OSCILLATION OF NONLINEAR DIFFERENCE EQUATIONS WITH DELAYED ARGUMENT

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ABSTRACT. The following difference equation with delayed argument

\[ \Delta^2 u(k) + F(k, u(\tau(k))) = 0 \]

is considered, where \( F : \mathbb{N} \times \mathbb{R} \to \mathbb{R} \), \( \tau : \mathbb{N} \to \mathbb{N} \), \( \tau(k) \leq k \) for \( k \in \mathbb{N} \), \( \lim_{k \to +\infty} \tau(k) = +\infty \) and \( \Delta u(k) = u(k + 1) - u(k) \), \( \Delta^2 = \Delta \circ \Delta \). In the paper sufficient (necessary and sufficient) conditions are established for all proper solutions of the above equation to be oscillatory.

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1. INTRODUCTION

Consider the difference equation

\[ \Delta^2 u(k) + F(k, u(\tau(k))) = 0, \quad (1.1) \]

where \( F : \mathbb{N} \times \mathbb{R} \to \mathbb{R} \), \( \tau : \mathbb{N} \to \mathbb{N} \), \( \Delta u(k) = u(k + 1) - u(k) \) and \( \Delta^2 = \Delta \circ \Delta \). Everywhere it will be assumed that

\[ F(k, x) \text{ sign } x \geq 0 \quad \text{for} \quad k \in \mathbb{N} \quad \text{and} \quad x \in \mathbb{R}, \quad (1.2) \]

\[ \tau(k) \leq k \quad \text{for} \quad k \in \mathbb{N}, \quad \lim_{k \to +\infty} \tau(k) = +\infty \quad (1.3) \]

and

\[ \sup \{|F(i, x)| : i \geq k\} > 0 \quad \text{for} \quad k \in \mathbb{N}, \quad x \neq 0. \quad (1.4) \]

For any \( n \in \mathbb{N} \), denote \( N_n = \{n, n + 1, \ldots\} \).

Definition 1.1. For \( n \in \mathbb{N} \) put \( n_0 = \min\{\tau(k) : k \in N_n\} \). A function \( u : N_{n_0} \to \mathbb{R} \) is said to be a proper solution of (1.1), if it satisfies (1.1) on \( N_n \) and

\[ \sup \{|u(i)| : i \geq k\} > 0 \quad \text{for} \quad k \in \mathbb{N}. \]