EXPERIMENTAL VALIDATION OF THEORETICAL MODELS FOR THE LINEAR AND NONLINEAR VIBRATIONS OF IMMERSED ROTORS

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ABSTRACT

Vibration of rotating shafts has been studied for different gap geometries, ranging from bearing configurations to pump systems. This paper deals with the rotor-flow dynamics of immersed shafts under moderate confinement—clearance gap about \( \delta = \frac{H}{R} = 0.1 \) (where \( H \) is the average gap and \( R \) is the rotor radius). Following simplified assumptions, analytical models for the linearized forces, for both centered and eccentric immersed rotors have been developed as well as a theoretical nonlinear model which fully describes the nonlinear flow terms. These models were supported by encouraging results from preliminary experiments. In the present paper, we discuss some recent and representative results of an extensive series of tests performed on a small-scale model, in order to assert the validity of our theoretical models.

From the overall experimental programme, the following conclusions emerged: (1) The linearized bulk-flow model is adequate, provided the dissipative effects are duly accounted for using an empirical friction coefficient to empirically model the turbulent stresses. Such predictions are quite accurate if the system is working at low rotor eccentricities and far from the instability boundaries. (2) However, for large rotor eccentricities and for dynamic regimes near the linear instability, the fully nonlinear model leads to better predictions. Obviously, these effects are instrumental to obtaining reasonable predictions for all post-stable motion regimes. (3) When discrepancies arise, the nonlinear model was usually found to be conservative.

INTRODUCTION

The effects of co-rotating annular flows on rotor vibrations have been studied extensively by many researchers, mostly in connection with bearings and seals [see for instance (Childs, 1993)]. However, flow structure interaction can also lead to significant effects in less common devices. In this paper we will address the dynamical behavior of immersed rotors, such as found in fast-breeder nuclear reactor pumps (for circulating the liquid sodium), and other applications. In such components, the clearance ratio \( \delta = \frac{H}{R} \) (where \( H \) is the average gap and \( R \) is the rotor radius) is typically about 0.1—one or two orders of magnitude higher than the clearances found in bearings and seals. As a consequence, the flow is quite turbulent, inertial effects are then of prime importance and cannot be neglected as assumed in the basic Reynolds equation approach. Also, the shaft length subject to fluid forces is quite significant, these combined effects leading to specific rotor-dynamic properties.

Using simplified assumptions for the flow, we have developed analytical models for the linearized forces, for both centered and eccentric rotors [(Axisa and Antunes, 1992), (Antunes et al., 1996) and (Moreira et al., 2000)]. Because the flow was modeled as being two-dimensional, it was possible to extend our analytical solutions to fully account for the nonlin-
Table 1. MAIN PHYSICAL PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor length, ( L ) (m)</td>
<td>0.365</td>
</tr>
<tr>
<td>Rotor radius, ( R ) (m)</td>
<td>0.0251</td>
</tr>
<tr>
<td>Annular gap, ( H ) (m)</td>
<td>0.002</td>
</tr>
<tr>
<td>Clearance ratio, ( \delta = \frac{H}{R} )</td>
<td>0.08</td>
</tr>
<tr>
<td>Length-to-diameter ratio, ( \frac{L}{D} )</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 2. PHYSICAL AND MODAL PARAMETERS OF THE PLANAR EXPERIMENTS.

<table>
<thead>
<tr>
<th>Eccentricity, ( \varepsilon )</th>
<th>P1</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural mass, ( M_s ) (kg)</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Mass ratio ( \gamma = M_s/M_a )</td>
<td>1.2</td>
<td>1.3</td>
</tr>
<tr>
<td>X str. stiff., ( K_s^X ) (N/m)</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>Y str. stiff., ( K_s^Y ) (N/m)</td>
<td>5.1</td>
<td>5.0</td>
</tr>
<tr>
<td>Freq. in air (Hz)</td>
<td>7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>Freq. in water (Hz)</td>
<td>5.1</td>
<td>5.0</td>
</tr>
<tr>
<td>Damping in air (%)</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Damping in water (%)</td>
<td>3.7</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Figure 1. ORBITAL SETUP.

Figure 2. PLANAR SETUP DETAILS.

cardan coupling for torque transmission. To avoid surface effects and flow ventilation at higher spinning velocities, a large-clearance labyrinth was used. Rotor vibrations were measured with eddy-current displacement transducers, located near the support plate. Excitation in planar experiments was provided by a non-contact electro-mechanical shaker and, in orbital experiments, by two electro-mechanical shakers, driven by filtered random noise. The excitation force was measured using piezo-electric transducers. Accelerometers were also used in order to perform an accurate sensor calibration and improve measurement accuracy. In Figure 2, some details of sensor setup can be observed (planar experiments). The modal parameters of the coupled system and the non-linear dominant response frequencies and damping were identified using the ERA method—Eigensystem Realization Algorithm (Juan, 1994).

The main physical parameters of the test rig are shown in Table 1. The experimental modal parameters of the two planar and the two orbital tested configurations, which are discussed here, were labeled respectively P1, P2, O1 and O2. The corresponding parameters can be seen in Tables 2 and 3.
### Table 3. PHYSICAL AND MODAL PARAMETERS OF ORBITAL EXPERIMENTS.

<table>
<thead>
<tr>
<th></th>
<th>O1</th>
<th>O2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity, $\varepsilon$</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Structural mass, $M_s$ (kg)</td>
<td>7.1</td>
<td>7.1</td>
</tr>
<tr>
<td>Mass ratio $\gamma = M_a/M_s$</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>$X$ str. stiff., $K^X_{\gamma}$ ($N/m$)</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>$Y$ str. stiff., $K^Y_{\gamma}$ ($N/m$)</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>Freq. in air (Hz)</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>Freq. in water (Hz)</td>
<td>4.5</td>
<td>4.4</td>
</tr>
<tr>
<td>Damping in air (%)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Damping in water (%)</td>
<td>4.0</td>
<td>4.1</td>
</tr>
</tbody>
</table>

**THEORETICAL PREDICTIONS**

For the complex modal computations of the rotor-flow coupled system, the improved linear model [deduced from the linearized forms of equations (8)] for rotors subject to dissipative annular flows developed by Moreira et al., 2000, was used. In addition to the forward and backward whirling modes predicted by the classic linear theory the new improved model also presents a zero-frequency eigenvalue related to the co-rotating flow.

**Nonlinear numerical simulations** were based in the model for nonlinear orbital motions, developed by Moreira et al., 1999. The rotodynamic nonlinear equations for this model, are as follows

\[
\begin{align*}
\mathbf{X} &= \mathbf{F}_1 [\mathbf{X}, \mathbf{X}, \mathbf{Y}, \mathbf{Y}, \mathbf{C}, F^\text{ext}_X, t] \quad (1) \\
\mathbf{Y} &= \mathbf{F}_2 [\mathbf{X}, \mathbf{X}, \mathbf{Y}, \mathbf{Y}, \mathbf{C}, F^\text{ext}_Y, t] \quad (2) \\
\mathbf{C} &= \mathbf{F}_3 [\mathbf{X}, \mathbf{X}, \mathbf{Y}, \mathbf{Y}, \mathbf{C}, F^\text{ext}_X, F^\text{ext}_Y, t] \quad (3)
\end{align*}
\]
where \( X, Y \) define the rotor motion, \( C = C(t) \) is an auxiliary flow variable, \( F_X^{\text{ext}}, F_Y^{\text{ext}} \) represent autonomous forces and \( t \) time. The nonlinear formulation, for the fluidelastic forces \( F_X(t) \) and \( F_Y(t) \) (and an additional auxiliary equation) which lead to equations (1), (2) and (3) was deduced (under simplifying assumptions) from the continuity equation for incompressible flow and the momentum equation (projected in the tangential direction),

\[
0 = \frac{\partial h}{\partial t} + \frac{1}{R} \frac{\partial (hu)}{\partial \theta},
\]

\[
0 = \rho \left( \frac{\partial (hu)}{\partial t} + \frac{1}{R} \frac{\partial (hu^2)}{\partial \theta} \right) + \tau_s + \tau_r + \frac{h}{R} \frac{\partial p}{\partial \theta},
\]

where \( h \) is the annular gap depth

\[
h(\theta, t) = H - X(t) \cos \theta - Y(t) \sin \theta,
\]

\( \rho \) is the fluid density, \( p \) is the gap-averaged pressure and \( \tau_s \) and \( \tau_r \) the shear stresses at the rotor and stator walls.

After obtaining the velocity field from equation (4)

\[
u(\theta, t) = \frac{R \left( \frac{X(t)}{H} \sin \theta - \frac{Y(t)}{H} \cos \theta + C(t) \right)}{H - X(t) \cos \theta - Y(t) \sin \theta},
\]

the fluidelastic forces \( F_X(t) \) and \( F_Y(t) \) (and the additional auxiliary equation) can be deduced from equation (5):

\[
\begin{align*}
F_X(t) &= -\rho R^2 L_k \left[ \frac{X(t)}{H} \sin \theta - \frac{Y(t)}{H} \cos \theta + C(t) \right], \\
F_Y(t) &= -\rho R^2 L_1 \left( \frac{X(t)}{H} \sin \theta - \frac{Y(t)}{H} \cos \theta + C(t) \right) + \rho R L_2 \left( \frac{X(t)}{H} \sin \theta - \frac{Y(t)}{H} \cos \theta + C(t) \right) + R^2 L_3 \left( \frac{X(t)}{H} \sin \theta - \frac{Y(t)}{H} \cos \theta + C(t) \right), \\
\rho I_1(t) + \frac{\rho}{R} I_2(t) + I_3(t) &= 0,
\end{align*}
\]

where \( I_1(t), I_2(t), I_3(t) \) depend on the following azimuthal integrals

\[
G_{ij}^k (H, X, Y) = \int_0^{2\pi} \frac{\sin^i \theta \cos^j \theta}{(H - X \cos \theta - Y \sin \theta)^k} d\theta
\]

for \( 0 \leq i, j, k \leq 4 \).

All details of these theoretical formulations are presented in Moreira et al., 1999 and Moreira et al., 2000.

Figure 6. PLANAR CONFIGURATION P1: \( \epsilon = 0 \)
\(-, \text{LINEAR THEORY}; \times, \text{NON-LINEAR SIMULATIONS}; \ast, \text{EXPERIMENTS}\).

Figure 7. PLANAR CONFIGURATION P1: \( \epsilon = 0 \)
\(-, \text{LINEAR THEORY}; \times, \text{NON-LINEAR SIMULATIONS}; \ast, \text{EXPERIMENTS}\).
RESULTS AND DISCUSSION

The main results of our tests and computations are resumed in several figures which show the modal frequencies and damping values of the system as a function of the rotor eccentricity and spinning velocity. In these figures we display the real part $\sigma_n$ as well as the imaginary part $\omega_n$ of the complex eigenvalues $\lambda_n = \sigma_n + i\omega_n$ of the linearized model. Also the corresponding identified response frequencies and damping values from experiments and numerical simulations are displayed.

The predicted modal behavior of eccentric planar and orbital configurations $P2$ and $O2$ are illustrated in Figures 3 and 4.

In addition to the forward and backward whirling modes, there is a zero-frequency mode, related to the co-rotating flow, which can be identified in the figures exhibiting a non zero damping. The computed modes in Figures 3 and 4 are identified in the plots using the following codes: Planar configurations—$V$ for the vibrating mode and $Z$ for the zero-frequency eigenvalue; Orbital configurations—$F$ and $B$ respectively for the forward and backward whirling modes and $Z$ for the zero-frequency mode.

Note that, because system dynamics are strongly dependent on the rotor eccentricity, the linearized predictions of rotor dynamics, were computed using an estimate of the actual eccentricity at each spinning velocity, which was obtained from nonlinear simulations. This estimate proved to be accurate as confirmed by the results shown in Figure 5.

The main theoretical and experimental results of this paper are summarized in Figures 6–9 (planar tests) and 10–13 (orbital tests).

Planar Configurations

In Figures 6, 7, 8 and 9 one can observe the identified response frequencies and damping values which were identified from the experiments and nonlinear simulations. These are superimposed to the modal frequencies and damping predictions stemming from the linear theory, for both centered and eccentric planar configurations $P1$ and $P2$.

In planar centered configuration $P1$, linear and nonlinear theory agree very well with experiments below the predicted instability onset. In this range the dynamical behavior of the system is linear. Linear instability is predicted at about 620 rpm. Above this range and below 1000 rpm the system becomes linearly unstable and so only the nonlinear simulations yields results which are similar to experiments. In Figure 14 one can detect a deterioration of the values displayed by the coherence function (in experiments) above 900 rpm. Post-stable motions are very well predicted by the nonlinear theory.

In planar configuration $P2$ (static reduced eccentricity equal to 0.5) linear theory predicts instability by flutter at about 520 rpm, which is a lower value than obtained from both the experiments and non-
linear theory. Indeed high-amplitude motions were obtained only at about 600 rpm (nonlinear numerical simulations) and 800 rpm (experiments).

From Figure 8 one can notice that the system vibrating frequency depart from linear predictions at about 500 rpm confirming the significance of the nonlinear effects. Therefore, we have found that nonlinear flow forces are very significant for the eccentric configuration, even at relatively low spinning velocities. This is attested by the low values of the coherence function (in experiments) displayed in Figure 15.

Orbital Configurations

The identified response frequencies and damping from experiments and nonlinear simulations, for the orbital configurations 01 and 02 as well as the modal frequencies and damping values from the linear theory, are shown in Figures 10, 11, 12 and 13.

In the orbital centered configuration 01, linear and nonlinear formulations agree very well below 600 rpm. In this range both formulations give acceptable predictions of the experimental results. Indeed the system behavior is essentially linear, as confirmed by the high values of the coherence function shown in Figure 16. These plots were computed using the system responses obtained from the numerical simulations of the nonlinear flow model.

Flutter instability is predicted at about 610 rpm by both linear and nonlinear formulations. However, large amplitude motions were experimentally obtained only at about 900 rpm. The theoretical model fails to predict the onset of instability, but is nevertheless conservative. We suspect that such behavior might be due to three-dimensional flow end-effects, related to the relatively low $L/D$ ratio of our experimental rig. Indeed, the simplified two-dimensional flow model used for our predictions is inadequate if three-dimensional flow effects are significant. At the present time it is difficult to quantify the magnitude of such effects.

In the eccentric orbital configuration 02 the linearized and nonlinear formulations agree only up to 300 rpm. In Figure 17 one can observe high coherence values displayed below 400 rpm. Beyond this range, as in the planar case, nonlinear effects become significant. Indeed, the linearized predictions fail completely for all spinning velocities higher than 400 rpm.

Interestingly, linear instability is predicted for the backward mode at about 360 rpm. However, both experiments and nonlinear computations agree that the post-stable nonlinear system will display high-amplitude forward whirling orbits. The qual-
Conclusions

Besides the fact that the nonlinear model can predict the rotor dynamic eccentricity, which strongly governs the system dynamics and whose knowledge is essential for linear predictions, the present results show that:

1. Both linear and nonlinear formulations agree with experiments when the spinning velocity and the dynamic eccentricity are low enough. In this case the system dynamics are practically linear.

2. For higher spinning velocities and at significant dynamic eccentricities, nonlinear effects become dominant. The results suggest that experiments were performed under linear instability conditions, the response being stabilized by nonlinear effects. The importance of nonlinear flow forces was confirmed for eccentric configurations, well below the instability boundary.

3. It appears that post-stable whirling motion regimes (limit cycles) were also reasonably well predicted by the nonlinear flow model.
4. However, nonlinear numerical simulations lead to lower apparent damping values than experimentally identified. This may be due to three-dimensional flow effects not accounted for by the theoretical assumptions of the model. As a result of the lower damping exhibited, the nonlinear model is conservative. Instability is typically predicted at velocities which are $10\% - 30\%$ lower than the experimental observations.

In spite of some verified qualitative differences most probably due to the 3D flow effects already mentioned, it can be concluded that the nonlinear flow model leads to better predictions of the system dynamics.

**NOMENCLATURE**

- $C(t)$: auxiliary flow variable.
- $F_X, F_Y$: nonlinear fluid-elastic forces.
- $F_{ext}, F_{ext}$: external forces.
- $h(\theta, t)$: annular gap depth.
- $H$: average annular gap.
- $K_s^X, K_s^Y$: structural stiffness.
- $L$: rotor length.
- $M_a$: added mass: $\pi \rho H^3 L / H$.
- $M_s$: structural mass.
- $p(\theta, t)$: gap-averaged pressure.
- $R$: rotor radius.
- $t$: time.
- $X(t), Y(t)$: rotor motions.
- $\gamma$: mass ratio: $M_a / M_s$.
- $\delta$: clearance ratio: $H / R$.
- $\varepsilon$: reduced static eccentricity.
- $\lambda_n$: eigenvalue of the flow-struc. system.
- $\theta$: azimuthal angle.
- $\rho$: fluid density.
- $\sigma_n$: real part of the eigenvalue $\lambda_n$.
- $\tau_r, \tau_s$: shear stresses.
- $\omega_n$: imaginary part of $\lambda_n$. 

Figure 15. PLANAR CONFIGURATION P2: $\varepsilon = 0.5$ TRANSFER AND COHERENCE FUNCTIONS (EXPERIMENTS).

Figure 16. ORBITAL CONFIGURATION O1: $\varepsilon = 0$ TRANSFER AND COHERENCE FUNCTIONS (NONLINEAR SIMULATIONS).

Figure 17. ORBITAL CONFIGURATION O1: $\varepsilon = 0.5$ TRANSFER AND COHERENCE FUNCTIONS (NONLINEAR SIMULATIONS).
REFERENCES


