Abstract

Fluid-coupling effects lead to a complex dynamical behavior of immersed spent fuel assembly storage racks. Predicting their responses under strong earthquakes is of prime importance for the safety of nuclear plant facilities.

In the near-past we introduced a simplified linearized model for the vibrations of such systems, in which gap-averaged velocity and pressure fields were described analytically in terms of a single space-coordinate for each fluid inter-rack channel. Using such approach it was possible to generate and assemble a complete set of differential-algebraic equations describing the multi-rack fluid coupled system dynamics.

Because of the linearization assumptions, we achieved computation of the flow-structure coupled modes, but also time-domain simulations of the system responses. However, nonlinear squeeze-film and dissipative flow effects, connected with very large amplitude responses and/or relatively small water gaps, cannot be properly accounted unless the linearization assumption is relaxed. Such was the aim of a previous paper.

Here, our nonlinear model is introduced and tested in order to expose the significance of the squeeze-film and dissipative effects. The response of a fuel storage pool with several racks submitted to strong seismic excitations applied in different directions are also presented and discussed.

Keywords: Fluid structure interaction, nonlinear flow effects, squeeze-film, spent fuel storage racks, seismic response, differential-algebraic equations, squeeze-film.

1 Introduction

Spent fuel storage racks are welded honeycomb stainless steel structures (generally described as blocks with rectangular walls) with large surfaces (the area of a rack wall

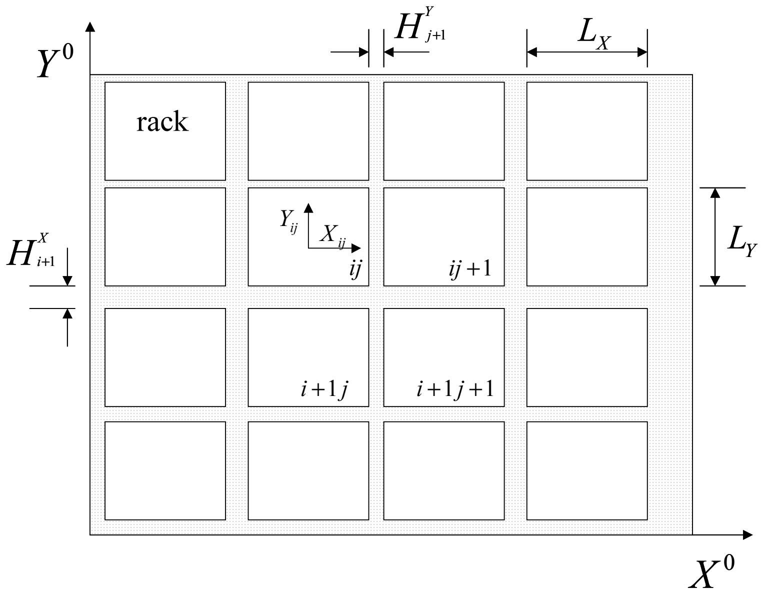

Figure 1: Geometrical parameters.

can be of the order of 10 m^2). They are used to accommodate spent nuclear fuel assemblies. The spent fuel racks are placed on a fuel pool freely on the floor and are separated by small water gaps (sometimes down to the order of 10 mm). The fuel storage pool is filled with water several meters above the top of the racks. Fluid effects induce strong coupling between immersed nuclear fuel racks, when they are subjected to earthquake excitations (see for instance [1, 2, 3, 4]). Therefore, during a seismic event, spent fuel storage racks may bend, slide, twist and uplift. Note that, racks are designed with short aspect ratios so that they would not tilt over. Undoubtedly understanding the complex dynamic behaviour of immersed spent fuel assemblies storage racks under earthquake is of prime importance for the safety of nuclear plant facilities.

In the near-past we introduced a simplified linearized 2-D multirack model for the fluid-coupled vibratory responses of nuclear fuel racks based on the following main simplifying assumptions [5]: (i) Three-dimensional flow effects were neglected; (ii) Gaps between the fuel assemblies and between these and the container were small when compared with the longitudinal length-scales; (iii) Dynamical fluid effects were linearized. From these assumptions, we postulated a simplified flow inside the channels, such that the gap-averaged velocity and pressure fields were described in terms of a single space- coordinate, for each fluid channel. Time-domain simulations of the system responses to seismic excitations were also produced and, despite the simplifications introduced, the model yielded qualitatively similar predictions when compared with other recently published work. However, nonlinear squeeze-film and dissipative effects, connected with very large amplitude responses, cannot be properly modeled unless assumption (iii) is relaxed. In [6] the above-mentioned model was generalized to account for nonlinear flow effects namely squeeze-film and dissipative effects, connected with very large amplitude responses. Although algebraically involved, the proposed methodology can be automatically implemented on a symbolic computer environment, leading to a system of DAE's which is then solved through an adequate time-step integration solver.

In the present paper, our nonlinear model [6] is explored by performing two sets of numerical simulations. In the first set and using a small-storage pool with a single centered rack the significance of the squeeze-film and dissipative effects are exposed. In the second set of numerical simulations, the response to a seismic excitation of a storage pool with 10 racks regularly stored is tested and discussed.

2 Theoretical Model

Consider a pool with $M \times N$ nuclear spent fuel racks arranged in M lines and N columns, which will be described using matrix notation.

The dimensions along the principal directions of each rack cross-section are L_X and L_Y. The X- and Y-direction channels (between each pair of racks or between a wall and a rack) are denoted as

$$H_j^X, \; 1 \;\; \leq \;\; i \leq M+1, \tag{1}$$
$$H_j^Y, \; 1 \;\; \leq \;\; j \leq N+1. \tag{2}$$

In Fig. 1 one can see the main geometrical parameters, for a quite general system configuration.

The motion of each rack $\left(\tilde{X}_{ij}(t), \tilde{Y}_{ij}(t)\right)$,with respect to a global frame can be defined as

$$\tilde{X}_{ij}(t) \;\; = \;\; X_{ij}^0 + X_{ij}(t), \tag{3}$$
$$\tilde{Y}_{ij}(t) \;\; = \;\; Y_{ij}^0 + Y_{ij}(t), \tag{4}$$

where $\left(X_{ij}^0, Y_{ij}^0\right)$ are the coordinates of the geometric centers with respect to the global frame (lower and left walls of the pool container), that is, $\left(X_{ij}(t), Y_{ij}(t)\right)$ are the local coordinates of each rack. So, the actual time varying X-direction $h^X(t)$ and Y-direction gaps $h^Y(t)$ gaps can be defined as

$$h_{ij}^X(t) = H_i^X + Y_{i-1j}(t) - Y_{ij}(t), \begin{cases} 1 \leq i \leq M+1 \\ 1 \leq j \leq N \end{cases} \tag{5}$$

and

$$h_{ij}^Y(t) = H_j^Y + X_{ij}(t) - X_{ij-1}(t), \begin{cases} 1 \leq i \leq M \\ 1 \leq j \leq N+1 \end{cases} \tag{6}$$

where

$$Y_{0j} = Y_{M+1j} = X_{i0} = X_{iN+1} = 0$$

by definition.

Following [7], a simplified flow model inside the X-direction and Y-direction channels can be developed from the above-mentioned assumptions [6]. With this approach the gap-averaged velocity and the pressure fields are described, for an incompressible flow, in terms of a single space coordinate and the continuity and momentum equations, in each channel. Exact integration of the continuity and momentum equation lead to the pressures $p_{ij}^{X}(x,t)$ and $p_{ij}^{Y}(y,t)$ along X-direction and Y-direction channels. The $X-$ and $Y-$ direction fluid forces acting (per unit length) on each rack was found as

$$F_{ij}^{X}(t) = \int_{-L_Y/2}^{L_Y/2} \left(p_{ij}^{Y}(y,t) - p_{ij+1}^{Y}(y,t)\right) dy, \tag{7}$$

$$F_{ij}^{Y}(t) = \int_{-L_X/2}^{L_X/2} \left(p_{i+1j}^{X}(x,t) - p_{ij}^{X}(x,t)\right) dx, \tag{8}$$

for $1 \leq i \leq M$ and $1 \leq j \leq N$. Note that the $2 \times M \times N$ motion dependent flow forces Eqs. (7)-(8) generated by this approach still contain the unknown integration functions $C_{ij}^{X}(t)$, $C_{ij}^{Y}(t)$, $p_{ij}^{X}(0,t)$ and $p_{ij}^{Y}(0,t)$.

Unknowns	Number
$X_{ij}(t)$	MN
$Y_{ij}(t)$	MN
$C_{ij}^{X}(t)$	$(M+1)N$
$C_{ij}^{Y}(t)$	$M(N+1)$
$p_{ij}^{X}(0,t)$	$(M+1)N$
$p_{ij}^{Y}(0,t)$	$M(N+1)$
Total	$6MN+2(M+N)$

Table 1: Total Number of Unknowns.

However, between rack or rack/wall positions ij, $ij+1$, $i+1j$ and $i+1j+1$, one can establish the additional equations we need to obtain this terms, namely, $(M+1)\times(N+1)-1$ linearly independent equations of compatibility of flow (mass conservation for all nodes but one), $4\times M\times N-(M-1)\times(N-1)$ linearly independent equations of compatibility of pressure (in all corners of each rack except $(M-1)\times(N-1)$ corners) and finally one last equation setting a reference for the pressure.

For simplicity, it was assumed that the racks are linear oscillating systems, with structural mass M_{s}, damping C_{s} and stiffness K_{s} (all these parameters being per unit length). Then, we can deduce the following fluid-structure coupled model:

$$M_{\mathrm{s}}\ddot{X}_{ij} + C_{\mathrm{s}}\dot{X}_{ij} + K_{\mathrm{s}}X_{ij} = F_{ij}^{X} + F_{ij,\mathrm{aut}}^{X}, \tag{9}$$

$$M_{\mathrm{s}}\ddot{Y}_{ij} + C_{\mathrm{s}}\dot{Y}_{ij} + K_{\mathrm{s}}Y_{ij} = F_{ij}^{Y} + F_{ij,\mathrm{aut}}^{Y}, \tag{10}$$

for $1 \leq i \leq M$ and $1 \leq j \leq N$ where F_{ij}^{X}, F_{ij}^{Y}, $F_{ij,\mathrm{aut}}^{X}$ and $F_{ij,\mathrm{aut}}^{Y}$ represent, respectively, the above-deduced fluid forces and the external autonomous forces per unit length. Here, the structural parameters have been assumed identical for both directions. However, dealing with asymmetrical systems bring no further difficulties whatsoever.

In Table 1 and 2 we summarize the above-mentioned unknowns and equations defining our nonlinear model for the flow-coupled vibratory responses of the system. In [6] one can find all the details of the derivations of our nonlinear model.

Equations	Number of Equations
Flow compatibility	$(M+1)(N+1)-1$
Pressure compatibility	$4MN-(M-1)(N-1)$
Reference for the pressure	1
Fluid-structure coupled equations	$2MN$
Total	$6MN+2(M+N)$

Table 2: Total Number of Equations.

All these equations represent a set of differential-algebraic equations (DAE's). That is, among those equations, some of them are pure algebraic constrains between unknowns. Note that this class of equations arise naturally in many applications but present numerical and analytical difficulties which do not occur with systems of ordinary differential equations [8]. In our case the DAE's developed can be classified as an implicit nonlinear differential-algebraic system of equations

Note that these equations can be written and established for generic systems of $M \times N$ racks entirely on a symbolic computer environment, as it was done here for the illustrative computations.

3 Numerical Simulations

Define the set of differential-algebraic equations corresponding to our model as

$$\mathbf{F}\left(\dot{\mathbf{v}}, \mathbf{v}, t\right) = 0 \tag{11}$$

	Pool A	Pool B
Racks $(M \times N)$	1×1	2×5
L_X (m)	2	2
L_Y (m)	2	2
H_i^X	0.2	0.02
H_j^Y	0.2	0.02

Table 3: Main Geometrical Parameters.

Structural mass, M_s (kg)	32000
Structural damping, C_s (N s/m)	8000
Structural stiffness, K_s (N/m)	5×10^6
Modal frequency in air, f_s (Hz)	2
Reduced damping in air, ζ	0.01
Water density, ρ (kg /m^3)	1000
friction factor, f	0.01

Table 4: Physical Modal Parameters.

where $\mathbf{v}$ is the vector of unknowns. The simplest first order backward difference formula is the implicit Euler method

$$\mathbf{F}\left(\frac{\mathbf{v}_{n+1}-\mathbf{v}_n}{t_{n+1}-t_n}, \mathbf{v}_{n+1}, t_{n+1}\right) = 0 \tag{12}$$

in which equation (11) is approximated by finite differences [8]. In the present work we used a fourth and fifth-order generalization of (12) coded in MATLAB [9].

All numerical simulations were performed with the main geometrical, physical and structural modal parameters presented in Tables 3 and 4. The reference time-step used, less than $\triangle t = 0.005$, was one order of magnitude smaller than $1/(2f_{\max})$, with $f_{\max} \approx 20$ Hz (maximum frequency of the system and/or excitation in our computation).

As one can observe from Table 3, two sets of numerical simulations were performed. One, using a small-storage pool with a single centered rack (pool A) and other set using a pool with ten racks regularly stored in 2 lines and 5 columns (pool B).

4 Results and Discussion

4.1 Pool A

The response of a single centered rack – pool A – to a strong impulsive excitation applied along the direction X is displayed in Fig. 2. In this figure we display also the

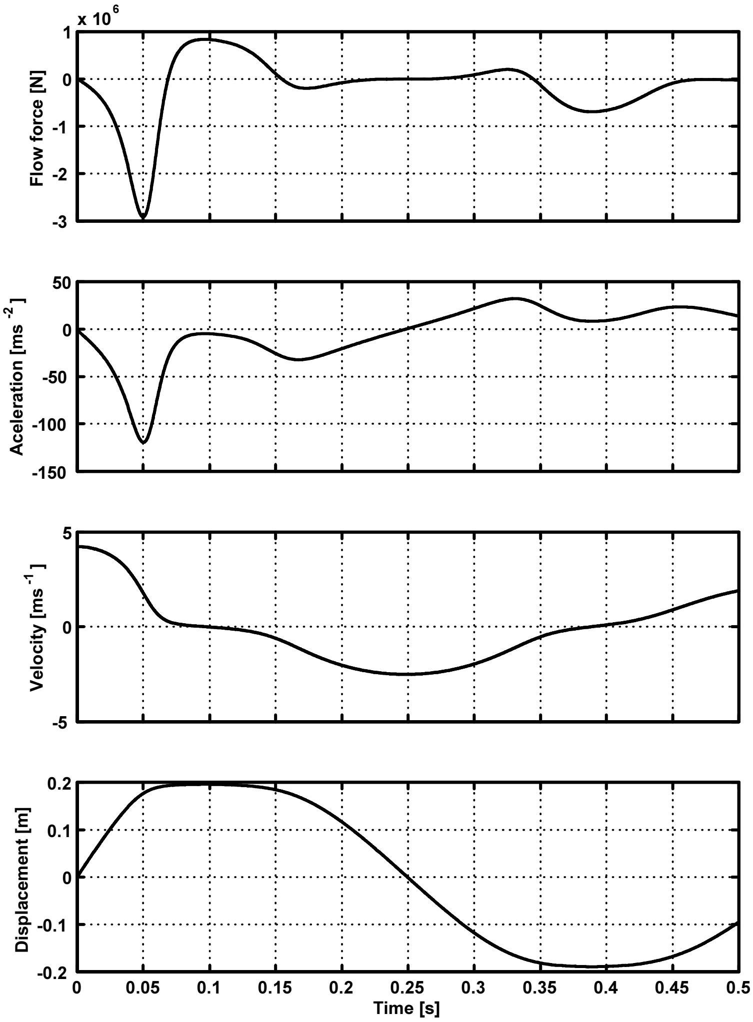

Figure 2: Pool A: Rack response to a strong impulse, $f = 0.01$.

flow force exerted on the rack as well as its acceleration and velocity. The simulation was performed using a flow skin friction factor $f = 0.01$ and a steady fluid gap of 0.2 m. Observing the rack response in we note clearly the occurrence of squeeze-film phenomena, namely, for instance, between 0.05 and 0.16 s. The flow forces, which were computed in each time-step, are maximum at beginning and end of the film-squeezing. Clearly dissipative effects of the flow friction are obvious.

4.2 Pool B

In the second set of numerical simulations, the response to a seismic excitation of a storage pool with 10 racks regularly stored in 2 lines and in 5 columns– pool B, are tested. Note that steady fluid gap is now 0.02 m. The seismic excitations used were based on the Loma Prieta earthquake in October 17, 1989, projected, in each simulation, along one of 7 different directions varying in steps of 15^o from the X-direction (0^o degrees) up to the Y-direction (90^o degrees). Note that, due the symmetrical arrangement of the racks in the pool, as well as, the symmetry of the structural modal parameters, the behaviour of our system under the same seismic excitation projected along different multiple of 15^o, falling between 90^o and 360^o, can be inferred from the above mentioned simulations.

The E-W and N-S traces (accelerogram) were obtained and supplied by the Natural Sciences Laboratory at U. C. Santa Cruz and is displayed in Fig. 3.

Angle	$\max \lvert X_{ij} \rvert$	$\max \lvert Y_{ij} \rvert$	$\max \sqrt{X_{ij}^2 + Y_{ij}^2}$
0^o	0.0373	0.0336	0.0428
15^o	0.0409	0.0359	0.0435
30^o	0.0391	0.0323	0.0410
45^o	0.0405	0.0291	0.0433
60^o	0.0530	0.0388	0.0542
75^o	0.0533	0.0386	0.0539
90^o	0.0451	0.0276	0.0452

Table 5: Maximum displacements observed.

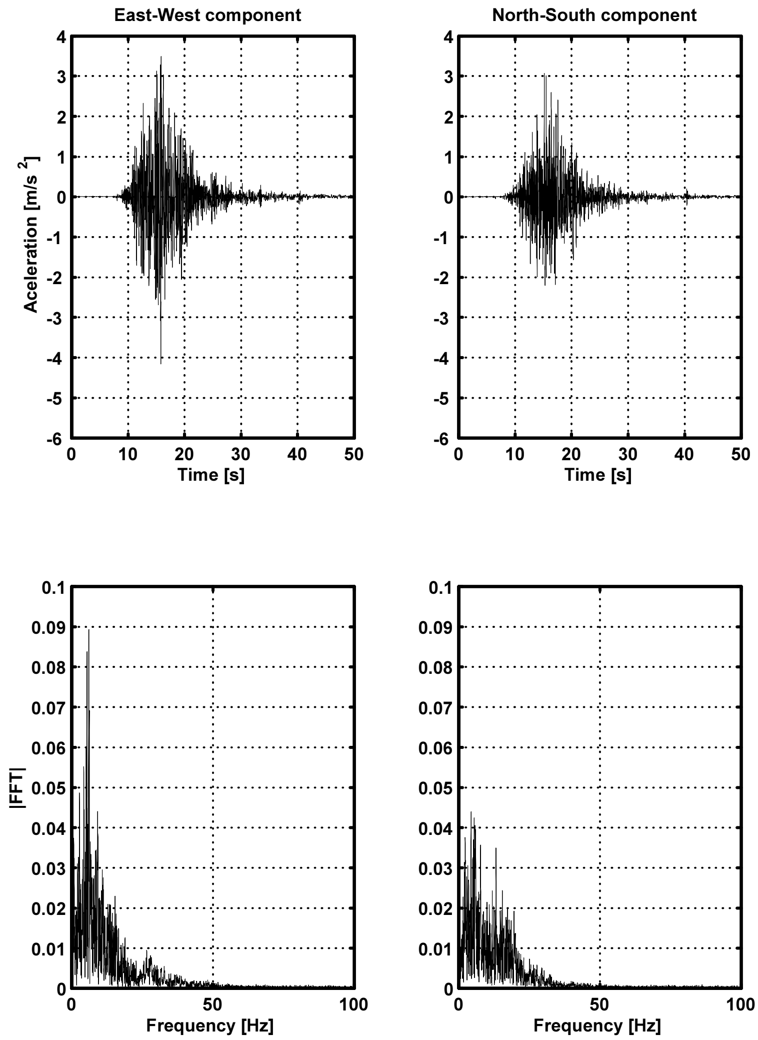

Figure 3: Accelerograms and Fourier transforms of the Loma Prieta earthquake in October 17, 1989.

Figure 4: Scaled trajectories of the racks (angle of 30^o).

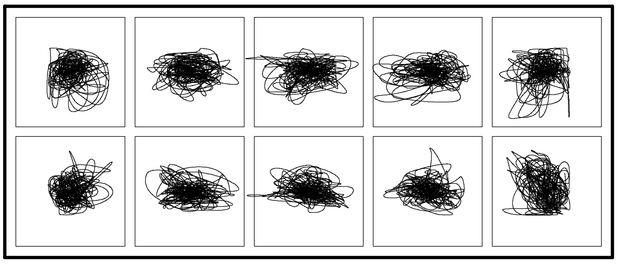

Figure 5: Scaled trajectories of the racks (angle of 60^o).

The maximum displacement of the racks in each symmetry direction and the maximum displacement with respect to the initial position of each rack is presented in Table 5. One can observe that for angles of 30^o and 60^o degrees the maximum displacements are respectively minimum and maximum. Notice that although the maximum displacements appear larger than the average fluid gap, which is 0.02 m, due the compliant motion of the racks, there is never penetration, as it should be.

In Figs. 4 and 5 one can observe the scaled trajectories (in order to make them clearly observable) of each rack for the above-mentioned projection angles of 30^o and 60^o. The occurrence of the phenomena of squeeze film can be perceived in both simulations, namely, for instance, observing the trajectories near the walls of rack $(1, 5)$ in Fig. 4 or rack $(1, 1)$ in Fig. 5. The phenomenon of squeeze-film is more significant in Fig. 5. For this projection angle of the seismic excitation the different coupled pseudo-modes of our system were more efficiently exited.

The corresponding spectral response of rack $(1, 1)$ for such simulations, can be observed in Figs. 6 and 7. The spectral response display significant different coupled pseudo-modes between 0.5 Hz and 1 Hz. Note that in air the modal frequency of the structure was 2 Hz. This difference is due to the fluid added-mass effect. Observe also, in Figs. 6 and 7, that the overall spectral response of rack $(1, 1)$ is diluted over a larger frequency interval (between 0.2 Hz and 2 Hz). This energy spread is due to the nonlinear flow forces related to the large amplitude responses and squeeze-film phenomena.

Overall it can be stated, based on the results of Table 5, that the earthquake ”direction” (projection angle) is not a crucial aspect, for the system studied here. However we feel that the methodology illustrated can be used in other related systems.

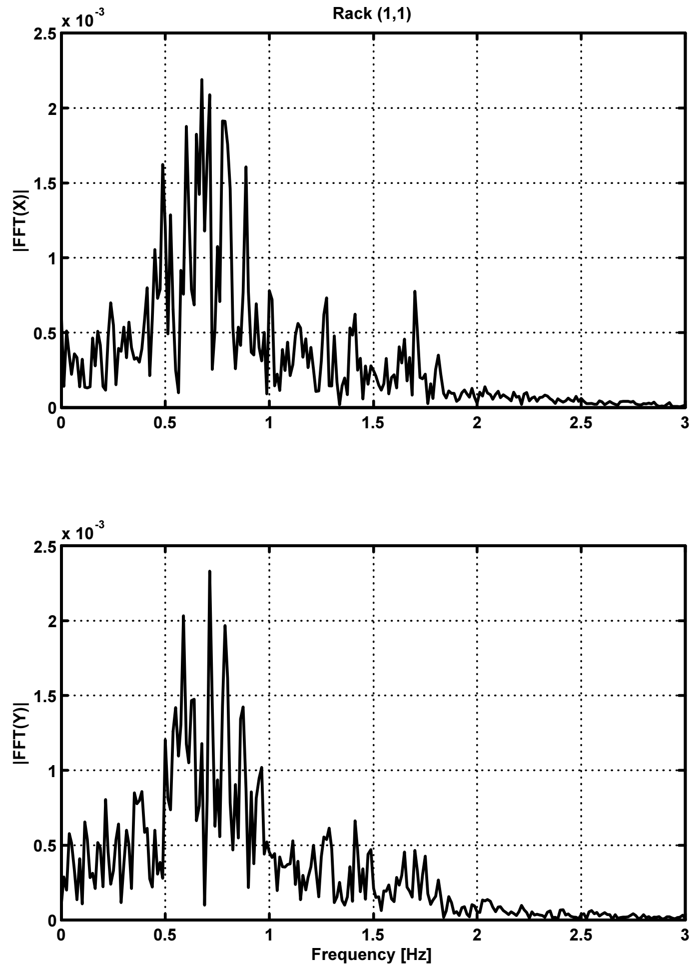

Figure 6: Spectral response of rack $(1, 1)$ to the seismic excitation applied along an angle of 30^o.

5 Conclusion

In this paper we explore a nonlinear model for fluid-coupled vibrations of spent nuclear racks, based on the main simplifying assumptions: (i) 3-D effects were neglected, (ii) small gaps between the fuel assemblies and between these and the container, when compared with the longitudinal length scales.

Although algebraically involved, the proposed approach can be automatically implemented on a symbolic computer environment, leading to a system of DAE's which is then solved through an adequate time-step integration solver.

This nonlinear 2-D model accounts for squeeze-film and dissipative phenomena related with large amplitude responses and/or small fluid gaps.

Regardless of the fact that neglecting 3-D flow effects can lead to an overestimation

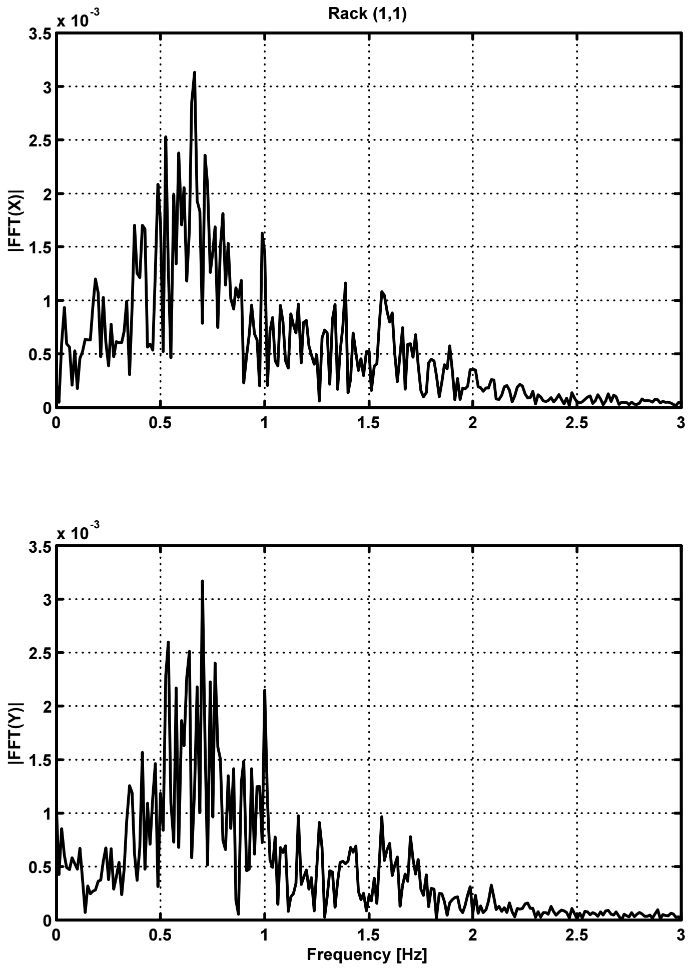

Figure 7: Spectral response of rack $(1,1)$ to the seismic excitation applied along an angle of 60^o.

of the true flow added mass effects [2, 10] this model can produce realistic predictions of the displacements and squeeze-film forces taking place on immersed spent fuel racks, when excited by a seismic event, contributing to a better understanding of the complex dynamic behaviour of such systems.

Future work will address 3-D dimensional flow aspects, which are undoubtedly significant in the present context.

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