linear differential equations to be bounded together with their first derivatives and investigate its asymptotic behavior.

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1 Introduction

In the asymptotic theory of second order nonlinear differential equations, an interesting problem is that of the investigation of solutions with prescribed asymptotic behavior via solutions of the equation $x''(t) = 0$. This problem has been studied during the last four decades. For recent contributions, we refer to [6]-[8], [12], [13], [18], [19], [21]-[28] and the references cited therein.

In the present paper, we are concerned with the second order nonlinear differential and integro-differential equations of the form

$$(a(t)(x'(t))^\alpha)' = f(t, x(t)) + \int_{t_0}^t k(t, s)h(s, x(s))ds, \quad t_0 \geq 0, \quad (1)$$

and the more general equation

$$(a(t)(x'(t))^\alpha)' + q(t)x^\beta(t) = p(t)x^\gamma(t) + f(t, x(t)) + \int_{t_0}^t k(t, s)h(s, x(s))ds, \quad (2)$$

where α, β, γ are ratios of positive odd integers, $\alpha \geq 1$, $p, q : [t_0, \infty) \rightarrow [0, \infty)$, $h : [t_0, \infty) \times [t_0, \infty) \rightarrow [0, \infty)$, $f, h : [t_0, \infty) \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous. $\text{sgn} f(t, x)$ and $h(t, x)$ are the same as $\text{sgn } x$.

Our purpose in this paper is to investigate solutions of equations (1) and (2) which behave asymptotically at ∞ like solutions of the equation $(a(t)(x'(t))^\alpha)' = 0$. We also establish some necessary and sufficient conditions for solutions of a special case of equation (1) to be bounded together with their first order derivatives and study asymptotic properties. The obtained results extend, improve and unify the existing results appeared in the literature.

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