COMPARISON AND OSCILLATION THEOREM FOR
SECOND-ORDER NONLINEAR NEUTRAL
DIFFERENCE EQUATIONS OF MIXED TYPE

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ABSTRACT. In this paper, we establish some comparison theorems for the oscillation of second order neutral difference equations of mixed type

\[ \Delta (a_n \Delta (x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2})^\alpha) + q_n x_{n-\tau_1}^{\beta} + p_n x_{n+\tau_2}^{\beta} = 0, \]

where \( \alpha \) and \( \beta \) are ratio of odd positive integers, \( \sigma_1, \sigma_2, \tau_1 \) and \( \tau_2 \) are positive integers. Our results are new even if \( p_n = c_n = 0 \). Examples are provided to illustrate the results.

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1. INTRODUCTION

In this paper, we shall study the oscillatory behavior of the second order nonlinear neutral difference equation of mixed type

(1.1) \[ \Delta (a_n \Delta (x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2})^\alpha) + q_n x_{n-\tau_1}^{\beta} + p_n x_{n+\tau_2}^{\beta} = 0, \]

where \( n \geq n_0 \in \mathbb{N} \), subject to the following conditions:

(H1) \( \{a_n\} \) is a positive sequence for all \( n \geq n_0 \) and \( \sum_{n=n_0}^{\infty} \frac{1}{a_n} = \infty \);

(H2) \( \{b_n\} \) and \( \{c_n\} \) are nonnegative sequences such that \( 0 \leq b_n \leq b \) and \( 0 \leq c_n \leq c \), where \( b \) and \( c \) are constants;

(H3) \( \{p_n\} \) and \( \{q_n\} \) are nonnegative real sequences and not eventually zero for many values of \( n \);

(H4) \( \sigma_1, \sigma_2, \tau_1 \) and \( \tau_2 \) are nonnegative integers and \( \alpha \) and \( \beta \) are ratio of odd positive integers.

We put \( z_n = (x_n + b_n x_{n-\sigma_1} + c_n x_{n+\sigma_2})^\alpha \). By a solution of equation (1.1), we mean a real sequence \( \{x_n\} \) defined for all \( n \geq n_0 - \max \{\sigma_1, \tau_1\} \), and satisfies equation (1.1) for