INTRODUCTION

National standards with design rules for stiffened flanges used in steel box girder bridges were almost non-existent until the early nineteen seventies. These design rules were based on the elastic critical stress ($\sigma_{cr}$) obtained by the linear buckling theory of plates with some correction factors to take into account the favourable effect of the post-buckling strength reserve [1]. Research work to establish new design rules for plated structures was strongly increased in the early nineteen seventies as a result of well-known accidents during construction involving box girder bridges [1]. Although the behaviour and strength of stiffened plates have been extensively studied over the last forty years, there are still some inconsistencies and gaps between the results of some studies and the results obtained through the design rules established in the European and American standards [2,3], which are usually adopted by the bridge designers to solve the problem of predicting the ultimate strength of stiffened plates.

As the current design rules laid down in the European and American standards do not recognize the effect of boundary conditions for the in-plane displacements at the edges to obtain the ultimate strength of stiffened plates, the use of these design rules can give inaccurate results and in some cases against safety.

This paper presents a design method for stiffened flanges used in steel box girder bridges based on a set of design curves that take into account all relevant parameters regarding the ultimate strength of stiffened flanges, including the real boundary conditions for the in-plane displacements at the edges. These design curves were developed and calibrated based on the results obtained with nonlinear analysis using the semi-analytical and finite element methods and they were validated by comparison with the results of experimental tests in accordance with the target failure probability of the European standard EN-1990 [4]. Finally, it is presented a comparison between the ultimate strength obtained using the proposal design curves with that obtained using the nonlinear analysis and the current bridge design rules.

1 STIFFENED PLATES USED IN STEEL BOX GIRDER BRIDGES

The most widely used bridge structural elements that resort to the use of steel plates are the plate and box girders. Steel plates used in these structural elements are outstand flanges, internal flanges and webs.

Outstand flanges have a free longitudinal edge and they are used as upper and lower flanges of the plate girders and narrow top flanges in open top box girders of composite sections. Internal flanges are used as bottom flanges of box girders and wide top flange in closed box girders of steel sections. In wide flanges and webs of considerable height it is common to use longitudinal stiffeners to reduce the problems associated with the effects of buckling.

The study of the behaviour and strength of steel plates with longitudinal stiffeners is very complex because it depends on a large number of variables, such as the shape and amplitude of the initial geometric imperfections, the residual stresses, the boundary conditions for the in-plane and out-of-plane displacements, the load conditions and the geometric and material data. Recent numerical studies [5,6] showed that the behaviour and strength of steel plates are very sensitive to in-plane...
displacements boundary conditions and the European and American standards [2,3] do not provide a definition and clear guidance for the proper use of the simplified rules available to designers. These rules may give considerable errors, particularly when applied to the safety verification of stiffened plates where the transverse in-plane displacements at longitudinal edges cannot be considered as uniform (in-plane displacements perpendicular to the longitudinal edges constrained to remain straight).

In European standard [2] the resistance assessment of webs and internal flanges with longitudinal stiffeners used in steel box girder bridges is performed through the same criterion and independent of the in-plane displacement boundary conditions of the plate. Just apparently both elements may have the same in-plane displacement boundary conditions. Indeed, the webs are usually connected to flanges that possess sufficient rigidity to consider uniform the in-plane displacements at longitudinal edges, while the flanges are connected to webs that usually do not provide this type of constraint and it is more conservative to consider free the in-plane displacements at longitudinal edges, as shown in the work of Ferreira [5] through a comparison between numerical and experimental results.

The design proposal presented in this paper for stiffened flanges aims to fill a gap in the current European bridge design rules, which should not be applied to stiffened plates with the fully free transverse in-plane displacements at longitudinal edges.

2 DESIGN PROPOSAL

2.1 General

The design proposal is a model based on design curves (arithmetic expressions) describing the influence of all relevant parameters on the ultimate strength of stiffened flanges used in steel box girder bridges. It was developed and calibrated based on the results obtained with the semi-analytical and finite element methods and it was validated by comparison with the results of experimental test in accordance with the target failure probability of EN-1990 [4].

These design curves consider that the plate is subject to compressive longitudinal direct stress with a stress ratio $\psi$ higher than 0.5 (ratio between the lower compressive longitudinal stress to the higher compressive longitudinal stress), the transverse stiffeners provide rigid support lines, the longitudinal stiffeners are fully effective and equally spaced and the plates have the longitudinal in-plane displacements constrained to remain straight at loaded edges (transverse edges) and the other in-plane displacements fully free. This design model does not consider the effects of local buckling of the stiffener elements and tripping of the stiffener. These effects can be avoided by adopting minimum values for the geometric properties of the longitudinal stiffeners as used in European and American standards [2,3].

2.2 Design procedure

The design proposal is a design approach based on the reduced section concept, where the mean compressive stress at peak load (or ultimate strength $\sigma_u$) of a stiffened plate is estimated through the yield strength of the effective cross-sectional area according to

$$\frac{\sigma_u}{f_y} = \rho_{\text{glob}} \frac{A_{\text{eff,loc}}}{A},$$

where $f_y$ represents the yield stress, $\rho_{\text{glob}}$ represents the global reduction factor, $A_{\text{eff,loc}}$ represents the effective local cross-sectional area which takes into account the local buckling effects and $A$ represents the gross cross-sectional area of the stiffened plate.

From Eq. (1) it can be noted that the effective cross-sectional area ($A_{\text{eff}} = \rho_{\text{glob}} A_{\text{eff,loc}}$) which takes into account the local and global buckling effects is obtained by reducing the cross-sectional area of the stiffened plate in two different steps, as illustrated in Fig. 1.
In the first step the effect of local buckling in the panels between longitudinal stiffeners is taken into account and an effective width of these panels is adopted to obtain an effective local cross-sectional area \( (A_{\text{eff},\text{loc}}) \) according to

\[
A_{\text{eff},\text{loc}} = \sum_{i} (\rho_{\text{loc},i} b_{p,i}) + n_{st} A_{sl},
\]

where \( n_{st} \) represents the number of longitudinal stiffeners, \( \rho_{\text{loc}} \) represents the local reduction factor, \( b_{p} \) represents the width of the panels between longitudinal stiffeners, \( t \) represents the plate thickness and \( A_{sl} \) represents the gross cross-sectional area of a single longitudinal stiffener (without the contribution of the plate). The local reduction factor \( (\rho_{\text{loc}}) \) takes into account the favourable effect from post-buckling strength reserve and the unfavourable effect of the initial imperfections and it is defined for each panel between longitudinal stiffeners by

\[
\rho_{\text{loc}} = \frac{\lambda_{p,\text{norm}} - 0.055(3 + \psi)}{\lambda_{p,\text{norm}}^2} \leq 1,
\]

where \( \lambda_{p,\text{norm}} \) represents the relative slenderness of the panels. For a relative slenderness of the panels lower than 0.673 the local reduction factor should be considered equal to 1. Eq. (3) is the same criterion used in European standard [2].

In the second step, which takes into account the effect of global buckling in the stiffened plate, a global reduction factor \( (\rho_{\text{glob}}) \) is considered to reduce the effective local cross-sectional area \( (A_{\text{eff},\text{loc}}) \). The global reduction factor is given by

\[
\rho_{\text{glob}} = \max \left( \chi_c; \rho_{p,\infty} \right),
\]

where \( \chi_c \) represents the reduction factor for the column buckling behaviour and \( \rho_{p,\infty} \) represents the reduction factor for the plate buckling behaviour estimated for a long stiffened plate (stiffened plate where the elastic critical stress \( \sigma_{cr} \) no longer depends of the plate length \( a \)).

Column buckling behaviour is considered in stiffened plates with insignificant post-buckling resistance. This behaviour is modelled by removing the supports along the longitudinal edges of the plate considering the cross-section of the column equivalent to the plate composed of a single stiffener with an effective width of the adjacent panels to the stiffener. The reduction factor for the column buckling behaviour is obtained by

\[
\chi_c = \min \left( 0.66 \frac{\lambda_{c,\text{norm}}}{\phi_c + \sqrt{\phi_c^2 - \lambda_{c,\text{norm}}^2}} \right) \leq 1,
\]

where \( \lambda_{c,\text{norm}} \) represents the relative slenderness of the equivalent column and \( \phi_c \) represents the parameter that depends on the initial imperfections, the relative slenderness of the equivalent column \( (\lambda_{c,\text{norm}}) \), the relative flexural stiffness of the stiffened plate \( \gamma \) (ratio between the second
moment of area of the stiffened plate \( I_{st} \) to the second moment of area for bending of the plate corrected by the effect of the Poisson coefficient \( br^2/[12(1-\nu^2)] \) and the number of longitudinal stiffeners \( (n_{st}) \).

The first term in Eq. (5) \((0.66\lambda_{c,norm})\) is the critical criterion in stiffened flanges with low values of equivalent column relative slenderness \( (\lambda_{c,norm}) \) and it is equal to the reduction factor established in the American standard [3] for the column buckling behaviour of stiffened plates. The other term in Eq. (5) is the determinant criterion in stiffened plates with intermediate and high values of equivalent column relative slenderness \( (\lambda_{c,norm}) \) and it is equal to the reduction factor established in the European standard [2] for the column buckling behaviour of stiffened plates with an improvement that takes into account the relative flexural stiffness of the stiffened plate \( (\gamma) \) and the number of longitudinal stiffeners \( (n_{st}) \).

Fig. 2 presents the reduction factor for the column buckling behaviour \( (\chi_c) \) obtained using the design proposal. It is also shown the yield, the elastic buckling and the European standard criteria. An equivalent geometric imperfection of 1/400 of the plate length \( (a) \) was used.

Plate buckling behaviour is considered in stiffened plates with post-buckling resistance. The design proposal uses this type of behaviour to define the minimum ultimate strength of the stiffened plate, which is estimated considering a long plate. The reduction factor for the plate buckling behaviour of a long stiffened plate is obtained by

\[
\rho_{p,\infty} = \begin{cases} 
\frac{1}{4\xi} & \text{for } \lambda_{p,norm} \leq 2\xi \\
\frac{1}{\lambda_{p,norm} - \frac{\xi}{\lambda_{p,norm}}} & \text{for } \lambda_{p,norm} > 2\xi 
\end{cases}
\]

where \( \lambda_{p,norm} \) represents the relative slenderness of the stiffened plate and \( \xi \) represents the parameter that depends on the compressive longitudinal stress ratio \( (\psi) \), the panel slenderness \( \lambda_p \) (ratio between the width of the panels between stiffeners \( b_p \) to the plate thickness \( t \)), the relative cross-sectional area \( \delta \) (ratio between the cross-sectional area of the stiffeners without any contribution of the plate \( (n_{st}A_{sl}) \) to the cross-sectional area of the plate \( (bt) \)), the relative flexural stiffness of the stiffened plate \( (\gamma) \) and the number of longitudinal stiffeners \( (n_{st}) \).

The reduction factor for the plate buckling behaviour estimated for a long stiffened plate \( (\rho_{p,\infty}) \) is governed by the parameter \( \xi \) and Eq. (6) becomes the well-known criterion proposed by Winter when this parameter is equal to 0.22. Based on the geometric parameters of the stiffened flanges identified in the survey of stiffened flanges used in real steel box girder bridges presented in the work of Ferreira [5] the minimum and maximum values of the parameter \( \xi \) are 0.19 and 0.64 respectively. Fig. 3 presents the reduction factor for the plate buckling behaviour of a long stiffened plate \( (\rho_{p,\infty}) \) obtained using the design proposal with different values of the parameter \( \xi \). It is also shown the yield, elastic buckling and Winter criteria.
Fig. 3 shows that the application of the design proposal results in a lower ultimate strength ($\sigma_u$) of the stiffened flange with fully free in-plane displacements at longitudinal edges than that obtained by the current European bridge design rules, which is consistent with the results obtained using nonlinear finite element simulations and with the results of experimental tests presented in the work of Ferreira [5].

The complete definition of the procedures and variables used in the design proposal and the statistical evaluation using experimental test and numerical results within the framework of a probabilistic reliability theory in accordance with EN-1990 [4] can be found in the work of Ferreira [5]. It is noted that the numerical value for the partial safety factor ($\gamma_M$) obtained from the statistical evaluation is 1.10, which is the recommended numerical value for the partial factor for resistance of members to instability assessed by member checks ($\gamma_M$) on bridges [7].

2.3 Comparison with nonlinear analysis and current bridge design rules

The comparison between the ultimate strength ($\sigma_u$) obtained by the design proposal with that obtained by the nonlinear analyses and current bridge design rules established in the European and American standards [2,3] was performed considering 20 stiffened flanges. The 20 stiffened flanges were obtained based on two types of cross-sections, SF1 and SF2, and considering for each cross-section the plate aspect ratio $\phi$ (ratio between the plate length $a$ to the plate width $b$) ranging from 0.5 to 5.0 at intervals of 0.5. Both cross-sections have five longitudinal equally spaced and single sided stiffeners, the stiffened flanges with the cross-section SF1 have a relative flexural stiffness $\gamma$ $\left(12I_{st}(1-\nu^2)/(bt^3)\right)$ of 65 and the stiffened flanges with the cross-section SF2 have a relative flexural stiffness $\gamma$ of 130. All stiffened flanges have a relative cross-sectional area $\delta$ $\left(n_{st}A_{sl}/(bt)\right)$ of 0.5, a plate slenderness $\lambda_{plt}$ ($b/t$) of 150 and a panel slenderness $\lambda_{p}$ ($b/p/l$) of 25.

The nonlinear analyses were performed using the semi-analytical model presented in the work of Ferreira [5] considering a yield stress ($f_y$) of 355 Nmm$^{-2}$, Young’s modulus ($E$) of 2.1x10$^5$ Nmm$^{-2}$ and Poisson’s coefficient ($\nu$) of 0.3.

The stiffened flanges were considered simply supported under longitudinal uniform compression ($\sigma$) with the following two cases for the in-plane displacement boundary conditions: (i) in-plane displacements perpendicular to the edges constrained to remain straight in all edges (designated as case CC) and (ii) in-plane displacements perpendicular to the edges constrained to remain straight at loaded edges and free at unloaded edges (designated as case CF).

Fig. 4 presents the comparison between the ultimate strength ($\sigma_u$) obtained by the design proposal, the semi-analytical model and current bridge design rules. Fig. 4(a) shows the ultimate strength for the stiffened flanges with the cross-section SF1 and Fig. 4(b) shows the ultimate strength for the stiffened flanges with the cross-section SF2. The results are presented in terms of normalized strength (ratio between the mean compressive stress at peak load $\sigma_u$ to the yield stress $f_y$) for different values of the plate aspect ratio $\phi$ ($a/b$).

From the analysis of Fig. 4 it can be noted that the ultimate strength ($\sigma_u$) obtained by the design proposal presents a much better agreement with the nonlinear analysis results obtained by the semi-
analytical model considering the case CF for the in-plane displacement boundary conditions than the ultimate strength obtained by the current bridge design rules.

![Graph comparing ultimate strength](image)

Fig. 4. Comparison between the ultimate strength obtained by the design proposal (DCP), the semi-analytical model (SAM) and current bridge design rules for the stiffened flanges with the cross-section: (a) SF1 and (b) SF2

This observation is particularly clear in the case of long plates, where the design rules established by the European standard leads to nonconservative results and the design rules established by the American standard leads to extremely conservative results.

3 SUMMARY AND ACKNOWLEDGMENT

A design proposal for stiffened flanges used in steel box girder bridges based on a set of design curves that take into account all relevant parameters is presented. The design proposal predicts the ultimate strength ($\sigma_u$) in a quick and simple way and provides viable and conservative results, which thus proves its usefullness as a design tool.

From this study the main conclusions can be summarized as follows: (i) the ultimate strength ($\sigma_u$) obtained by the design proposal is in close agreement with the results obtained using the semi-analytical model considering fully free in-plane displacements at longitudinal edges and (ii) it presents a much better agreement with the results obtained using the semi-analytical model considering fully free in-plane displacements at longitudinal edges than the ultimate strength obtained with the current bridge design rules.

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REFERENCES

DESIGN CURVES FOR STIFFENED PLATES WITH THE UNLOADED EDGES FREE FROM STRESSES USED IN STEEL BOX GIRDERS

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KEYWORDS: Stiffened plates; Steel Bridges; Design curves; Numerical results; In-plane displacements boundary conditions.

ABSTRACT
National standards with design rules for stiffened flanges used in steel box girder bridges were almost non-existent until the early nineteen seventies. These design rules were based on the elastic critical stress ($\sigma_{cr}$) obtained by the linear buckling theory of plates with some correction factors to take into account the favourable effect of the post-buckling strength reserve [1].

Research work to establish new design rules for plated structures was strongly increased in the early nineteen seventies as a result of well-known accidents during construction involving box girder bridges [1]. Although the behaviour and strength of stiffened plates have been extensively studied over the last forty years, there are still some inconsistencies and gaps between the results of some studies and the results obtained through the design rules established in the European and American standards [2,3], which are usually adopted by the bridge designers to solve the problem of predicting the ultimate strength of stiffened plates.

As the current design rules laid down in the European and American standards do not recognize the effect of boundary conditions for the in-plane displacements at the edges to obtain the ultimate strength of stiffened plates, the use of these design rules can give inaccurate results and in some cases against safety.

This paper presents a design method for stiffened flanges used in steel box girder bridges based on a set of design curves that take into account all relevant parameters regarding the ultimate strength of stiffened flanges, including the real boundary conditions for the in-plane displacements at the edges. These design curves were developed and calibrated based on the results obtained with nonlinear analysis using the semi-analytical and finite element methods and they were validated by comparison with the results of experimental tests in accordance with the target failure probability of the European standard EN-1990 [4]. A comparison between the ultimate strength obtained using the proposal design curves with that obtained using the nonlinear analysis and the current bridge design rules it is also presented.

The study of the behaviour and strength of steel plates with longitudinal stiffeners is very complex because it depends on a large number of variables, such as the shape and amplitude of the initial geometric imperfections, the residual stresses, the boundary conditions for the in-plane and out-of-plane displacements, the load conditions and the geometric and material data. Recent numerical studies [5,6] showed that the behaviour and strength of steel plates are very sensitive to in-plane displacements boundary conditions and the European and American standards [2,3] do not provide a definition and clear guidance for the proper use of the simplified rules available to designers. These rules may give considerable errors, particularly when applied to the safety verification of stiffened plates where the transverse in-plane displacements at longitudinal edges cannot be considered as uniform (in-plane displacements perpendicular to the longitudinal edges constrained to remain straight).
In European standard [2] the resistance assessment of webs and internal flanges with longitudinal stiffeners used in steel box girder bridges is performed through the same criterion and independent of the in-plane displacement boundary conditions of the plate. Just apparently both elements may have the same in-plane displacement boundary conditions. Indeed, the webs are usually connected to flanges that possess sufficient rigidity to consider uniform the in-plane displacements at longitudinal edges, while the flanges are connected to webs that usually do not provide this type of constraint and it is more conservative to consider free the in-plane displacements at longitudinal edges, as shown in the work of Ferreira [5] through a comparison between numerical and experimental results.

The design proposal presented in this paper for stiffened flanges aims to fill a gap in the current European bridge design rules, which should not be applied to stiffened plates with the fully free transverse in-plane displacements at longitudinal edges.

The design proposal is a design approach based on the reduced section concept, where the mean compressive stress at peak load (or ultimate strength $\sigma_u$) of a stiffened plate is estimated through the yield strength of the effective cross-sectional area according to

\[
\frac{\sigma_u}{f_y} = \rho_{glob} \frac{A_{eff, loc}}{A},
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where $f_y$ represents the yield stress, $\rho_{glob}$ represents the global reduction factor, $A_{eff, loc}$ represents the effective local cross-sectional area which takes into account the local buckling effects and $A$ represents the gross cross-sectional area of the stiffened plate.

**CONCLUSIONS**

The design proposal predicts the ultimate strength ($\sigma_u$) in a quick and simple way and provides viable and conservative results, which thus proves its use-fullness as a design tool.

From this study the main conclusions can be summarized as follows: (i) the ultimate strength ($\sigma_u$) obtained by the design proposal is in close agreement with the results obtained using the semi-analytical model considering fully free in-plane displacements at longitudinal edges and (ii) it presents a much better agreement with the results obtained using the semi-analytical model considering fully free in-plane displacements at longitudinal edges than the ultimate strength obtained with the current bridge design rules.

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