OSCILLATION CRITERIA OF DIFFERENCE EQUATIONS WITH SEVERAL DEVIATING ARGUMENTS

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Abstract. New sufficient conditions for the oscillation of all solutions of difference equations with several deviating arguments are presented. Corresponding difference equations of both retarded and advanced type are studied. The significance of the conditions established are demonstrated by comparing with known oscillation conditions. Examples illustrating the results are also given.

1. Introduction

In this paper we study the oscillation of all solutions of the difference equation with several variable retarded arguments of the form

$$\Delta x(n) + \sum_{i=1}^{m} p_i(n)x(\tau_i(n)) = 0, \quad n \in \mathbb{N}_0, \ m \in \mathbb{N}$$ \hfill (E_R)

and the (dual) difference equation with several variable advanced arguments of the form

$$\nabla x(n) - \sum_{i=1}^{m} p_i(n)x(\sigma_i(n)) = 0, \quad n \in \mathbb{N}, \ m \in \mathbb{N},$$ \hfill (E_A)

where \((p_i(n)), \ 1 \leq i \leq m\) are sequences of nonnegative real numbers, \((\tau_i(n)), \ 1 \leq i \leq m\) are sequences of integers such that

$$\tau_i(n) \leq n - 1 \quad \forall n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n \to \infty} \tau_i(n) = \infty, \ 1 \leq i \leq m \quad (1.1)$$

\((\sigma_i(n)), \ 1 \leq i \leq m\) are sequences of integers such that

$$\sigma_i(n) \geq n + 1 \quad \forall n \in \mathbb{N}, \quad 1 \leq i \leq m. \quad (1.2)$$

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