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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO MIXED TYPE DIFFERENTIAL EQUATIONS

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Abstract. This work concerns the asymptotic behavior of solutions to the differential equation

$$\dot{x}(t)+\sum_{i=1}^{m}a_i(t)x(r_i(t))+\sum_{j=1}^{n}b_j(t)x(\tau_j(t))=0,$$

where $a_j(t)$ and $b_j(t)$ are real-valued continuous functions and $r_j(t)$ and $\tau_j(t)$ are non-negative functions such that

$$r_i(t)\le t,\ t\ge t_0,\quad \lim_{t\to\infty}r_i(t)=\infty,\ i=1,\dots,m;$$
$$\tau_j(t)\ge t,\ t\ge t_0,\quad \lim_{t\to\infty}\tau_j(t)=\infty,\ j=1,\dots,n.$$

1. Introduction

In recent years, the theory of delay differential equations with advanced and retarded arguments (mixed type) has provided a natural framework for mathematical modeling of many real world phenomena, namely optimal control problems [6], nerve conduction theory [3], the slowing down of neutrons in nuclear reactors [9], models for economic dynamics [6, 7] and the description of traveling waves in a spatial lattice [4, 5]. See Bellman and Cooke [1] for more applications of differential equations of mixed type. The concept of delay is related to the memory of systems, where past events influence the current behavior. The concept of advance is related to a potential future events which are known at the current time, and which could be useful for decision making. It is well known that the solutions of these types of equations cannot be obtained in closed form. In the absence of a closed form, a viable alternative is studying the qualitative behavior of solutions. As a first step, we need existence and uniqueness of solutions which can be a complicated issue for mixed type equations.

In this article we study the asymptotic behavior of the advanced and retarded differential equation

$$x'(t)+\sum_{i=1}^{m}a_i(t)x(r_i(t))+\sum_{j=1}^{n}b_j(t)x(\tau_j(t))=0,\quad t\ge t_0, \tag{1.1}$$

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