The 8th International Congress on Sound and Vibration
2-6 July 2001, Hong Kong, China

ANALYSIS OF THE NONLINEAR ORBITAL MOTIONS OF IMMERSED ROTORS USING A SPECTRAL/GALERKIN APPROACH

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Abstract

Recently, we developed a symbolic-numerical formulation for the nonlinear planar motion of rotors under fluid confinement, based on a spectral/Galerkin approach, for gap geometries of about $\delta = H/R \approx 0.1$ — where H is the average annular gap and R is the rotor radius. Results showed a quite good agreement between the class of approximate models generated, the corresponding analytical exact planar model and experiments. Note that this methodology can be almost entirely automated on a symbolic computing environment.

In the present paper this symbolic-numerical spectral/Galerkin procedure is extended in order to deal with nonlinear orbital motions — $X(t)$ and $Y(t)$ taking place in orthogonal directions.

Numerical simulations performed over a centered rotor configuration maintained by non-isotropic supports ($\beta = \frac{K_X^{st}}{K_X^{st}} \approx 2.4$, where K_X^{st} and K_Y^{st} stands for the structural stiffnesses), which exhibit rich rotordynamics, show a quite good agreement between this type of approximate models and the corresponding analytical exact (but quite involved) model developed in the past by the authors.

INTRODUCTION

Rotor dynamics under moderate fluid confinement — that is, in equipments with clearance ratios of about $\delta = \frac{H}{R} \approx 0.1$ — have been studied since Black (1969), Fritz (1970) and Hirs (1973). Further relevant work was presented by Ramsden et al. (1974 and 1975), Childs (1983), Nelson (1985) and Nordmann et al. (1985). Thorough analysis using linearized flow equations on the gap-averaged fluctuating quantities were performed by Grunenwald et al. (1991), Axisa & Antunes (1992), Grunenwald (1994) and Antunes et al. (1996).

Theoretical formulations for the nonlinear planar motions — with $X(t)$ taking place in one single direction — and for nonlinear orbital motions of rotors — with $X(t)$ and $Y(t)$ taking place in orthogonal directions— were developed and validated by Antunes et al. (1999) and Moreira et al. (2000a, 2000b).

Recently a symbolic-numerical method based on a spectral/Galerkin approach was applied to obtain approximate nonlinear planar models for the rotor dynamics [Moreira et al. (2000c)]. Here, the same symbolic-numerical spectral/Galerkin procedure is extended in order to cover nonlinear orbital motions.

In this approach the gap-averaged tangential flow velocity $u(t,\theta)$ and the gap-averaged pressure $p(t,\theta)$ are approximated, in the Navier-Stokes equations, by truncated Fourier series with time-dependent coefficients. The Galerkin method [see, for instance, Zwillinger (1997) and Reddy (1986)] applied over the residuals so obtained, generate a set of simultaneous ordinary differential algebraic equations for the time-dependent coefficients. This procedure can be almost entirely automated working on a symbolic computing environment.

Finally, the set of differential-algebraic equations deduced is solved numerically in parallel with the dynamics equations for the structure yielding an approximate solution for the motions $X(t)$ and $Y(t)$.

MODEL FORMULATION

Flow Formulation

Consider the geometry of the fluid annulus represented in Figure 1, where θ and t are respectively the azimuth and time, R is the shaft radius and $u(\theta,t)$ is the gap-averaged tangential flow velocity. For orbital motions, the annular gap depth $h(\theta,t)$ is very well approximated by

$$h(\theta,t) = H - X(t)\cos\theta - Y(t)\sin\theta, \tag{1}$$

where H is the average annular gap.

Under the same simplifying assumptions adopted in Antunes et al. (1996), which are — (i) the flow is modeled as being two-dimensional and incompressible, (ii) the radial gradients in the velocity and pressure fields are neglected, (iii) the dissipative effects due to turbulent shear stresses at the walls are modeled using semiempirical loss-of-head terms — one can obtain the Navier-Stokes equations, e.g. the continuity equation for incompressible flow and the momentum equation (projected in the tangential direction),

$$\frac{\partial h}{\partial t} + \frac{1}{R}\frac{\partial (hu)}{\partial \theta} = 0, \ \rho\left\{\frac{\partial (hu)}{\partial t} + \frac{1}{R}\frac{\partial (hu^2)}{\partial \theta}\right\} + \tau_s + \tau_r + \frac{h}{R}\frac{\partial p}{\partial \theta} = 0 \tag{2, 3}$$

where ρ is the constant fluid density and $p(\theta, t)$ is the gap-averaged pressure.

The shear stresses at the rotor and stator walls, in equation (3), are given by

$$\tau_s = \frac{1}{2}\rho u |u| f_s, \ \tau_r = -\frac{1}{2}\rho(\Omega R - u)|\Omega R - u| f_r \tag{4, 5}$$

where f_r and f_s are empirical friction coefficients, which depend on the flow Reynolds number and on wall rugosity.

Among several empirical correlations, those suggested by Wend (1993) and Hirs (1973) are formulated as $f = a(\mathrm{Re})^b$ where coefficients a and b are obtained from experiments.

Besides the natural 2π azimuthal periodicity of the gap-averaged tangential velocity $u(t, \theta)$ and pressure $p(t, \theta)$ we will assume that those functions have continuous space derivatives, enabling us to represent the velocity and pressure fields by the corresponding Fourier series

$$u(t,\theta) = a_0^u(t) + \sum_{n=1}^{\infty}\left[\begin{array}{c} a_n^u(t)\cos(n\theta) + \\ +b_n^u(t)\sin(n\theta)\end{array}\right], \ p(t,\theta) = a_0^p(t) + \sum_{n=1}^{\infty}\left[\begin{array}{c} a_n^p(t)\cos(n\theta) + \\ +b_n^p(t)\sin(n\theta)\end{array}\right]. \tag{6, 7}$$

In the following we will be interested in the dynamic flow forces

$$F_X(t) = -LR\int_0^{2\pi} p(\theta,t)\cos\theta d\theta, \ F_Y(t) = -LR\int_0^{2\pi} p(\theta,t)\sin\theta d\theta, \tag{8, 9}$$

where L is the immersed length of the rotor. Note that (8)-(9) can also be represented as

$$F_X(t) = -\pi LRa_1^p(t), \ F_Y(t) = -\pi LRb_1^p(t), \tag{10,11}$$

where $a_1^p(t)$ and $b_1^p(t)$ are the corresponding coefficients of (7).

Let

$$u(N) = a_0^u(t) + \sum_{n=1}^{N}\left[\begin{array}{c} a_n^u(t)\cos(n\theta) + \\ +b_n^u(t)\sin(n\theta)\end{array}\right], \ p(M) = a_0^p(t) + \sum_{n=1}^{M}\left[\begin{array}{c} a_n^p(t)\cos(n\theta) + \\ +b_n^p(t)\sin(n\theta)\end{array}\right] \tag{12, 13}$$

be approximate solutions of $u(t, \theta)$ and $p(t, \theta)$. One can substitute (12) and (13) in equations (2) and (3) to obtain the residuals

$$R_c = R_c(u(N)), \ R_m = R_m(u(N), p(M)) \tag{14, 15}$$

Because (12) and (13) are approximate solutions of $u(t, \theta)$ and $p(t, \theta)$ the residuals do not identically vanish. However, (14) and (15) can be minimized imposing their orthogonality with respect to the vectorial subspaces generated, respectively, by

$$\{1, \cos(\theta), \cos(2\theta), \ldots, \cos(q\theta), \sin(\theta), \sin(2\theta)\ldots, \sin(q\theta)\}$$

where q is N or M, respectively.

This can be done by the Galerkin method [see, Zwillinger (1997)] using the orthogonality conditions

$$\begin{aligned} \langle\cos(n\theta), R_c\rangle &= 0, \ 0 \le n \le N, \ \langle\sin(n\theta), R_c\rangle = 0, \ 1 \le n \le N, & (16, 17) \\ \langle\cos(n\theta), R_m\rangle &= 0, \ 0 \le n \le M, \ \langle\sin(n\theta), R_m\rangle = 0, \ 1 \le n \le M, & (18, 19) \end{aligned}$$

where the inner product is defined as

$$\langle w(\theta), z(\theta)\rangle = \int_0^{2\pi} w(\theta)\, z(\theta)\, d\theta.$$

That is, conditions (16)—(19) generate a set of $2 \times (N+M)+1$ simultaneous nonlinear ordinary first order differential and algebraic equations for

$$\{X(t), Y(t)\} \cup \left\{ \begin{array}{l} a_n^u(t),\ n=0,\dots,N \\ b_n^u(t),\ n=1,\dots,N \end{array} \right\} \cup \left\{ \begin{array}{l} a_n^p(t),\ n=1,\dots,M \\ b_n^p(t),\ n=1,\dots,M \end{array} \right\} \tag{20}$$

whose solution minimize the residuals (14) and (15), as we referred.

The Coupled System

The motion equations of the rotor-flow coupled system read

$$M^{st}\ddot{X} + C^{st}\dot{X} + K_X^{st}X = F_X + F_X^{ext},\ M^{st}\ddot{Y} + C^{st}\dot{Y} + K_Y^{st}Y = F_Y + F_Y^{ext}, \tag{21, 22}$$

where M^{st}, C^{st} and K_X^{st}, K_Y^{st} stand respectively for the rigid rotor mass, the damping coefficient and the stiffness in each direction of the flexible isotropic rotor fixture, F_X^{ext} and F_Y^{ext} stand for any external forcing functions and F_X and F_Y are given by the nonlinear dynamic flow forces described by equations (10)—(11).

Recalling equations (10)—(11) and letting $\dot{X} = Z$ and $\dot{Y} = W$, equations (21)—(22) can be written as a second set of four additional first order ordinary differential equations

$$M^{st}\dot{Z} + C^{st}Z + K_X^{st}X + \pi L R a_1^p(t) = F_X^{ext}, \tag{23}$$

$$M^{st}\dot{W} + C^{st}W + K_Y^{st}Y + \pi L R b_1^p(t) = F_Y^{ext}, \tag{24}$$

$$\dot{X} - Z = 0,\ \dot{Y} - W = 0. \tag{25, 26}$$

Notice that the $5+2\times(N+M)$ ordinary differential and algebraic equations (DAE's) generated by conditions (16)—(19) and those corresponding to equations (23)—(26) are the same number as the unknown time functions:

$$\{X(t), Y(t), Z(t), W(t)\} \cup \left\{ \begin{array}{l} a_n^u(t),\ n=0,\dots,N \\ b_n^u(t),\ n=1,\dots,N \end{array} \right\} \cup \left\{ \begin{array}{l} a_n^p(t),\ n=1,\dots,M \\ b_n^p(t),\ n=1,\dots,M \end{array} \right\} \tag{27}$$

The equations generated by conditions (16)—(19) and equations (23)—(29) constitute the approximate nonlinear rotor dynamics model, called here $N \times M$ model. The level of accuracy of this model depends on the choice of N and M.

Note that among the generated equations, which constitute the above mentioned approximate nonlinear rotor dynamics $N \times M$ model, there are a set of $2 \times N - 2$ pure algebraic equations, all of them related with the continuity equation (2). Besides, the time derivatives of

$$a_n^p(t), n=1,\dots,M \text{ and } b_n^p(t), n=1,\dots,M$$

do not appear in any of the equations. Both reasons explain why our $N \times M$ model lead to a set of differential-algebraic equations (DAE's). This class of problems arise naturally in many applications but present numerical and analytical difficulties which do not occur with systems of ordinary differential equations [Brenen et al. (1996)].

L (Rotor length [m])	0.250	Eccentricity, ε	0
R (Rotor radius [m])	0.044	Struct. mass, M^{st} (kg)	7.2
H (Annular gap [m])	0.0062	Damping, C^{st} (Ns/m)	50
$\delta = \frac{H}{R}$(Clearance ratio)	0.14	Struct. stiff., K_X^{st}; K_Y^{st} ($\frac{N/m}{10^4}$)	3.8; 1.6
$\frac{L}{D}$, Length-to-diameter ratio	2.8	$f = f_r = f_s$ (friction coef.)	0.01

Table 1: *Geometric and physical rotor parameters*

NUMERICAL SIMULATIONS

To assert the validity of the proposed approximate methodology numerical simulations were performed in parallel with the exact analytical orbital model from Moreira et al. (2000a). The significant parameters used are shown in Table 1.

Note that the chosen centered rotor configuration — a configuration maintained by non-isotropic supports — displays rich rotordynamics, inducing a range of linear instability at low spinning velocities (namely between 700 rpm and 1000 rpm).

Define the set of differential algebraic equations, corresponding to the $N \times M$ model, as $\mathbf{F}(t, \mathbf{v}, \dot{\mathbf{v}}) = 0$ where $\mathbf{v}$ is the vector of unknowns. In the present work we used a fourth and fifth-order generalization of the implicit first order backward differentiating formula

$$\mathbf{F}\left(t_{n+1}, \mathbf{v}_{n+1}, \frac{\mathbf{v}_{n+1} - \mathbf{v}_n}{t_{n+1} - t_n}\right) = 0$$

which was coded in MATLAB by Roberts (1998).

On the other hand, the analytical exact model was solved using an explicit algorithm of the Runge-Kutte type, with an error-controlled time-step based on the fourth and fifth-order approximations [Press et al., (1992); Shampine, (1994)]. All nonlinear simulations were performed with a time-step of $\Delta t = 0.002$, one order of magnitude smaller than $1/(2 \times f_{\max})$, with $f_{\max} \approx 30$ Hz (maximum frequency of interest).

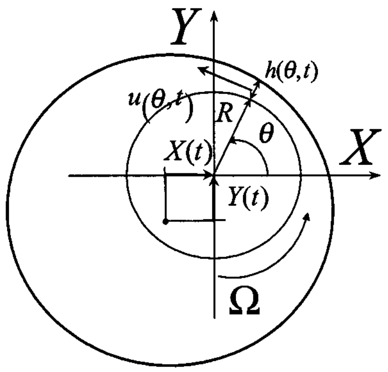

Figure 1: *Geometry of the fluid annulus*

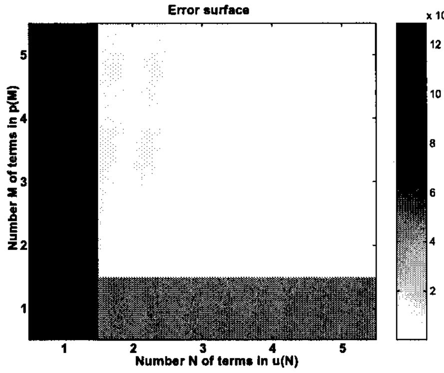

Figure 2: *Mean square error of the displacement*

Two different types of nonlinear simulations were performed. The first type, for the purpose of comparison with the analytical exact orbital model or time domain prediction, was made for spinning velocities between 800 and 1000 rpm, during 2 seconds of transient temporal simulation without external excitation. The second type, for the purpose of modal identification, was made applying a random excitation during the initials 10 seconds of 15 seconds of total simulation, in order to obtain five seconds of free response to perform identification tasks (for each spinning velocity). The external excitation used in this type of simulation was a Gaussian random force of about 0.25 N rms low-pass filtered outside the frequency range $0 \sim 30$ Hz of interest. The ERA method—Eigensystem Realization Algorithm [Juan (1994)] — was used for system identification.

Note that preliminary computations showed that the chosen configuration is linearly unstable between 710 rpm, and 1070 rpm, leading to nonlinear regimes with limit cycles.

In Figure 2 one can see the mean square error of the computed displacement, as predicted by different $N \times M$ models, with $1 \leq N \leq 5$ and $1 \leq M \leq 5$. Comparison was made with the exact analytical orbital model for a spinning velocity of 800 rpm. For this orbital configuration the error surface suggest that, to decrease the model error, the number of terms N and M should increase in a regular-identical fashion. Typically, $N = M$ for $N, M \geq 3$ seems a good model structure. This result, and the need to able to detect minor nonlinearities in the behavior of our system, motivated usage of the 5×5 model, in all other nonlinear simulations we present here.

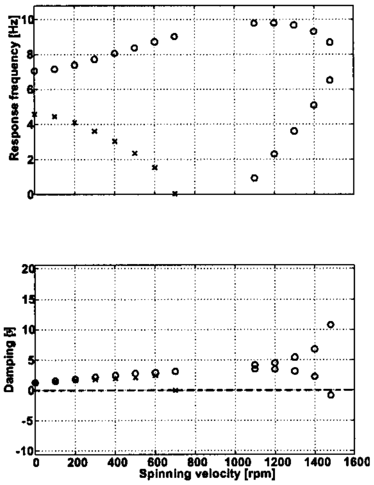

Figure 3: *Frequency and damping*

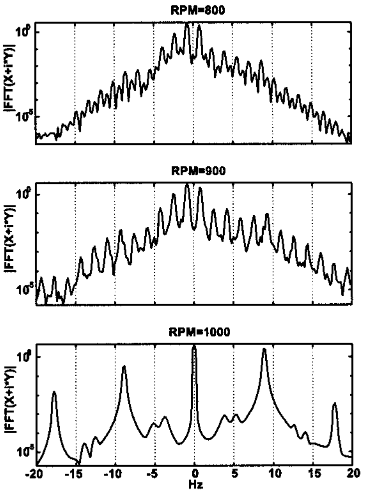

Figure 4: *Response spectra*

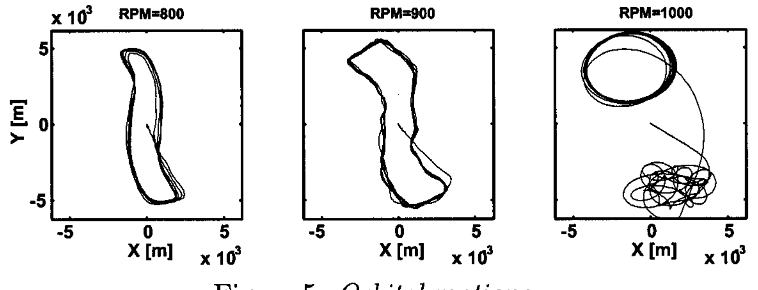

Figure 5: *Orbital motions*

Figure 3 displays predictions of the response frequency and damping, for different spinning velocities, performed by the 5×5 model (×-forward whirling mode; O-backward whirling mode). In this figure one can also see, using superimposed solid lines (—), the corresponding modal frequency and damping curves, computed from a linear model recently developed [see Moreira et al. (2000c)]. The solid lines display, respectively, the frequency $\frac{\nu_n}{2\pi}$ and the damping $-\sigma_n$, where ν_n and σ_n are the imaginary and real parts of the complex eigenvalues $\lambda_n = \sigma_n + i\nu_n$ of the linearized model. Note that the modal curves show an additional zero-frequency mode, discussed in Moreira et al. (2000c), which is related to the co-rotating flow. Observe that linear theory predicts instability by divergence between 710 rpm and 1070 rpm. Outside this range, linear theory and nonlinear simulations show a quite good agreement. However between 710 rpm and 1070 rpm the 5×5−model exhibit high-amplitude limit-cycles. This fact can be observed in Figures 4 and 5 in which a backward whirling high amplitude motion for spinning velocities of 800 and 900 rpm and a forward whirling high amplitude motion for 1000 rpm are predicted.

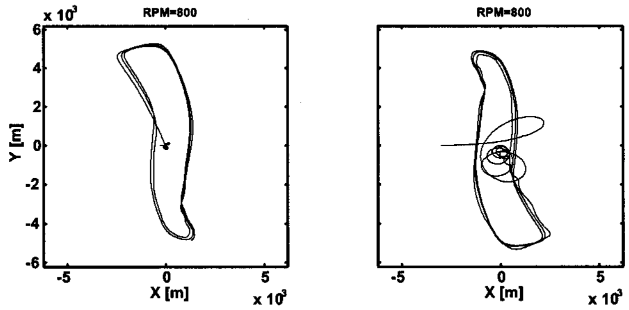

Figure 6: *Orbital motions showing the presence of two attractors*

Finally, in Figure 6 one can observe two plots displaying for the same regime (800 rpm), two different limit cycles originated by two different initial conditions. This phenomenum ilustrate the interesting rotordynamics exhibited by the chosen system configuration.

CONCLUSIONS

In this paper the authors develop a symbolic-numerical method based on a spectral/Galerkin approach for the study of the nonlinear rotor dynamics under fluid confinement.

The proposed methodology is able to generate approximate models on a symbolic computing environment, which can be used in a straightforward manner to obtain accurate nonlinear orbital dynamical formulations. These enable the study, understanding and prediction of nonlinear rotor-dynamics.

ACKNOWLEDGMENTS

This project has been endorsed by the Portuguese FCT and POCTI, with funding participation through the EC programma FEDER.

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